GCE

Further Mathematics B MEI

Y435/01: Extra pure

A Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.

5. **Annotations**

Annotation	Meaning
✓and✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.
BP	Blank Page
Seen	
Highlighting	

6. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed

reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

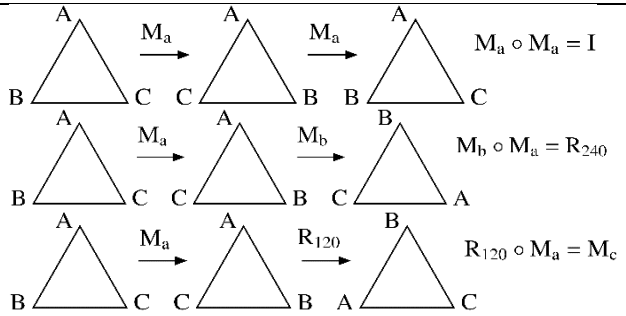
Question			Answer	Marks	AO	Guidance	
1	(a)		$\frac{\partial z}{\partial x} = 12 + 6y$	B1	1.1	For both **B** marks accept eg $\frac{\partial f}{\partial x}$ or z_x or f_x .	For both **B** marks accept d or δ but must be correct indication of both parameters (ie $\partial x/\partial z =$ or $\partial z =$ is **B0**).
			$\frac{\partial z}{\partial y} = -30 + 6x$	B1	1.1	Both could be embedded in a vector.	Ignore z component.
			$\frac{\partial z}{\partial x} = 0 \Rightarrow 12 + 6y = 0 \Rightarrow y = -2$	M1	1.1	Setting one partial derivative to 0 and finding the value of one variable	
			$\frac{\partial z}{\partial y} = 0 \Rightarrow -30 + 6x = 0 \Rightarrow x = 5$	M1	1.1	Setting the other partial derivative to 0 and finding the value of the other variable	This mark can only be awarded if it is clear that both derivatives are 0 at the SP (eg stated or by giving one SP with the found co-ordinates).
			$y = -2, x = 5 \Rightarrow z = 60$ (so the only SP is (5, –2, 60))	A1	1.1	SP found and no others. Do not ISW. Must be coordinates not vector.	Answer in format $x =..., y =..., z = ...$ is sufficient if not invalidated.
				[5]			
1	(b)		$24x + b \equiv 12x - 30a + 6ax = (12 + 6a)x - 30a$ so $12 + 6a = 24$ (or $b = -30a$)	M1	3.1a	Substituting $y = a$ into the equation of the surface, equating to $24x + b$ and correctly comparing constants or x coefficients (soi by correct answers).	Condone $12x + 6ax = 24x$ for **M1**. Could find a correct equation by choosing a value of x (eg $x = 0$).
			$a = 2$	A1	2.2a		
			$a = 2 \Rightarrow b = -60$	A1FT	2.2a	FT –30×their value of a. **SCB1** (after **M0** or **M1**) if a and b not properly identified (eg so 2, –60).	Can be embedded so $f(x, 2) = 24x - 60$ is sufficient for **A1A1**. Ignore answers at "$a =$" and "$b =$" in PAB unless no answers given or a and/or b not identified in main working.
				[3]			
1	(c)		$z = 12 \Rightarrow 12 = 12x - 30y + 6xy$			or $2 = 2x - 5y + xy$ oe	
			$y = \frac{12x - 12}{30 - 6x} = \frac{2x - 2}{5 - x} = -2 + \frac{8}{5 - x}$	M1	3.1a	Substituting $z = 12$ and rearranging to a useful form eg $y = g(x)$.	May see $y = \frac{12 - 12x}{6x - 30}$ oe or $x = \frac{12 + 30y}{12 + 6y} = \frac{2 + 5y}{2 + y} = 5 - \frac{8}{2 + y}$ oe
			(so vertical asymptote is) eg $x = 5$	A1	2.2a	Either asymptote obtained.	
			(...and horizontal asymptote is) $y = -2$ www	A1	2.2a	Both asymptotes obtained with no errors in working.	Condone if asymptotes not correctly labelled.
				[3]			

