

Please write clearly in block capitals.	
Centre number	Candidate number
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A-level **MATHEMATICS**

Paper 2

Tuesday 11 June 2024

Afternoon

Time allowed: 2 hours

Materials

- You must have the AQA Formulae for A-level Mathematics booklet
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do not write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use		
Question	Mark	
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Section A

Answer **all** questions in the spaces provided.

1 One of the equations below is the equation of a circle.

Identify this equation.

[1 mark]

Tick (\checkmark) one box.

$$(x + 1)^2 - (y + 2)^2 = -36$$

$$(x+1)^2 - (y+2)^2 = 36$$



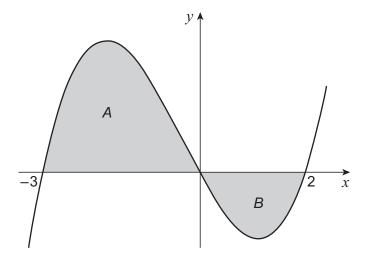
$$(x + 1)^2 + (y + 2)^2 = -36$$



$$(x + 1)^2 + (y + 2)^2 = 36$$



The graph of y = f(x) intersects the *x*-axis at (-3, 0), (0, 0) and (2, 0) as shown in the diagram below.



The shaded region A has an area of 189

The shaded region B has an area of 64

Find the value of $\int_{-3}^{2} f(x) dx$

Circle your answer.

[1 mark]

$$-253$$

$$-125$$

Turn over for the next question



3	Solve the inequality	(1-x)(x-4) < 0	[1 mark]
	Tick (✓) one box.		
	${x:x<1} \cup {x:x>4}$		
	${x:x<1} \cap {x:x>4}$		
	$\{x: x < 1\} \cup \{x: x \ge 4\}$		
	${x:x<1} \cap {x:x\geq 4}$		

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4	Use logarithms to solve the equation	
	5x-2 = 71570	
	Give your answer to two decimal places. [3 mar	ks]

Turn over for the next question



find $\frac{dy}{dx}$ [3 marks]	5 (Given that		
find $\frac{dy}{dx}$			$v = \frac{x^3}{}$	
[3 marks]		- dv	$\sin x$	
[3 marks]	f	find $\frac{dy}{dx}$		
				[3 marks]
				



It is given that	
$(2\sin\theta + 3\cos\theta)^2 + (6\sin\theta - \cos\theta)^2 = 30$	
and that θ is obtuse.	
Find the exact value of $\sin \theta$.	
Fully justify your answer.	
	[6 marks]



7 On the first day of each month, Kate pays £50 into a savings account.

Interest is paid on the total amount in the account on the last day of each month.

The interest rate is 0.2%

At the end of the nth month, the total amount of money in Kate's savings account is $\mathfrak{L}T_n$

Kate correctly calculates T_1 and T_2 as shown below:

$$T_1 = 50 \times 1.002 = 50.10$$

$$T_2 = (T_1 + 50) \times 1.002$$

= $((50 \times 1.002) + 50) \times 1.002$
= $50 \times 1.002^2 + 50 \times 1.002$
 ≈ 100.30

7 (a) Show that T_3 is given by

$$T_3 = 50 \times 1.002^3 + 50 \times 1.002^2 + 50 \times 1.002$$

[1 mark]

7 (b)	Kate uses her method to correctly calculate how much money she can expect to have
	in her savings account at the end of 10 years .

7 (b) (i)	Find the amount of money Kate expects to have in her savings account at the end
	of 10 years .

[3 marks]



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7 (b) (ii)	The amount of money in Kate's savings account at the end of 10 years may not be the amount she has correctly calculated.	
	Explain why.	
	[1 mark]	
	Turn over for the next question	



A zookeeper models the median mass of infant monkeys born at their zoo, up to the age of 2 years, by the formula

$$y = a + b \log_{10} x$$

where y is the median mass in kilograms, x is age in months and a and b are constants.

The zookeeper uses the data shown below to determine the values of a and b.

