



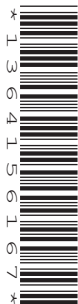
Oxford Cambridge and RSA

**Thursday 16 May 2024 – Afternoon**

**AS Level Mathematics B (MEI)**

**H630/01 Pure Mathematics and Mechanics**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- the Printed Answer Booklet
- a scientific or graphical calculator

**QP**

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ ms}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **70**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

**Formulae AS Level Mathematics B (MEI) (H630)****Binomial series**

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Sample variance**

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

$$\text{Standard deviation, } s = \sqrt{\text{variance}}$$

**The binomial distribution**

$$\text{If } X \sim B(n, p) \text{ then } P(X=r) = {}^nC_r p^r q^{n-r} \text{ where } q = 1-p$$

$$\text{Mean of } X \text{ is } np$$

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

- 1 The triangle ABC has an obtuse angle at A. The angle at B is  $15^\circ$ . The length of AC is 10 cm and the length of BC is 13 cm.

Calculate the size of the angle at A.

[2]

- 2 Two forces  $\mathbf{F}_1$  N and  $\mathbf{F}_2$  N are given by  $\mathbf{F}_1 = -6\mathbf{i} + 2\mathbf{j}$  and  $\mathbf{F}_2 = -8\mathbf{i} + \mathbf{j}$ .

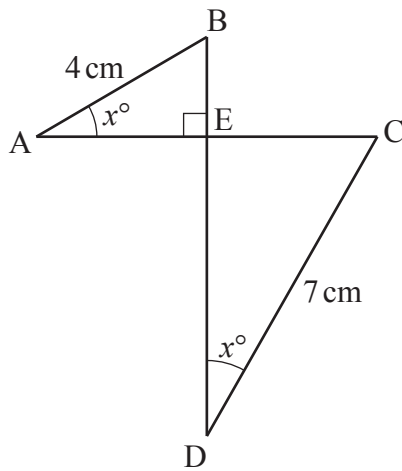
Show that the magnitude of the resultant of these two forces is  $\sqrt{205}$  N.

[2]

- 3 Prove that, when  $n$  is an even number,  $n^3 + 4$  is a multiple of 4 but not a multiple of 8.

[3]

- 4 The perpendicular lines AC and BD intersect at E as shown in the diagram. The point E is the midpoint of AC. The angles BAC and BDC are each equal to  $x^\circ$ . The lengths of AB and CD are 4 cm and 7 cm respectively.



Determine the value of  $x$ .

[4]

**5 In this question you must show detailed reasoning.**

(a) Show that the gradient of the curve  $y = \sqrt{x}\left(\frac{1}{x^2} - 2x\right)$  at the point  $\left(\frac{1}{4}, \frac{31}{4}\right)$  is  $-\frac{99}{2}$ . [4]

(b) Find the equation of the tangent to the curve at  $\left(\frac{1}{4}, \frac{31}{4}\right)$  giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. [2]

**6** The polynomial  $x^3 - 4x^2 + 10x - 21$  is denoted by  $f(x)$ .

(a) Use the factor theorem to show that  $(x - 3)$  is a factor of  $f(x)$ . [2]

(b) The polynomial  $f(x)$  can be written as  $(x - 3)(x^2 + bx + c)$  where  $b$  and  $c$  are constants.

Find the values of  $b$  and  $c$ . [2]

(c) Show that  $x = 3$  is the only real root of the equation  $f(x) = 0$ . [2]

**7** The velocity of a particle moving in a straight line is modelled by  $v = 0.6t^2 - 2.1t + 1.5$  where  $v$  is the velocity in metres per second and  $t$  is the time in seconds.

(a) Determine the times at which the particle is stationary. [2]

(b) Find the acceleration of the particle at the first of the times at which it is stationary. [2]

(c) Find the distance travelled by the particle between the times at which it is stationary. [2]

**8** A circle with centre  $C$  has equation  $x^2 + y^2 - 6x - 16y + 48 = 0$ .

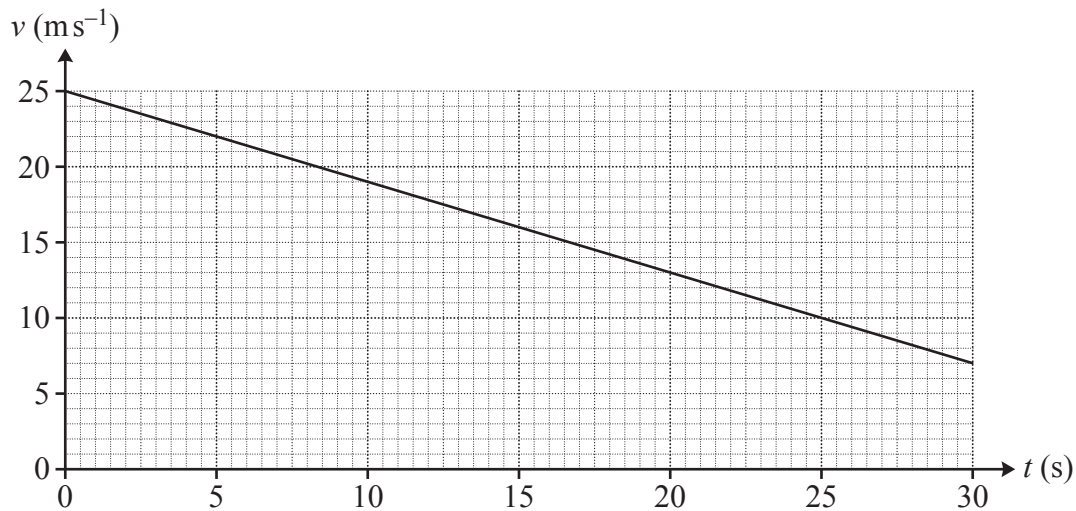
(a) Find the coordinates of  $C$ . [2]