Question			Answer	Marks	AO	Guidance	
1	(d)		$x = 3, y = 2 \Rightarrow z = 12$	**B1**	**1.1**	Clear statement of value of z or sight of eg $\begin{pmatrix}3\\2\\12\end{pmatrix}$.	z must be evaluated.
			$\nabla g = \begin{pmatrix}12+6y\\-30+6x\\-1\end{pmatrix} = \begin{pmatrix}24\\-12\\-1\end{pmatrix} \Rightarrow$ eg $\mathbf{r}.\begin{pmatrix}24\\-12\\-1\end{pmatrix} = ...$	**M1**	**1.1**	Forming **grad** g to find normal at S using $x = 3$ and $y = 2$ in the expressions for f_x and f_y from **(a)**, and using it. Condone use of f rather than g. Can be implied by 2 correct coordinates. Must be used in equation of plane, not line.	**grad** g must be formed as a vector; stating the components is not sufficient. z co-ordinate must be –1. Must realise that **grad** g gives the normal to S for **M1** so do not ignore subsequent work on ∇g.
			$\mathbf{r}.\nabla g = \begin{pmatrix}3\\2\\12\end{pmatrix}.\begin{pmatrix}24\\-12\\-1\end{pmatrix} = 72 - 24 - 12 = 36$				
			$\therefore$ equation is $\mathbf{r}.\begin{pmatrix}24\\-12\\-1\end{pmatrix} = 36$ oe	**A1FT**	**2.5**	FT 48–"z" miscalculated from f(3, 2). Not $z = -1$ unless clearly from f(3, 2). Or any non-zero multiple. Must be in vector form; $\mathbf{r}.\mathbf{n} = p$, $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$ or $\mathbf{r} = \begin{pmatrix}3\\2\\12\end{pmatrix} + \lambda\begin{pmatrix}0\\-1\\12\end{pmatrix} + \mu\begin{pmatrix}1\\0\\24\end{pmatrix}$ oe. Any 'free' scalar products must be evaluated but ISW once acceptable form reached.	If (**B1**)**M0** then **SC1** for using $z - c = \frac{\partial f}{\partial x}(x-a) + \frac{\partial f}{\partial y}(y-b)$ to derive the correct answer. Or eg $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}24\\-12\\-1\end{pmatrix} = 36$ or $\left(\mathbf{r} - \begin{pmatrix}3\\2\\12\end{pmatrix}\right).\begin{pmatrix}24\\-12\\-1\end{pmatrix} = 0$
				[3]			

Question	Answer	Marks	AO	Guidance	
1 (e)	$\nabla g = \begin{pmatrix} 36 \\ -30 \\ -1 \end{pmatrix} \Rightarrow (\mathbf{r} =)\ \mathbf{a} + \lambda \begin{pmatrix} 36 \\ -30 \\ -1 \end{pmatrix}$	**M1**	**1.1**	Forming **grad** g using $x = 0$ and $y = 4$ in the expressions for f_x and f_y from **(a)** and proper attempt to form the equation of the normal line. Attempt at **grad** can be implied by two correct coordinates. Not **grad** g from **(d)**. Condone use of f rather than g.	z co-ordinate must be –1. Must realise that **grad** g gives the normal at A for **M1** so do not ignore subsequent work on ∇g. **grad** must be direction vector of line. If "$\nabla g =$" not seen, **M1** can be awarded but see guidance for **A1**.
	$\left(\begin{pmatrix} 0 \\ 4 \\ -120 \end{pmatrix} + \lambda \begin{pmatrix} 36 \\ -30 \\ -1 \end{pmatrix}\right) . \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix} = 52$ or $3(36\lambda)+3(4-30\lambda)-2(-120-\lambda) = 52$	**M1**	**2.1**	Intersecting (**r** =) **a** + λ**b** (with $\mathbf{a} = \begin{pmatrix} 0 \\ 4 \\ -120 \end{pmatrix}$ and **b** = their new **grad** g) with the given plane. Need not be seen in vector form. Not **grad** g from **(d)**. Not [3, 3, –2].	If **M1M0** then **SCB1** for **both** verifying that (–360, 304, –110) lies on the plane **and** using $\lambda = -10$ (however derived) to verify that it also lies on the normal line. Both verifications need to be **complete**.
	$\therefore 12 + 240 + \lambda(108 - 90 + 2) = 52$ $\lambda = -10 \Rightarrow \mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -120 \end{pmatrix} - 10 \begin{pmatrix} 36 \\ -30 \\ -1 \end{pmatrix} = \begin{pmatrix} -360 \\ 304 \\ -110 \end{pmatrix}$ (so point is (–360, 304, –110))	**A1**	**2.2a**	AG Correct parameter for their line must be derived from cartesian equation and substituted in. Condone proof completed by finding position vector. AG so must see correctly derived value of λ found and correctly substituted in. Expect to see $20\lambda + 252 = 52$ => $\lambda = -200/20$ => $\lambda = -10$	Accept statement that parameter is to be substituted as evidence of substitution. Derivation of correct position vector or "$x = ..., y = ..., z = ...$" is sufficient. Do not award **A1** if "$\nabla g =$" never seen in any form or described or identified in some way as normal to the surface.
		[3]			

