GCE

Further Mathematics B MEI

Y434/01: Numerical methods

A Level

Mark Scheme for June 2024

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1.   Mark strictly to the mark scheme.

2.   Marks awarded must relate directly to the marking criteria.

3.   The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
-       if there is nothing written at all in the answer space and no attempt elsewhere in the script
-       OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
-       OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.
Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there

may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   • When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
   • When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
   NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   • If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   • If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   • if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.

If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)		$\frac{0.6753-0.5842}{2.1-1.9}$ **oe**	**M1**	**1.1a**	central difference method
			0.4555 or 0.456 or 0.46	**A1**	**1.1**	if **M0** allow **SC1** for 0.444 from forward difference method
						allow **B2** for correct answer unsupported
				[2]		
1	(b)		$0.05 \times$ their 0.4555	**M1**	**1.1**	may see [f(2.05) ≈] 0.6309 + 0.05 × their 0.4555 ;
						may be implied by
						f(2.05) ≈ 0.653675, 0.6537 or 0.6539
			$-$ 0.023 to $-$ 0.022775	**A1FT**	**1.1**	accept 0.022775 to 0.023; **B2** for correct answer unsupported
				[2]		
2	(a)		$\frac{0.8-\tanh(1)}{\tanh(1)}$ or $\frac{1-\tanh(2)}{\tanh(2)}$	**M1**	**1.1**	allow signs reversed in numerator; may be implied by one correct
						answer or the negative of one correct answer
			may be implied by			
			eg $\frac{0.8-0.761\ldots}{0.761\ldots}$ or $\frac{1-0.964\ldots}{0.964\ldots}$			
			$0.0504 \le R_A \le 0.05043$	**A1**	**1.1**	
			$0.0373 \le R_B \le 0.037315$	**A1**	**1.1**	if **A0A0** allow **SC B1** for awrt –0.0504 **and** awrt –0.0373
						or SC B1 for 0.050 **and** 0.037
				[3]		
2	(b)		$\frac{0.8\div1-0.76159\div0.96403}{0.76159\div0.96403}$ **soi**	**M1**	**1.1**	allow signs reversed in numerator; may be implied by $-$ 0.0126
						NB $\frac{0.8-0.7900128}{0.7900128}$
			$0.0126 \le R_C \le 0.012642$	**A1**	**1.1**	
				[2]		
2	(c)		$R_C \approx R_A - R_B$ **oe**	**B1**	**2.2b**	allow $R_C = R_A - R_B$; allow equivalent in words
				[1]		

Question			Answer	Marks	AO	Guidance
3	(a)		[1] $(x_1 =) 1.24(220796762)$ $(x_2 =) 1.14(067093071)$ $(x_7 =) 1.1689(9332106)$ $(x_8 =) 1.1685(9725396)$ $(x_9 =) 1.168754891$ $(x_{10} =) 1.168692142$ $(x_{11} =) 1.168717118$ $(x_{12} =) 1.168707177$ $[\alpha =] 1.169$	M1 A1 [2]	1.1 2.2a	must see 1.24…, 1.14…rounded or truncated to 2 dp or more **NB** 1.1800(4699025) 1.1642(2624512) 1.1704(9925209) 1.1679(9852762) for **A1**, must see two consecutive iterates from x_7 to x_{12} or better eg x_9 and x_{10} rounded or truncated to 4 dp or more **NB** $(x_{13} =) 1.168711134$ $(x_{14} =) 1.168709559$ $(x_{15} =) 1.168710186$ $(x_{16} =) 1.168709936$
3	(b)		because $g'(\beta) > 1$ **oe**	B1 [1]	2.4	must refer to gradient of g(x) at β.