Age in months (x)	3	24
Median mass (y)	6.4	12

8 (a)	The zookeeper uses	the data for monkeys aged	3 months to write the corre	ct equation
-------	--------------------	---------------------------	-----------------------------	-------------

$$6.4 = a + b \log_{10} 3$$

8 (a) (i)	Use the data for monkeys aged 24 months to write a second equation.	[1 mark]

8 (a) (ii) Show that

$$b = \frac{5.6}{\log_{10} 8}$$

[3	marks]



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8 (a) (iii)	Find the value of a .	
	Give your answer to two decimal places. [1 mark]	
	[Timark]	
	Question 8 continues on the next question	



8 (b)	Use a suitable value for x to determine whether the model can be used to pre median mass of monkeys less than one week old.	
		[2 marks]



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9 (a) (i)	Find the binomial expansion of $(1 + 3x)^{-1}$ up to and including the term in x^2	[2 marks]
9 (a) (ii)	Show that the first three terms in the binomial expansion of	
	$\frac{1}{2-3x}$	
	form a geometric sequence and state the common ratio.	[5 marks]



9 (b)	It is given that $\frac{36x}{(1+3x)(2-3x)} = \frac{P}{(2-3x)} + \frac{Q}{(1+3x)}$	
	where P and Q are integers.	
	Find the value of P and the value of Q	
		[3 marks]



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(c) (i)	Using your answers to parts (a) and (b), find the binomial expansion of		
	$\frac{12x}{(1+3x)(2-3x)}$		
	up to and including the term in x^2		
		[2 marks]	



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9 (c) (ii)	Find the range of values of x for which the binomial expansion of
	$\frac{12x}{(1+3x)(2-3x)}$
	is valid. [1 mark]

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$f(x) = x^2 + 2\cos x \text{ for } -\pi \leq x \leq \pi$ Determine whether the curve with equation $y = f(x)$ has a point of inflection at the point where $x = 0$ Fully justify your answer. [4 marks]	The function f is defined by
where $x = 0$ Fully justify your answer.	$f(x) = x^2 + 2\cos x \text{ for } -\pi \le x \le \pi$



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11 (a)	A student states that 3 is the smallest value of k in the interval $3 < k < 4$				
	Explain the error in the student's statement.				[1 mark]
					[
11 (b)	The student's tea	cher says there is no	o smallest valu	${\sf e}$ of k in the interv	ral 3 < <i>k</i> < 4
	The teacher gives	s the following correct	ct proof:		
		there is a smallest number be <i>x</i>	number in the i	interval $3 \le k \le 4$ a	and let this
	Step 2: let $y = \frac{3}{2}$	$\frac{3+x}{2}$			
	Step 3: 3 < y < 3	x which is a contrad	iction.		
	Step 4: Therefore	re, there is no small	est number in i	nterval 3 < <i>k</i> < 4	
11 (b) (i)					
					[1 mark]



1 (b) (ii)	Prove that there is no largest value of k in the interval $3 < k < 4$	[4 magules
		[4 marks
	END OF SECTION A	
	TURN OVER FOR SECTION B	



Section B

Answer all questions in the spaces provided.

Two constant forces act on a particle, of mass 2 kilograms, so that it moves forward in a straight line.

The two forces are:

- a forward driving force of 10 newtons
- a resistance force of 4 newtons.

Find the acceleration of the particle.

Circle your answer.

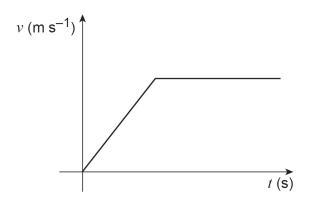
[1 mark]

 2 m s^{-2} 3 m s^{-2} 5 m s^{-2} 12 m s^{-2}



A car starting from rest moves forward in a straight line.

The motion of the car is modelled by the velocity-time graph below:



One of the following assumptions about the motion of the car is implied by the graph.

Identify this assumption.

[1 mark]

Tick (\checkmark) one box.

The car never accelerates.

The acceleration of the car is always positive.

The acceleration of the car can change instantaneously.

The acceleration of the car is never constant.



