OCR
Oxford Cambridge and RSA

Friday 14 June 2024 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 The difference table shows some values of x and the associated values of y.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	0.8			
1	0.7			
2	0.1			
3	11.3			
4	70.5			
5	250.6			

(a) Complete the copy of the difference table in the **Printed Answer Booklet**. **[2]**

(b) Explain whether a cubic polynomial would be suitable to model the relationship between y and x. **[1]**

2 A spreadsheet is used to find the approximate value of $\pi + \mathrm{e}$. The output is shown in the table.

	A	B
1	π	3.141 593
2	e	2.718 282
3	$\pi + \mathrm{e}$	5.859 874

The formula in cell B1 is $\boxed{= \mathrm{PI}()}$.

The formula in cell B2 is $\boxed{= \mathrm{EXP}(1)}$

and the formula in cell B3 is $\boxed{\mathrm{B1} + \mathrm{B2}}$.

(a) Show that the approximations to π and e displayed in cells B1 and B2 respectively have both been rounded. **[1]**

(b) Find the relative error when 3.141 593 is used to approximate π. **[2]**

(c) Explain why the value displayed in cell B3 is not equal to $2.718\,282 + 3.141\,593$. **[2]**

3 Following an injury a professional footballer has to undergo a rehabilitation programme before she can resume playing. As part of the programme she is required to take weekly fitness tests to assess her progress. When she achieves a score of 95% or over she is deemed fit enough to be part of the match day squad.

The footballer was ill at the end of week 3 and so did not take the fitness test. The percentage score she achieved, P, at the end of each week, w, is shown for $w = 1$, 2 and 4 in the table.

w	1	2	4
P	52.00	62.54	79.90

The medical staff at the footballer's club believe the data may be modelled by a polynomial.

(a) Use Lagrange's form of the interpolating polynomial to construct a polynomial of degree 2 to model the relationship between P and w. **[4]**

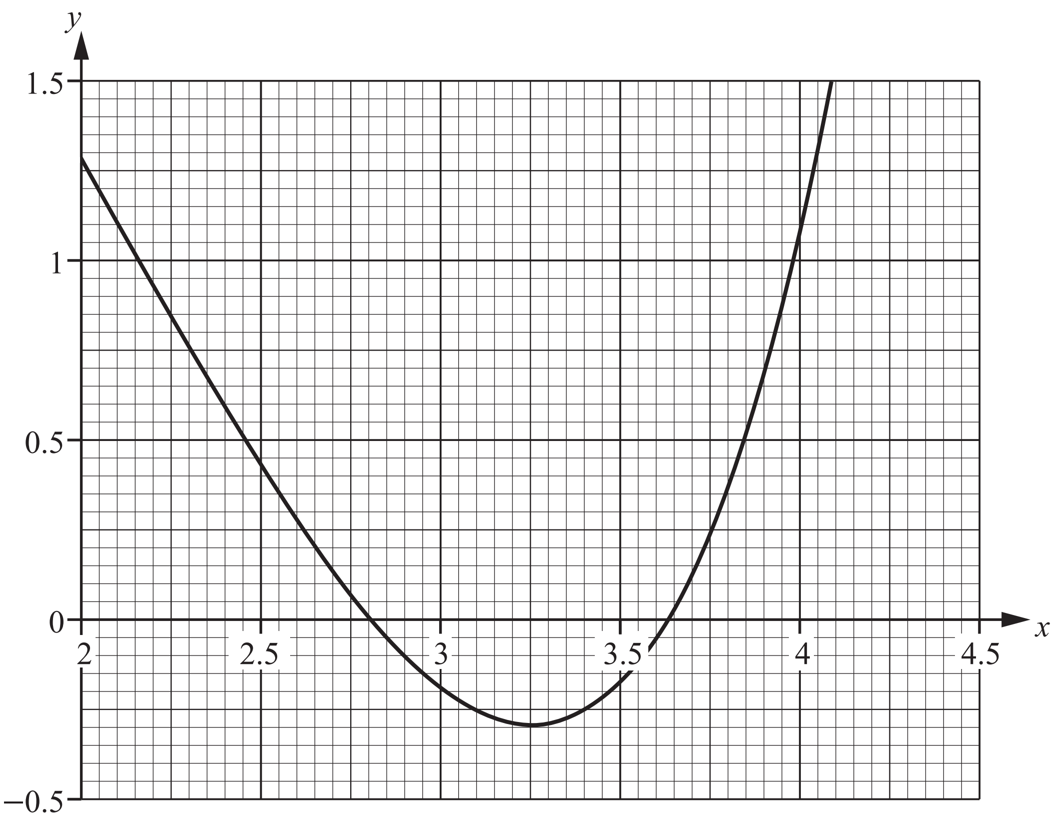
The club are due to play a match at the end of week 6 and another match at the end of week 7.

(b) Determine whether, according to the model, the footballer will be available to play in either of these matches. **[3]**

After returning to the match day squad, the medical staff continue to give the footballer a fitness test at the end of each week.

(c) Explain why the model is no longer viable from the end of week 9. **[1]**

4 The graph shows part of the graph of $y = f(x)$.



The equation $f(x) = 0$ has two positive roots, α and β, where $\alpha > \beta$.

The method of False Position is used to find α. A spreadsheet is used to find a sequence of approximations to α. The output is shown in **Table 4.1**.

Table 4.1

r	V	W	X	Y	Z	AA
3	a	$f(a)$	b	$f(b)$		
4	3	−0.18289	4	1.11963	3.14041	−0.26707
5	3.14041	−0.26707	4	1.11963	3.30597	−0.28855
6	3.30597	−0.28855	4	1.11963	3.44818	−0.22333
7	3.44818	−0.22333	4	1.11963	3.53995	−0.13196
8	3.53995	−0.13196	4	1.11963	3.58845	−0.06598
9	3.58845	−0.06598	4	1.11963	3.61136	−0.03027
10	3.61136	−0.03027	4	1.11963	3.62159	−0.01334
11	3.62159	−0.01334	4	1.11963	3.62604	−0.00577
12	3.62604	−0.00577	4	1.11963	3.62796	−0.00248

(a) On the copy of the graph in the **Printed Answer Booklet**, show how the method of False Position works to find the next **two** approximations to α using starting values of $x = a = 3$ and $x = b = 4$. **[2]**

The formula in cell W4 is $\boxed{= 0.2 * \text{EXP}(\text{V4}) - 0.8 * \text{V4}\text{^}2 + 3}$.

(b) Write down the equation which is being solved. **[2]**

The formula in cell V5 is $\boxed{= \text{IF}(\text{AA4} < 0, \text{Z4}, \text{V4})}$.

(c) Write down a similar formula for cell X5. **[1]**

(d) By considering the output in **Table 4.1**, state the value of α as accurately as possible. You must justify the precision quoted. **[1]**

The Newton-Raphson method is used with $x_0 = 2$ to find a sequence of approximations to β. The associated spreadsheet output is shown in **Table 4.2**.

Table 4.2

r	x_r	difference	ratio of differences
0	2		
1	2.741969	0.741969	
2	2.811136	0.069167	0.09322
3	2.814353	0.003217	0.04651
4	2.814361	7.67E-06	0.00238
5	2.814361	4.38E-11	5.7E-06

(e) Use the information in **Table 4.2** to state the value of β as accurately as possible. You must justify the precision quoted. **[1]**

(f) Explain what the ratios of differences tell you about the order of convergence of the sequence of approximations to β. **[2]**

(g) State **two** advantages of using the Newton-Raphson method instead of the method of False Position for finding the roots of an equation. **[2]**

5 **Table 5.1** shows some values of x and the associated values of $f(x)$.

Table 5.1

x	5	5.0001	5.001	5.01	5.1
$f(x)$	12.182 494	12.183 103	12.188 587	12.243 559	12.807 104

(a) Use the forward difference method to obtain four approximations to $f'(5)$. Write your answers in the copy of **Table 5.2** in the **Printed Answer Booklet**. **[5]**

Table 5.2

h	approximation to $f'(5)$
0.1	
0.01	
0.001	
0.0001	

(b) State the value of $f'(5)$ as accurately as you can. You must justify the precision quoted. **[1]**

Ali uses a smaller value of h to find an approximation to $f'(5)$. Ali's spreadsheet output is shown in **Table 5.3**.

Table 5.3

h	approximation to $f'(5)$
1.00E-14	6.039 613

(c) Explain whether Ali's approximation to $f'(5)$ is likely to be more accurate or less accurate than your answer to part **(b)**. **[2]**

(d) Determine an approximation of the error in using $f(5)$ to approximate $f(5.02)$. **[2]**

6 The equation $5\log_3 x - x = 0$ has two roots, α and β.

You are given that $1 < \alpha < 2$.

(a) Use the iterative formula $x_{n+1} = \mathrm{g}(x_n)$, where $\mathrm{g}(x_n) = 3^{\frac{x_n}{5}}$ and $x_0 = 2$, to determine the value of α correct to **6** decimal places. **[3]**

You are given that $\beta \approx 10.9$.

The table shows some approximations to $\mathrm{g}'(10.9)$ found using the central difference method.

h	approximation to $\mathrm{g}'(10.9)$
0.1	2.410 093
0.01	2.409 901
0.001	2.409 899

(b) (i) Explain why it is preferable to use the central difference method instead of the forward difference method where possible. **[1]**

(ii) Show how the approximation with $h = 0.1$ is calculated. **[2]**

(iii) Write down the value of $\mathrm{g}'(10.9)$ as accurately as you can. You must justify the precision quoted. **[1]**

(c) Explain why your answer to part **b(iii)** suggests that the iterative formula

$$x_{n+1} = \mathrm{g}(x_n), \text{ where } \mathrm{g}(x_n) = 3^{\frac{x_n}{5}},$$

may not be used to successfully find β, however close the starting value is to β. **[1]**

(d) Use the relaxed iteration

$$x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n) \text{ with } \lambda = -0.269 \text{ and } x_0 = 10.9$$

to determine the value of β correct to **6** decimal places. **[3]**

7 A student uses the midpoint rule, the trapezium rule and Simpson's Rule to find a series of approximations to $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$. The associated spreadsheet output, together with some further analysis, is shown in the table.

r	F	G	H	I	J	K
1	n	M_n	T_n	S_{2n}	difference	ratio of differences
2	1	0.8660254	0.5	0.74401694		
3	2	0.81484183	0.6830127	0.77089879	0.0268819	
4	4	0.79598231	0.74892727	0.78029729	0.0093985	0.35
5	8	0.78917173	0.77245479	0.78359942	0.0033021	0.351
6	16	0.78673795	0.78081326	0.78476305	0.0011636	0.352
7	32	0.78587285	0.78377561	0.78517377	0.0004107	0.353
8	64	0.78556617	0.78482423	0.78531885	0.0001451	0.353

(a) Write down a suitable spreadsheet formula for cell I2. **[2]**

The formula in cell J3 is $\boxed{= \mathrm{I}3 - \mathrm{I}2}$.

(b) Write down a suitable spreadsheet formula for cell K4. **[1]**

(c) By considering the values in column H, explain what may be inferred about the shape of the curve $y = \mathrm{f}(x)$. **[1]**

(d) State the order of convergence of the sequence of values in column K. **[1]**

(e) Explain what the values of the ratios of differences tell you about the order of Simpson's Rule in this case. **[2]**

In this question you must show detailed reasoning.

(f) Determine the value of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$ as accurately as you can. You must justify the precision quoted. **[5]**

END OF QUESTION PAPER