Question			Answer	Marks	AO	Guidance	
2	(a)		$2u_{n+2} - 7u_{n+1} + 3u_n = 0$ & $u_n = \alpha r^n$ $\Rightarrow 2r^2 - 7r + 3 = 0$ $\Rightarrow r = 3$ or $r = \frac{1}{2}$ So GS is $(u_n =) A \times 3^n + B\left(\frac{1}{2}\right)^n$	**M1** **A1** **[2]**	**1.1** **1.1**	Deriving the auxiliary equation. 3 term quadratic *equation*. "= 0" could be implied by correct solution. or $A \times 3^n + \frac{B}{2^n}$ or $A \times 3^n + B \times \frac{1}{2^n}$ or $A \times 3^n + B \times 2^{-n}$ or $A \times 3^n + B \times 0.5^n$	If **M0** then **SCB1** for correct answer. Condone missing brackets around the fraction. If deduced from repeated substitution then must be fully correct for **M1A1**.
2	(b)		Trial function: Try $u_n = an^2 + bn + c$ $2[a(n+2)^2 + b(n+2) + c] -$ $7[a(n+1)^2 + b(n+1) + c] + 3(an^2 + bn + c)$ $= 2[an^2 + (4a + b)n + 4a + 2b + c] -$ $7[an^2 + (2a + b)n + a + b + c] + 3(an^2 + bn + c)$ $= (2a - 7a + 3a)n^2 + (8a + 2b - 14a - 7b + 3b)n +$ $(8a + 4b + 2c - 7a - 7b - 7c + 3c)$ $= -2an^2 + (-6a - 2b)n + (a - 3b - 2c)$ $= 20n^2 + 60n$ $\Rightarrow -2a = 20, -6a - 2b = 60$ and $a - 3b - 2c = 0$ $\Rightarrow a = -10, b = 0, c = -5$ So GS is $u_n = A \times 3^n + B\left(\frac{1}{2}\right)^n - 10n^2 - 5$,	**B1** **M1** **M1** **A1** **B1FT** **[5]**	**3.1a** **1.1** **1.1** **1.1** **1.1**	Correct general form for trial function. Either stated as u_n or as TF or clearly subbed in. Substituting their polynomial form correctly into LHS of recurrence relation. Expanding, collecting terms and comparing coefficients with correct RHS coefficients to derive three equations in a, b and c (may be progressively solved). FT their CF (GS from **(a)**) + PTF provided that CF has two arbitrary constants. Only ISW if this is labelled as GS.	Other terms eg dn^3 may be added; **B1** can be awarded when eg $d = 0$ stated (from working or assumption) May also see CF subbed in. $u_n - 7u_{n-1} + 3u_{n-2} = 20n^2 - 20n - 40$ **M1** can be earned after single minor slip in indexing. **M1** can be earned from their TF provided it does not lead to significant reduction in complexity. Must be "$u_n =$" Do not accept missing brackets in fraction for **B1FT**.