Question			Answer	Marks	AO	Guidance
3	(c)		[6.4] $(x_1 =)6.4037529$ $(x_2 =)6.4057774$ $(x_5 =)6.4077693$ $(x_6 =)6.4079387$ $(x_7 =)6.4080298$ $(x_8 =)6.4080788$ $(x_9 =)6.4081051$ $(x_{10} =)6.4081192$	M1	1.1	must see 6.403…and 6.405… rounded or truncated to 3 dp or more **NB** 6.4068675 6.4074539
			$[\beta =]6.408$	A1 [2]	2.2b	must see two consecutive iterates from x_5 to x_{12} or better eg x_8 and x_9 rounded or truncated to 4 dp or more **NB** $(x_{11} =)6.4081268$ $(x_{12} =)6.4081309$ $(x_{13} =)6.4081331$ $(x_{14} =)6.4081343$
3	(d)		to **accelerate** the **convergence oe** (of a sequence which already converges)	B1 [1]	1.2	must refer to convergence

Question			Answer	Marks	AO	Guidance
4	(a)		(see table below)	M1	1.1	at least 3 first differences correct
				A1	1.1	all correct

t	T	ΔT	$\Delta^2 T$
0	8.8		
		4.4	
1	13.2		-2.2
		2.2	
2	15.4		-2.2
		0	
3	15.4		-2.1
		-2.1	
4	13.3		

[2]

Question			Answer	Marks	AO	Guidance
4	(b)		because the second differences are **approximately equal oe** **or** because the third differences are 0 and 0.1 and 0.1 is approximately 0 **oe** **[1]**	B1	2.2b	allow eg **approximately constant** eg **roughly the same** do not allow eg roughly equivalent eg relatively similar
4	(c)		$8.8 + 4.4t - 2.2\frac{t(t-1)}{2!}$ $8.8 + 5.5t - 1.1t^2$ $T = 8.8 + 5.5t - 1.1t^2$ **[4]**	M1 A1 A1 A1	3.3 1.1 1.1 2.5	**FT** their differences from part **(a)** and one sign error in substitution; allow other variable two of three terms correct; allow other variable all three terms correct; allow other variable; may see eg y in terms of x fully correct with correct variables used

	Question		Answer	Marks	AO	Guidance
4	(d)		[$t = 5$,] $T = 8.8$ (which is close to 8.9) so good fit for October **isw**	**B1**	3.4	**FT** their quadratic following award of at least **M1**;
			[$t = 6$,] $T = 2.2$ (which is a long way from 7.5), so bad fit for November	**B1**	3.2b	if **B0B0** allow **SC1** for 8.8 and 2.2 seen **or** allow **SC1** for calculation of **both** values of T found **FT** their quadratic following award of at least **M1** in part **(c)**
				[2]		
4	(e)		$T = 10.3 + 5.5t - 1.1t^2$	**B1**	2.2a	**FT** their quadratic; must be T in terms of t
				[1]		
5	(a)		$e^x - x^2 - x - 2$	**M1**	1.1	allow one sign error
			$e^x - x^2 - x - 2 = 0$	**A1**	1.1	all correct
				[2]		
5	(b)		=(A2 + C2)/2 **oe**	**B1**	1.1	
				[1]		
5	(c)		=IF(F2>0,E2,C2) **or** =IF(F2<0,C2,E2)	**B1**	1.1	
				[1]		
5	(d)		x-values (column A, C, E)	**B1**	1.1	2.1875 -0.0598 2.25 0.17524 2.21875 0.0542[3]
			y-values (column B, D, F)	**B1**	1.1	
				[2]		
5	(e)		2.1875 < root < 2.21875 so root = 2.2 (because both endpoints round to 2.2, so any approximation in this interval will also round to 2.2)	**B1**	2.2a	allow 2.2 because eg 2.1875 and 2.21875 both agree to 1 dp eg the last two approximations agree to 1 dp **oe** eg the best 2 approximations agree to 2 sf **oe** eg A6 and E6 agree to 1dp **oe** do not allow 2.2 because eg A6 and C6 agree to 1 dp
				[1]		