A line has equation  $y = x - 2$  and intersects the circle at the points  $A$  and  $B$ . The midpoints of  $AC$  and  $BC$  are  $A'$  and  $B'$  respectively.

(b) Determine the exact distance  $A'B'$ . [8]

- 9 Two trains are travelling in the same direction on parallel straight tracks and train A overtakes train B. At time  $t$  seconds after the front of train A overtakes the front of train B the velocities of trains A and B are  $v_A \text{ m s}^{-1}$  and  $v_B \text{ m s}^{-1}$  respectively.

The velocity of train A is modelled by  $v_A = 25 - 0.6t$ . The velocity-time graph of train A is shown below.



- (a) A student argues that the speed of train A changes by  $18 \text{ m s}^{-1}$  in 30 seconds so its acceleration is  $0.6 \text{ m s}^{-2}$ .

Comment on the validity of the student's argument.

[1]

- (b) When the front of train A overtakes the front of train B, train B has a velocity of  $10 \text{ m s}^{-1}$ . The acceleration of train B is constant and is modelled as  $0.15 \text{ m s}^{-2}$ .

Write down the equation for  $v_B$  in terms of  $t$  that models the velocity of train B.

[1]

- (c) Draw the velocity-time graph of train B on the copy of the diagram in the Printed Answer Booklet.

[1]

- (d) Determine the distance between the fronts of the trains at the time when the trains are travelling at the same velocity.

[3]

- (e) Explain why the model for train A would not be valid for large values of  $t$ .

[1]

- 10** A boat pulls a water skier of mass 65 kg with a light inextensible horizontal towrope. The mass of the boat is 985 kg. There is a driving force of 2400 N acting on the boat. There are horizontal resistances to motion of 400 N and 1200 N acting on the skier and the boat respectively.

- (a) Draw a diagram showing all the horizontal forces acting on the skier and the boat. [2]
- (b) (i) Write down the equation of motion of the skier. [1]
- (ii) Find the equation of motion of the boat. [2]
- (c) Find the acceleration of the skier and the boat. [1]

The driving force of the boat is increased. The skier can only hold on to the towrope when the tension is no greater than her weight.

- (d) Determine her greatest acceleration, assuming that the resistances to motion stay the same. [2]

- 11** A student records the time a pendulum takes to swing for different lengths of pendulum.

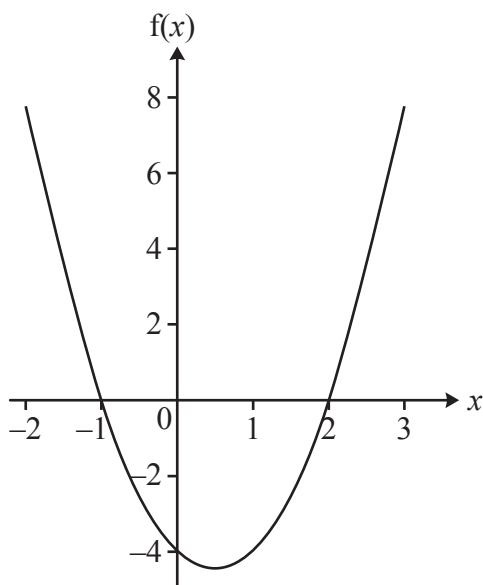
The student decides to plot a graph of  $\log_{10} T$  against  $\log_{10} l$  where  $T$  is the time in seconds that the pendulum takes to return to its start position and  $l$  is the length in metres of the pendulum. They use a model for  $\log_{10} T$  in terms of  $\log_{10} l$  of the form  $\log_{10} T = \log_{10} k + n \log_{10} l$ .

The student records the following data points.

$\log_{10} l$	-0.097	0.146
$\log_{10} T$	0.254	0.376

- (a) Determine the values of  $k$  and  $n$  that best model the data. Give your values correct to 2 significant figures. [4]
- (b) Using these values of  $k$  and  $n$ , write the student's model as an equation expressing  $T$  in terms of  $l$ . [2]

- 12 The diagram shows the graph of  $f(x) = k(x-p)(x-q)$  where  $k$ ,  $p$  and  $q$  are constants. The graph passes through the points  $(-1, 0)$ ,  $(0, -4)$  and  $(2, 0)$ .



- (a) Find  $f(x)$  in the form  $ax^2 + bx + c$ . [3]

A cubic curve has gradient function  $f(x)$ . This cubic curve passes through the point  $(0, 8)$ .

- (b) Find the equation of the cubic curve. [4]
- (c) Determine the coordinates of the stationary points of the cubic curve. [3]

**END OF QUESTION PAPER**



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