Question			Answer	Marks	AO	Guidance	
2	(c)		$n = 0 \Rightarrow -9 = A + B - 5$	M1	1.1	Using one initial condition in their GS from **(b)** to derive an equation in A and B.	$A + B = -4$
			$n = 1 \Rightarrow -12 = 3A + \frac{1}{2}B - 10 - 5$ So $6A + (-4 - A) = 6$	M1	1.1	Using another initial condition in their GS from **(b)** to derive a second equation in A and B and attempt to solve (can be implied by correct solution). Could be BC.	$6A + B = 6$ **M0M0A0** for using just the CF. $A + B = -9$, $3A + \frac{1}{2}B = -12$, $A = -3$, $B = -6$
			$\Rightarrow A = 2$, $B = -6$ so PS is $u_n = 2 \times 3^n - 6\left(\frac{1}{2}\right)^n - 10n^2 - 5$ cao	A1 [3]	1.1		"$u_n =$" and fraction in brackets but only penalise either once in **2(b)** and **2(c)**.
2	(d)		$u_2 = 2\times9 - 6/4 - 10\times4 - 5$ (= –28.5) $u_3 = 2\times27 - 6/8 - 10\times9 - 5$ (= –41.75) $u_4 = 2\times81 - 6/16 - 10\times16 - 5$ (= –3.375) NB $-28.5 = -\frac{57}{2}$, $-41.75 = -\frac{167}{4}$, $-3.375 = -\frac{27}{8}$	M1	3.1a	Using their PS (of the form $A\times3^n + B\left(\frac{1}{2}\right)^n + \alpha n^2 + \beta n + \gamma$) to calculate any three consecutive terms u_n, $n \geq 2$ (eg u_2, u_3 and u_4).	Values do not have to be correct provided that it is clear what is being attempted. **M0** if it is clear that the RR is being used to generate values.
			(So u_3, u_4, u_5, ... is an increasing sequence so the least value is u_3 =) –41.75	A1	2.2a	Values for u_2, u_3 and u_4 all correct and –41.75 explicitly selected. Not just "$n = 3$" as final answer. Correct value unsupported is 0/2.	If **M0** then **SC1** for correct values found from the RR and correct conclusion
			Alternative method: $u_{n+1} - u_n = 2\times3^{n+1} - 6\left(\frac{1}{2}\right)^{n+1} - 10(n+1)^2 - 5$ $-\left(2\times3^n - 6\left(\frac{1}{2}\right)^n - 10n^2 - 5\right)$ $= 6\times3^n - 2\times3^n - 3\left(\frac{1}{2}\right)^n + 6\left(\frac{1}{2}\right)^n - 20n - 10$ $= 4\times3^n + 3\left(\frac{1}{2}\right)^n - 20n - 10$ $n = 2 : u_3 - u_2 = -13.25$ (< 0) $n = 3 : u_4 - u_3 = 38.375$ (> 0) So u_3, u_4, u_5, ... is an increasing sequence so the least value is (u_3 =) –41.75	M1 A1 [2]		Using their PS to calculate, $u_3 - u_2$ and $u_4 - u_3$, perhaps from derived general rule. Values do not have to be correct provided that it is clear what is being attempted. All values correct. Correct value unsupported is 0/2.	

Question			Answer	Marks	AO	Guidance	
3	(a)			**B1** **B1** **B1** **[3]**	**2.2a** **2.2a** **2.2a**	Triangles need not be labelled for this **B1** but must be correct if shown. Triangles must be correctly labelled. Triangles must be correctly labelled.	The transformations do not need to be shown on the arrows throughout. If **B?B0B0** then **SCB1** if last two both correct but triangles not labelled.
3	(b)		$\circ$ | I | M_a | M_b | M_c | R_{120} | R_{240} I | **I** | **M_a** | **M_b** | **M_c** | **R_{120}** | **R_{240}** M_a | **M_a** | **I** | R_{120} | R_{240} | ***M_b*** | ***M_c*** M_b | **M_b** | R_{240} | **I** | R_{120} | ***M_c*** | ***M_a*** M_c | **M_c** | R_{120} | R_{240} | **I** | ***M_a*** | ***M_b*** R_{120} | **R_{120}** | ***M_c*** | ***M_a*** | ***M_b*** | *R_{240}* | *I* R_{240} | **R_{240}** | ***M_b*** | ***M_c*** | ***M_a*** | *I* | *R_{120}*	**B1** **B1** **B1** **B1** **[4]**	**2.2a** **2.2a** **2.2a** **2.2a**	**Identity row**, **column** and leading **diagonal** for **reflections** all correct. *Rotation block* all correct. Reflection combinations correct. ***Rotation/reflection combinations*** correct.	If **B0B0** for last two **B** marks then **SCB1** for fully correct table reflected in leading diagonal.
3	(c)		The order of $(G, \circ)$ is 6. (By Lagrange's theorem) the order of any subgroup of $(G, \circ)$ must be a factor of 6. 4 is not a factor of 6.	**B1** **[1]**	**2.2a**	or by Lagrange's theorem the order of any subgroup must be 1, 2, 3 (or 6) and so the order of a subgroup can't be 4. All three elements of the explanation must be present. Lagrange theorem must be stated or appealed to, not inferred. **B0** if incorrect statement seen.	Allow, "the order of G is 6, 4 is not a factor of 6 so by Lagrange, 4 cannot be the order of a subgroup." Condone, "The order of a subgroup must be a factor of the order of the group. 4 is not a factor of 6."