	Question		Answer	Marks	AO	Guidance
5	(f)		interval $\times 0.5^n$	M1	3.1a	interval may be 1 or an integer power of 0.5;
			$\ln(\text{interval} \times 0.5^n) < \ln 0.0005$ **oe**	M1	2.1	may see 5.96
			$[n > 5.96]$ so $n = 6$	A1	3.2a	may see $11 - 5 = 6$ or $11 - 4 = 7$
			allow$[n > 6.97]$ so $n = 7$			
				[3]		
			Alternatively			
			eg $[0.03125], 0.015625, 0.0078125, 0.003906,$ $0.001953, 0.0009765, 0.0004883$	M1		two bisections of maximum possible error; may see eg 0.5, 0.25, 0.125, 0.0625, 0.03125, …
			0.0009765 **and** 0.0004883 seen	M1		
			$n = 6$ **or** $n = 7$	A1		this mark is dependent on the award of **one M** mark; if **M0M0** allow **SC2** for $\frac{0.03125}{2^6} = 0.000488$ **oe** so $n = 6$
						or SC2 for $\frac{0.0625}{2^7} = 0.000488$ **oe** so $n = 7$
						or SC2 for $\frac{1}{2^{11}} = 0.000488$ **oe** so $n = 6$ or $n = 7$
				[3]		
6	(a)		because the other two *x*-values are both greater than 1	B1	2.4	allow because eg we don't have a pair of values of *x* and *y* (which are equally spaced) **either side** of $x = 1$ eg we need $f(0.5)$ eg we need $(0, f(0))$ eg we need a value for $x = 0$ (or 0.5)
				[1]		
6	(b)		$\frac{1.18921-1}{2-1}$ **or** $\frac{0.84089-1}{1.5-1}$ **soi**	M1	1.1	
			0.18921 or 0.1892 or 0.189	A1	1.1	**B2** for either correct answer unsupported
			or $- 0.31822$ or $- 0.3182$ or $- 0.318$			
				[2]		

Question			Answer	Marks	AO	Guidance
6	(c)	(i)	ratio of differences is **converging** to **0.5 oe**	B1	2.2b	
			this suggests order of method is 1 (since h is halved in successive approximations)	B1	2.4	allow eg the order of convergence of the method is 1 eg first order (convergence) eg method has first order convergence
				[2]		
	(c)	(ii)	this is to be expected since forward difference is usually a first order method	B1	1.2	
				[1]		
6	(d)		$X + D \times \frac{r}{1-r}$	M1	3.1a	allow $X \pm D \times \frac{r}{1-r}$ X is an approximation from the table, D is the associated difference and $r = 0.5$ or one of the (possibly rounded) ratios from the table may see $0.148304 + 0.000345 \times \frac{1}{1-r}$
			$0.148477 + 0.000173 \times \frac{r}{1-r}$ **or** $0.148304 + 0.000173 \times \frac{1}{1-r}$	M1	2.1	allow $0.148477 \pm 0.000173 \times \frac{r}{1-r}$ **or** $0.148304 \pm 0.000173 \times \frac{1}{1-r}$ allow $0.5 \leq r \leq 0.503$ **M1M1** may be implied by sight of correct value in range with evidence of use of correct formula
			0.14865 to 0.1486521	A1	1.1	**NB** may see may see $0.148304 + 0.000173 \times \frac{1}{1-r}$ with correct value of r
			accept 0.14865 or 0.1487 or 0.149 since extrapolation greatly increases accuracy **oe**	A1	3.2a	if **M0M0** allow **SC1** for partial extrapolation: $0.148477 + 0.000173 \times r = 0.1485635$ to 0.1485639 then **SC1** for accept 0.149 since extrapolation greatly increases accuracy **oe**