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14	The displacement, r metres, of a particle at time t seconds is			
	$r = 6t - 2t^2$			
14 (a)	Find the value of r when $t = 4$	[1 mark]		
14 (b)	Determine the range of values of $\it t$ for which the displacement is positive.	[2 marks]		



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Two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$, are acting on a particle of mass 3 kilograms.
It is given that
$\mathbf{F_1} = \begin{bmatrix} a \\ 23 \end{bmatrix}$ newtons and $\mathbf{F_2} = \begin{bmatrix} 4 \\ b \end{bmatrix}$ newtons
where a and b are constants.
The particle has an acceleration of $\begin{bmatrix} 4b \\ a \end{bmatrix}$ m s ⁻²
Find the value of a and the value of b





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16	In this question use $g=9.8~{\rm m~s^{-2}}$	
	An apple tree stands on horizontal ground.	
	An apple hangs, at rest, from a branch of the tree.	
	A second apple also hangs, at rest, from a different branch of the tree.	
	The vertical distance between the two apples is d centimetres.	
	At the same instant both apples begin to fall freely under gravity.	
	The first apple hits the ground after 0.5 seconds.	
	The second apple hits the ground 0.1 seconds later.	
	Show that d is approximately 54	morkol
	[4	marks]



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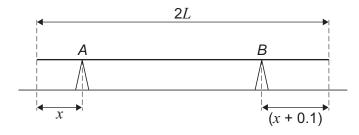
A uniform rod is resting on two fixed supports at points A and B.

A lies at a distance x metres from one end of the rod.

B lies at a distance (x + 0.1) metres from the other end of the rod.

The rod has length 2L metres and mass m kilograms.

The rod lies horizontally in equilibrium as shown in the diagram below.



The reaction force of the support on the rod at *B* is twice the reaction force of the support on the rod at *A*.

Show that

$$L - x = k$$

where k is a constant to be found.



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18	A particle is moving in a straight line through the origin ${\cal O}$		
	The displacement of the particle, r metres, from O , at time t seconds is given by		
	$r = p + 2t - qe^{-0.2t}$		
	where p and q are constants.		
	When $t = 3$, the acceleration of the particle is -1.8 m s^{-2}		
18 (a)	Show that $q \approx$ 82 [5 marks]		



(b)	The particle has an initial displacement of 5 metres.	Do ou
	Find the value of p	
	Give your answer to two significant figures. [2 marks]	
	Turn over for the next question	



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19	In this question use $g=9.8~{ m m~s^{-2}}$
	A toy shoots balls upwards with an initial velocity of 7 m s ⁻¹
	The advertisement for this toy claims the balls can reach a maximum height of 2.5 metres from the ground.
19 (a)	Suppose that the toy shoots the balls vertically upwards.
19 (a) (i)	Verify the claim in the advertisement. [2 marks]
19 (a) (ii)	State two modelling assumptions you have made in verifying this claim. [2 marks]



9 (b)	In fact the toy shoots the balls anywhere between 0 and 11 degrees from the vertical .		
	The range of maximum heights, h metres, above the ground which can be reached by the balls may be expressed as		
	$k < h \le 2.5$		
	Find the value of k		
	[4 marks]		

Turn over for the next question



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20	Two particles <i>P</i> and <i>Q</i> are moving in separate straight lines across a smooth horizontal surface.
	P moves with constant velocity (3 \mathbf{i} + 4 \mathbf{j}) m s ⁻¹
	Q moves from position vector (5 \mathbf{i} – 7 \mathbf{j}) metres to position vector (14 \mathbf{i} + 5 \mathbf{j}) metres during a 3 second period.
20 (a)	Show that <i>P</i> and <i>Q</i> move along parallel lines. [3 marks]



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20 (b)	Stevie says	
	Q is also moving with a constant velocity of (3 \mathbf{i} + 4 \mathbf{j}) m s ⁻¹	
	Explain why Stevie may be incorrect.	[1 mark]
	Question 20 continues on the next page	





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20 (c)	A third particle R is moving with a constant speed of 4 m s ⁻¹ , in a straight line, across the same surface.					
	P and R move along lines that intersect at a fixed point X					
	It is given that:					
	 P passes through X exactly 2 seconds after R passes through X 					
	 P and R are exactly 13 metres apart 3 seconds after R passes through X 					
	Show that <i>P</i> and <i>R</i> move along perpendicular lines.					
	[5 marks]					



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Two heavy boxes, M and N, are connected securely by a length of rope.