Question			Answer	Marks	AO	Guidance	
3	(d)		From Lagrange's theorem, all proper subgroups of $(G, \circ)$ must have order 2 or 3 (or 1). eg All of the proper non-trivial subgroups of $(G, \circ)$ are abelian since they are all of order 2 or 3 and so are abelian. But $(G, \circ)$ is not abelian since the table is not symmetric (about its leading diagonal) and so the claim is incorrect.	**B1** **B1** **B1** **[3]**	**2.1** **2.4** **2.2a**	Or these can be explicitly listed (in set or subgroup form) and stated as being the only proper (non-trivial) subgroups. Ignore reference to $(\{I\}, \circ)$. Or explicit example, eg $R_{240} = M_a \circ M_c \neq M_c \circ M_a = R_{120}$ Indication is required of understanding what it means for a group to be non-abelian.	$\{I, M_a\}$, $\{I, M_b\}$, $\{I, M_c\}$ and $\{I, R_{120}, R_{240}\}$. Condone e used for I. The presence of $\{I\}$ can be ignored Could use < > notation. ` Further guidance for marking: **B1** This group can only have (proper, non-trivial) subgroups of order 2 or 3 (justified by Lagrange or complete list explicitly given). **B1** Groups of order 2&3 are necessarily abelian. **B1** This group is not abelian (justified by example or table asymmetry).
3	(e)		$(G, \circ)$ has no elements of order 6. C_6 has (some or two or (at least) one) elements of order 6 (and hence there can be no isomorphism between the two groups.)	**B1** **[1]**	**2.2a**	Or $(G, \circ)$ has three elements of order 2 while C_6 has one. Answer must contain statements relating to numerical **order of element(s)** in **both** groups. **B0** if incorrect statement seen.	NB Answer must use order of elements, not order of subgroups or other explanation (eg "not abelian => not cyclic"). Not just "The orders of elements in the group do not match" but OK if supported by eg 1 2 3 6 G 1 3 2 0 C_6 1 1 2 2 Not just "No element generates G".

Question			Answer	Marks	AO	Guidance	
4	(a)		$\det(\mathbf{P}-\lambda\mathbf{I}) = \begin{vmatrix} 1-\lambda & 7 & 8 \\ -6 & 12-\lambda & 12 \\ -2 & 4 & 8-\lambda \end{vmatrix}$	M1	1.1	Formation of appropriate determinant. May be implied by 2nd **M1**.	
			$= (1-\lambda)\big((12-\lambda)(8-\lambda)-12\times 4\big)$ $-7\big(-6(8-\lambda)-12\times -2)\big)+8\big(-6\times 4-(12-\lambda)\times -2\big)$	M1	1.1	Attempt to expand determinant. Allow one arithmetic or algebraic error.	
			$= (1-\lambda)(96-20\lambda+\lambda^2-48)$ $-7(-48+6\lambda+24)+8(-24+24-2\lambda)$ $= (1-\lambda)(\lambda^2-20\lambda+48)-7(6\lambda-24)+8(-2\lambda)$ $= \lambda^2-20\lambda+48-\lambda^3+20\lambda^2-48\lambda-42\lambda+168-16\lambda$ $= -\lambda^3+21\lambda^2-126\lambda+216=0$ www	A1	1.1	AG. Must expand all brackets (except those with just ± or ±1 outside) before reaching AG.	Must have "= 0".
				[3]			
4	(b)	(i)	$\begin{pmatrix} 1 & 7 & 8 \\ -6 & 12 & 12 \\ -2 & 4 & 8 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$	M1	1.1	Finding **Pe**. Must specify "**Pe** =".	Verification must not start by assuming $\lambda = 3$.
			$= 3\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ (so it is an e-vector) and the e-value is 3	A1	1.1	and equating to λ**e** and comparing. Accept "$\lambda = 3$" as declaration.	Do not accept embedded answer.
			Alternative method: $\begin{pmatrix} 1-\lambda & 7 & 8 \\ -6 & 12-\lambda & 12 \\ -2 & 4 & 8-\lambda \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 3-\lambda \\ -6+2\lambda \\ 6-2\lambda \end{pmatrix}$ soi	M1	1.1	Finding (**P** – λ**I**)**e**. in terms of λ. Could be implied by <u>three</u> equations: $3-\lambda=0$, $-6+2\lambda=0$, $6-2\lambda=0$.	or $\begin{pmatrix} 1 & 7 & 8 \\ -6 & 12 & 12 \\ -2 & 4 & 8 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \lambda\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ giving <u>three</u> correct equations in λ from x, y and z components. Verification must not start by assuming $\lambda = 3$.
			$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ if $\lambda = 3$ (so it is an e-vector) so e-value is 3	A1	1.1	or declaring $\lambda = 3$ as solution to all three equations. .	

