OCR
Oxford Cambridge and RSA

Thursday 23 May 2024 – Afternoon

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

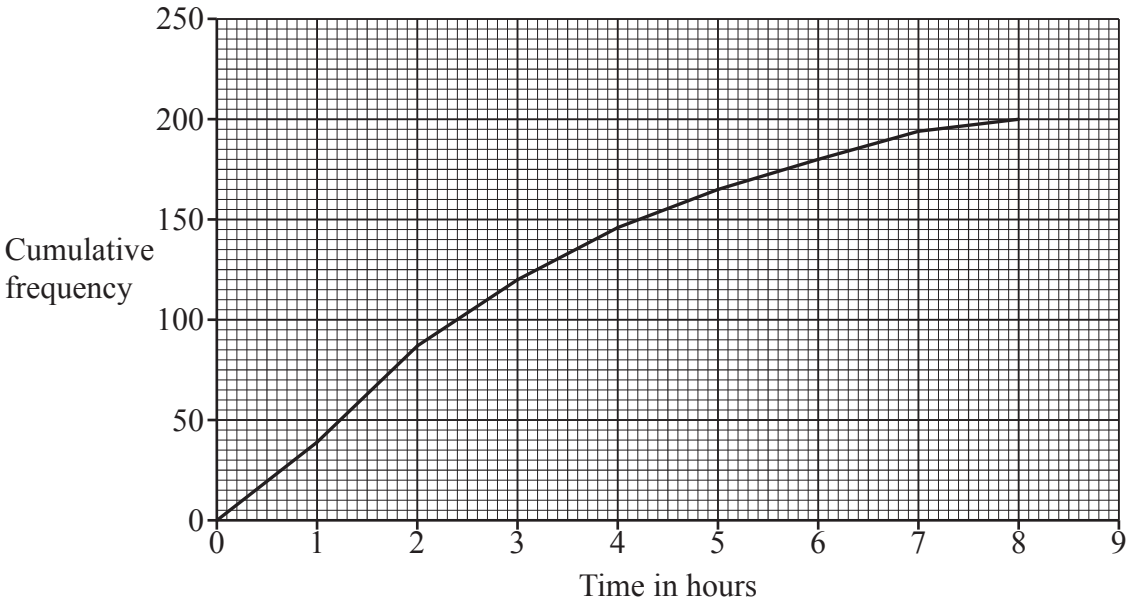
1 Express $2x(x+3)+5x^2-2(x-3)$ in the form ax^2+bx+c, where a, b and c are integers to be determined. **[3]**

2 **(a)** Find the discriminant of the equation $3x^2-2x+5=0$. **[2]**

(b) Use your answer to part **(a)** to find the number of real roots of the equation $3x^2-2x+5=0$. **[1]**

3 A student conducts an investigation into the number of hours spent cooking per week by people who live in village A. The student represents the data in the cumulative frequency diagram below.

Hours spent cooking per week by people who live in village A



(a) How many people were involved in the investigation? **[1]**

(b) Use the copy of the diagram in the **Printed Answer Booklet** to determine an estimate for the interquartile range. **[2]**

The student conducts a similar investigation into the number of hours spent cooking per week by 200 people who live in village B.

The interquartile range is found to be 3.9 hours.

(c) Explain whether the evidence suggests that the number of hours spent cooking by people who live in village B is more variable, equally variable or less variable than the number of hours spent cooking by people who live in village A. **[1]**

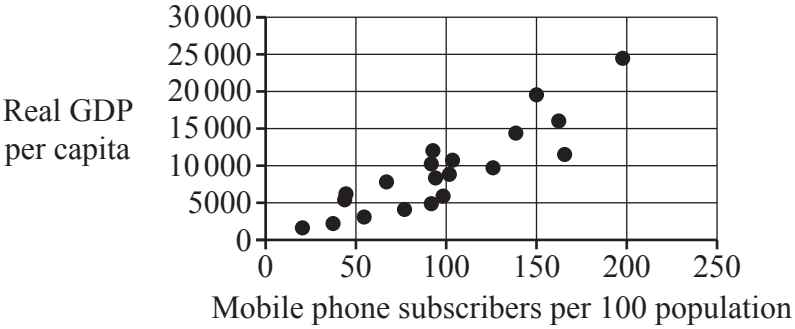
4 **In this question you must show detailed reasoning.**

Express $\dfrac{1+4\sqrt{3}}{2+\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers to be determined. **[3]**

5 The pre-release material contains information for countries in the world concerning real GDP per capita in US$ and mobile phone subscribers per 100 population. In an investigation into the relationship between these two variables, a student takes a sample of 20 countries in Africa. The student draws a scatter diagram for the data, which is shown in **Fig. 5.1**.

Fig. 5.1

Africa 1st sample

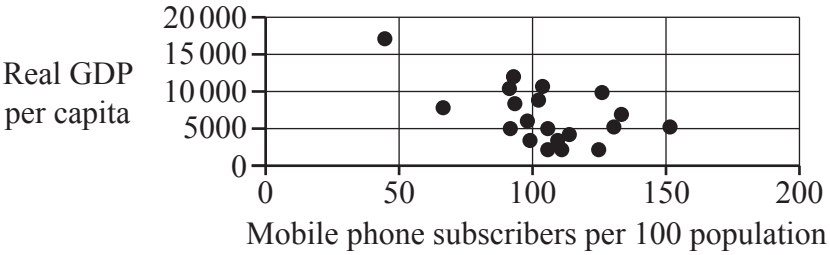


(a) What does **Fig. 5.1** suggest about the relationship between real GDP per capita and the number of mobile phone subscribers per 100 population? **[1]**

Another student collects a **different** sample of 20 countries from Africa, and draws a scatter diagram for the data, which is shown in **Fig. 5.2**.