The mass of M is 50 kilograms.

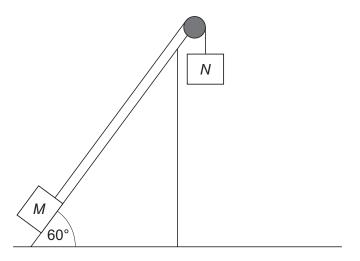
The mass of *N* is 80 kilograms.

M is placed near the bottom of a rough slope.

The slope is inclined at 60° above the horizontal.

The rope is passed over a smooth pulley at the top end of the slope so that *N* hangs with the rope vertical.

The boxes are initially held in this position, with the rope taut and running parallel to the line of greatest slope, as shown in the diagram below.



When the boxes are released, $\it M$ moves up the slope as $\it N$ descends, with acceleration $\it a$ m s⁻²

80g - T = 80a

The tension in the rope is T newtons.

21 (a) Explain why the equation of motion for N is

	[1 mark



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21 (b)	Show that the normal reaction force between $\it M$ and the slope is 25 $\it g$ newtons.	[1 mark]
	Question 21 continues on the next page	



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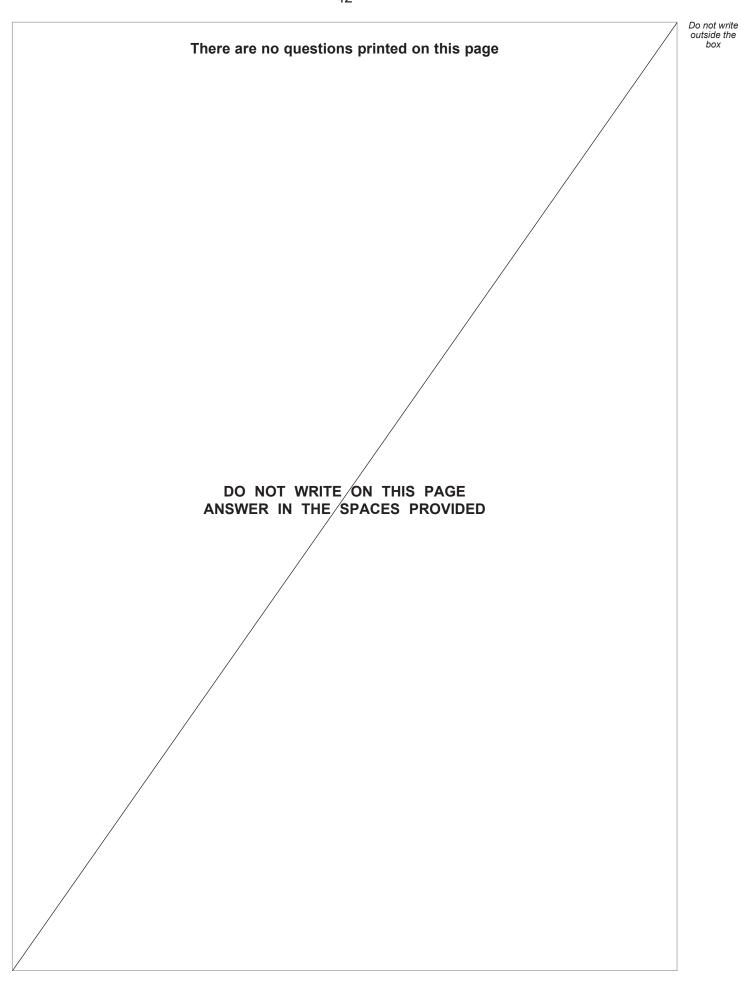
Show that	/44 E.D.	
	$a \ge \frac{(11 - 5\sqrt{3})g}{26}$	
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21 (d)	State one modelling assumption you have made throughout this question.	[1 mark]
	END OF QUESTIONS	







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