Question			Answer	Marks	AO	Guidance
				[4]		
7	(a)		<table><tr><td>n</td><td>M_n</td><td>T_n</td><td>S_{2n}</td></tr><tr><td>1</td><td>0.612547</td><td>1</td><td>0.74170</td></tr><tr><td>2</td><td>0.639735</td><td>0.80627</td><td>0.69525</td></tr></table>	**B1** **B1** **B1** [3]	1.1 1.1 1.1	award **B1** for each correct value given to **5** dp in the table. if only one **B** mark awarded, allow **SC1** for other 2 values given to higher precision if **B0B0B0** allow **SC1** for two values given to higher precision or **SC2** for all three values given to higher precision. **NB** $T_2 = 0.806274$ or 0.8062735 **NB** $S_2 = 0.741698$ **NB** $S_4 = 0.69524783$
7	(b)		0.7 because S_4 and S_2 agree to this precision	**B1** [1]	2.2b	allow eg $S_4 \approx S_2 \approx 0.7$ ignore further comments unless contradictory
7	(c)		spreadsheet stores values to greater accuracy than it displays student is working with the displayed values, which are less accurate, so the student's value is less accurate **oe**	**B1** **B1** [2]	1.2 2.4	 or spreadsheet works with the stored values so the answer in the spreadsheet is more accurate
7	(d)	(i)	appear to be **converging** to a constant [0.4] so first order convergence	**B1** [1]	2.2b	**or** slowly **decreasing**, so convergence (slightly) faster than 1st order

Question			Answer	Marks	AO	Guidance
7	(d)	(ii)	(ratio of differences is converging to 0.4 and) $0.25 < 0.4 < 0.5$	B1	2.2b	
			so **order** of **method** is between 1 and 2	B1	2.2a	allow eg order is higher than 1 and lower than 2 eg **order** of convergence of **method** is between 1 and 2 eg $1 < \text{order} < 2$ do not allow eg order is faster than 1 and slower than 2 eg convergence is between 1st and 2nd order eg convergence is higher than 1st order and lower than 2nd order eg convergence is faster than 1st order and slower than 2nd order eg convergence is between 1st and 2nd order
				[2]		
7	(d)	(iii)	this is unusual because Simpson's rule is usually (a) fourth order (method)	B1 [1]	2.5	allow eg unusual because Simpson's rule usually has fourth order convergence

Question			Answer	Marks	AO	Guidance
7	**(e)**		$S + D \times \frac{r}{1-r}$	**M1**	**3.1a**	allow $S \pm D \times \frac{r}{1-r}$
						S is value of S_{2n} from the table, *D* is the associated difference and *r* = 0.4 or one of the (possibly rounded) ratios from the table
			$0.659015 - 0.00066 \times \frac{r}{1-r}$	**M1**	**2.1**	allow $0.659015 \pm 0.00066 \times \frac{r}{1-r}$
						or $0.659675 \pm 0.00066 \times \frac{1}{1-r}$
						NB $0.659675 - 0.00066 \times \frac{1}{1-r}$ earns **M1M1**
						$0.4 \leq r \leq 0.406785$
						M1M1 may be implied by sight of correct value in range with evidence of use of correct formula
			0.6585624 to 0.65858	**A1**	**1.1**	
			accept 0.65856 or 0.6586 since extrapolation greatly improves accuracy **isw oe**	**A1**	**3.2a**	allow 0.659 is certain since extrapolated value and S_{128} (= 0.659015) agree to this precision; last (or best) Simpson's estimate
						if **M0M0** allow **SC1** for $\frac{16 \times 0.659015 - 0.659675}{15} = 0.658971$ then **SC1** for 0.659 seems secure by comparison with best Simpson's estimate **oe**
						if **M0M0** allow **SC1** for $0.659015 - 0.00066 \times r = 0.65874$ to 0.658751 then **SC1** for 0.659 is secure by comparison with last Simpson's estimate **oe**
				[4]		

APPENDIX

Exemplar responses for Q2(b)

Response	Mark

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on

01223 553998

Alternatively, you can email us on

support@ocr.org.uk

For more information visit

ocr.org.uk/qualifications/resource-finder

ocr.org.uk

Twitter/ocrexams

/ocrexams

/company/ocr

/ocrexams

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT