OCR
Oxford Cambridge and RSA

Friday 21 June 2024 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

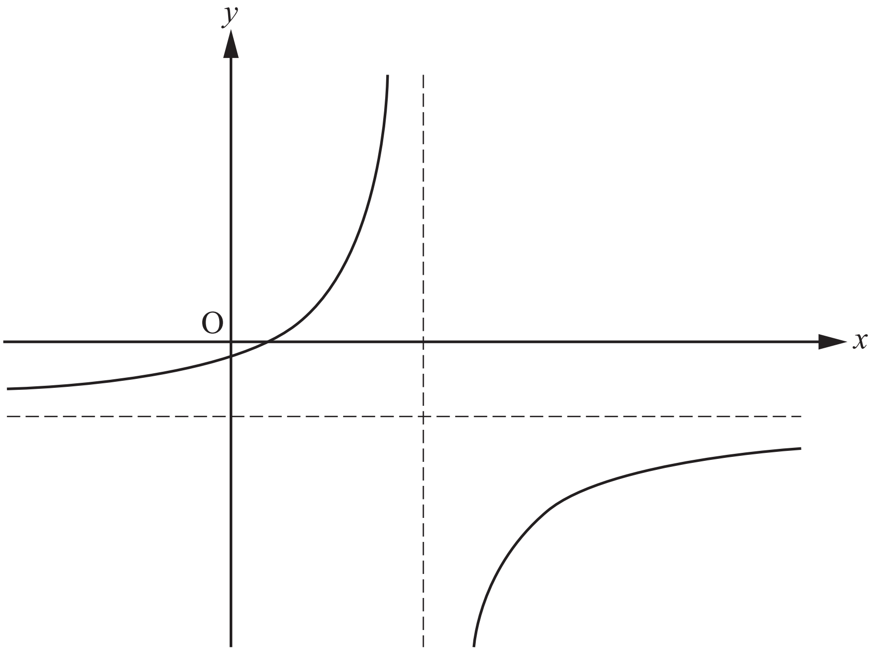
1 A surface, S, is defined in 3-D by $z = \mathrm{f}(x, y)$ where $\mathrm{f}(x, y) = 12x - 30y + 6xy$.

(a) Determine the coordinates of any stationary points on the surface. **[5]**

(b) The equation $z = \mathrm{f}(x, a)$, where a is a constant, defines a section of S.

Given that this equation is $z = 24x + b$, find the value of a and the value of b. **[3]**

The diagram shows the contour $z = 12$ and its associated asymptotes.



(c) Find the equations of the asymptotes. **[3]**

(d) By forming **grad** g, where $\mathrm{g}(x, y, z) = \mathrm{f}(x, y) - z$, find the equation of the tangent plane to S at the point where $x = 3$ and $y = 2$. Give your answer in vector form. **[3]**

The point $(0, 4, -120)$, which lies on S, is denoted by A.

The plane with equation $\mathbf{r}.\begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix} = 52$ is denoted by Π.

(e) Show that the normal to S at A intersects Π at the point $(-360, 304, -110)$. **[3]**

2 **(a)** Determine the general solution of the recurrence relation $2u_{n+2} - 7u_{n+1} + 3u_n = 0$. **[2]**

(b) Using your answer to part **(a)**, determine the general solution of the recurrence relation $2u_{n+2} - 7u_{n+1} + 3u_n = 20n^2 + 60n$. **[5]**

In the rest of this question the sequence $u_0, u_1, u_2, \ldots$ satisfies the recurrence relation in part **(b)**. You are given that $u_0 = -9$ and $u_1 = -12$.

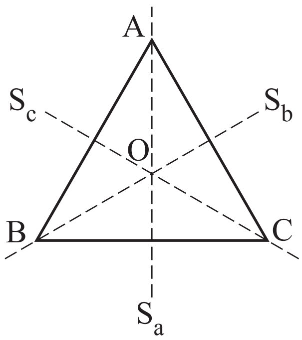
(c) Determine the particular solution for u_n. **[3]**

You are given that, as n increases, once the values of u_n start to increase, then from that point onwards the sequence is an increasing sequence.

(d) Use your answer to part **(c)** to determine, by direct calculation, the least value taken by terms in the sequence. You should show any values that you rely on in your argument. **[2]**

3 **Fig. 3.1** shows an equilateral triangle, with vertices A, B and C, and the three axes of symmetry of the triangle, S_a, S_b and S_c. The axes of symmetry are fixed in space and all intersect at the point O.

Fig. 3.1



There are six distinct transformations under which the image of the triangle is indistinguishable from the triangle itself, ignoring labels.

These are denoted by I, M_a, M_b, M_c, R_{120} and R_{240} where

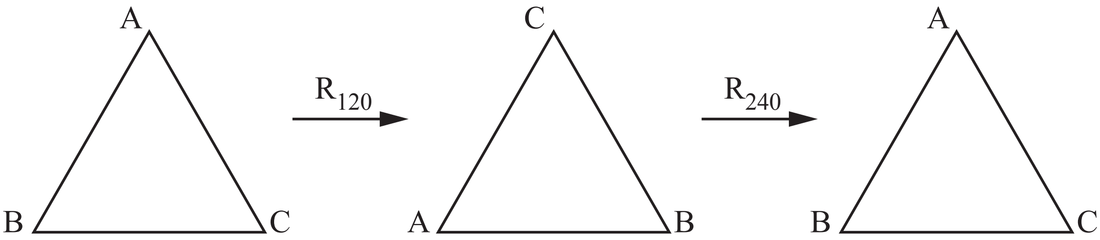
- I is the identity transformation
- M_a is a reflection in the mirror line S_a (and likewise for M_b and M_c)
- R_{120} is an anticlockwise rotation by 120° about O (and likewise for R_{240}).

Composition of transformations is denoted by $\circ$.

Fig. 3.2 illustrates the composition of R_{120} followed by R_{240}, denoted by $R_{240} \circ R_{120}$.

This shows that $R_{240} \circ R_{120}$ is equivalent to the identity transformation, so that $R_{240} \circ R_{120} = I$.

Fig. 3.2



(a) Using the blank diagrams in the Printed Answer Booklet, find the single transformation which is equivalent to each of the following.

- $M_a \circ M_a$
- $M_b \circ M_a$
- $R_{120} \circ M_a$

[3]

The set of the six transformations is denoted by G and you are given that (G, ○) is a group.

The table below is a mostly empty composition table for ○. The entry given is that for $R_{240} \circ R_{120}$.

First transformation performed is

followed by

○	I	M_a	M_b	M_c	R_{120}	R_{240}
I						
M_a						
M_b						
M_c						
R_{120}						
R_{240}					I	

(b) Complete the copy of this table in the **Printed Answer Booklet**. You can use some or all of the spare copies of the diagram in the **Printed Answer Booklet** to help. **[4]**

(c) Explain why there can be **no** subgroup of (G, ○) of order 4. **[1]**

(d) A student makes the following claim.

"If all the proper non-trivial subgroups of a group are abelian then the group itself is abelian."

Explain why the claim is incorrect, justifying your answer fully. **[3]**

(e) With reference to the order of elements in the groups, explain why (G, ○) is **not** isomorphic to C_6, the cyclic group of order 6. **[1]**

4 The matrix $\mathbf{P}$ is given by $\mathbf{P} = \begin{pmatrix} 1 & 7 & 8 \\ -6 & 12 & 12 \\ -2 & 4 & 8 \end{pmatrix}$.

(a) Show that the characteristic equation of $\mathbf{P}$ is $-\lambda^3 + 21\lambda^2 - 126\lambda + 216 = 0$. **[3]**

You are given that the roots of this equation are 3, 6 and 12.

(b) **(i)** Verify that $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ is an eigenvector of $\mathbf{P}$, stating its associated eigenvalue. **[2]**

(ii) The vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is an eigenvector of $\mathbf{P}$ with eigenvalue 6.

Given that $z = 5$, find x and y. **[3]**

You are given that $\mathbf{P}$ can be expressed in the form $\mathbf{EDE}^{-1}$, where $\mathbf{E} = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & 2 \end{pmatrix}$ and $\mathbf{D}$ is a diagonal matrix. The characteristic equation of $\mathbf{E}$ is $-\lambda^3 + 7\lambda^2 - 15\lambda + 9 = 0$.

(c) **(i)** Use the Cayley-Hamilton theorem to express $\mathbf{E}^{-1}$ in terms of positive powers of $\mathbf{E}$. **[2]**

(ii) Hence find $\mathbf{E}^{-1}$. **[1]**

(iii) By identifying the matrix $\mathbf{D}$ and using $\mathbf{P} = \mathbf{EDE}^{-1}$, determine $\mathbf{P}^4$. **[4]**

5 In this question you may assume that if p and q are distinct prime numbers and $p^{\alpha} = q^{\beta}$ where $\alpha, \beta \in \mathbb{Z}$, then $\alpha = 0$ and $\beta = 0$.

(a) Prove that it is **not** possible to find a and b for which $a, b \in \mathbb{Z}$ and $3 = 2^{\frac{a}{b}}$. **[2]**

(b) Deduce that $\log_2 3 \notin \mathbb{Q}$. **[2]**

END OF QUESTION PAPER

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