Fig. 5.2

Africa 2nd sample



(b) What does **Fig. 5.2** suggest about the relationship between real GDP per capita and the number of mobile phone subscribers per 100 population? **[1]**

(c) Explain whether either of the two scatter diagrams is likely to be representative of the true relationship between real GDP per capita and the number of mobile phone subscribers per 100 population, for countries in Africa. **[1]**

6 Determine the equation of the line which passes through the point (4, –1) and is perpendicular to the line with equation $2x+3y=6$.

Give your answer in the form $y=mx+c$, where m is a fraction in its lowest terms and c is an integer. **[4]**

7 Determine the coefficient of x^5 in the expansion of $(3-2x)^7$. **[4]**

8 **In this question you must show detailed reasoning.**

Determine the coordinates of the point of intersection of the line with equation $y=2x+3$ and the curve with equation $y^2-4x^2=33$. **[4]**

9 A fair six-sided die has its faces numbered 1, 3, 4, 5, 6 and 7. The die is rolled once.

A is the event that the die shows an even number.

B is the event that the die shows a prime number.

(a) Write down the value of p(A). **[1]**

(b) Write down the value of p(B). **[1]**

(c) Write down the value of p(A or B). **[1]**

The die is rolled again.

(d) Calculate the probability that the sum of the scores from the two rolls is even. **[3]**

10 The pre-release material contains information about the birth rate per 1000 people in different countries of the world. These countries have been classified into different regions.

The table shows some data for three of these regions: the mean and standard deviation (sd) of the birth rate per 1000, and the number of countries for which data was used, n.

Birth rate per 1000 by region

	Africa	Europe	Oceania
n	55	49	21
mean	29.3	10.0	17.8
sd	8.43	1.94	4.50

(a) Use the information in the table to compare and contrast the birth rate per 1000 in Africa with the birth rate per 1000 in Europe. **[2]**

(b) The birth rate per 1000 in Mauritius, which is in Africa, is recorded as 9.86. Use the information in the table to show that this value is an outlier. **[2]**

(c) Use your knowledge of the pre-release material to explain whether the value for Mauritius should be discarded. **[1]**

(d) The pre-release material identifies 27 countries in Oceania. Suggest a reason why only 21 values were used to calculate the mean and standard deviation. **[1]**

11 **In this question you must show detailed reasoning.**

The equation of a curve is $y = 5\sqrt{x} - x - 6$ for $x \geqslant 0$.

(a) Verify that the curve cuts the x-axis at $x = 4$ and at $x = 9$. **[2]**

The curve does not cut or touch the x-axis at any other points.

(b) Determine the exact area bounded by the curve and the x-axis. **[4]**

12 Data collected in the twentieth century showed that the probability of a randomly selected person having blue eyes was 0.08. A medical researcher believes that the probability in 2024 is less than this so they decide to carry out a hypothesis test at the 5% significance level.

(a) Write down suitable hypotheses for the test, defining the parameter used. **[2]**

(b) Assuming that the probability that a person selected at random has blue eyes is still 0.08, calculate the probability that 3 or fewer people in a random sample of 92 people have blue eyes. **[1]**

The researcher collects a random sample of 92 people and finds that 3 of them have blue eyes.

(c) Use your answer to part **(b)** to carry out the test, giving your conclusion in context. **[3]**

13 Determine the range of values of x for which $y = 4x^3 + 7x^2 - 6x + 8$ is a decreasing function. **[5]**

14 **In this question you must show detailed reasoning.**

Solve the equation $5 - \cos\theta - 6\sin^2\theta = 0$ for $0° < \theta < 360°$. **[6]**

Turn over for question 15

15 Ali and Sam are playing a game in which Ali tosses a coin 5 times. If there are 4 or 5 heads, Ali wins the game. Otherwise Sam wins the game. They decide to play the game 50 times.

(a) Initially Sam models the situation by assuming the coin is fair. Determine the number of games **Ali** is expected to win according to this model. **[2]**

Ali thinks the coin may be biased, with probability p of obtaining heads when the coin is tossed. Before playing the game, Ali and Sam decide to collect some data to estimate the value of p. Sam tosses the coin 15 times and records the number of heads obtained. Ali tosses the coin 25 times and records the number of heads obtained.

(b) Explain why it is better to use the combined data rather than just Sam's data or just Ali's data to estimate the value of p. **[1]**

Ali records 20 heads and Sam records 8 heads.

(c) Use the combined data to estimate the value of p. **[1]**

Ali now models the situation using the value of p found in part **(c)** as the probability of obtaining heads when the coin is tossed.

(d) Determine how many games Ali expects to win using this value of p to model the situation. **[2]**

(e) Ali wins 25 of the 50 games. Explain whether Sam's model or Ali's model is a better fit for the data. **[1]**

END OF QUESTION PAPER