Question			Answer	Marks	AO	Guidance	
				[2]			

Question			Answer	Marks	AO	Guidance	
4	(b)	(ii)	$\begin{pmatrix}1&7&8\\-6&12&12\\-2&4&8\end{pmatrix}\begin{pmatrix}x\\y\\5\end{pmatrix}=6\begin{pmatrix}x\\y\\5\end{pmatrix}$ or $\begin{pmatrix}6x\\6y\\6\times5\end{pmatrix}$	M1	3.1a	Using $\mathbf{Pv} = 6\mathbf{v}$ (or $(\mathbf{P} - 6\mathbf{I})\mathbf{v} = \mathbf{0}$)) and $z = 5$. Can be implied by two correct equations in x and y below. Can be awarded if z replaced by 5 later, even after errors.	or $\begin{pmatrix}-5&7&8\\-6&6&12\\-2&4&2\end{pmatrix}\begin{pmatrix}x\\y\\5\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$
			Any **two** of: $x + 7y + 40 = 6x$ $-6x + 12y + 60 = 6y$ $-2x + 4y + 40 = 30$ correct	M1	1.1	Multiplying the vector into the matrix and equating 2 of the components to find 2 correct simultaneous equations in x and y soi by answers. Need not be simplified.	$-5x + 7y = -40$ $(-5x + 7y + 40 = 0)$ $-x + y = -10$ $(-6x + 6y + 60 = 0)$ $-x + 2y = -5$ $(-2x + 4y + 10 = 0)$
			$\Rightarrow x = 15$ & $y = 5$ (so the eigenvector with an eigenvalue of 6 with $z = 5$ is $15\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$).	A1	3.2a	Allow embedded. Not $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ but ISW once $x = 15$ and $y = 5$ seen.	
				[3]			
4	(c)	(i)	$-\mathbf{E}^3 + 7\mathbf{E}^2 - 15\mathbf{E} + 9\mathbf{I} = \mathbf{0}$	M1	1.1	Demonstrating knowledge of C-H theorem. Presence of **I** and "$= \mathbf{0}$" (condone 0) are required. ie $9\mathbf{I}$ must be seen. Do not award **M1** if $\div\mathbf{E}$ seen.	Condone use of eg **M** rather than **E** if recovered. Otherwise, **SCB1** if fully correct answer in "**M**".
			$9\mathbf{I} = \mathbf{E}^3 - 7\mathbf{E}^2 + 15\mathbf{E} \Rightarrow \mathbf{E}^{-1} = \frac{1}{9}(\mathbf{E}^2 - 7\mathbf{E} + 15\mathbf{I})$	A1	1.1	Rearranging and multiplying by $\mathbf{E}^{-1}$ to find an expression for $\mathbf{E}^{-1}$ in terms of $\mathbf{E}^2$, $\mathbf{E}$ (or $\mathbf{E}^1$) and $\mathbf{I}$ (or $\mathbf{E}^0$). Do not ISW. C-H statement can be used to 'eliminate' **I** to reach $\mathbf{E}^{-1} = \frac{1}{9}(\mathbf{E}^2 - 7\mathbf{E} + \frac{5}{3}(\mathbf{E}^3 - 7\mathbf{E}^2 + 15\mathbf{E}))$ $= \frac{1}{27}(3\mathbf{E}^2 - 21\mathbf{E} + 5\mathbf{E}^3 - 35\mathbf{E}^2 + 75\mathbf{E})$ $= \frac{1}{27}(5\mathbf{E}^3 - 32\mathbf{E}^2 + 54\mathbf{E})$ $= \frac{5}{27}\mathbf{E}^3 - \frac{32}{27}\mathbf{E}^2 + 2\mathbf{E}$	$\mathbf{E}^{-1} = \frac{1}{9}\mathbf{E}^2 - \frac{7}{9}\mathbf{E} + \frac{5}{3}\mathbf{I}$ Like terms must be collected. If **M0** then **SCB1** for correct answer. Condone uncancelled fractions. If decimals, must be fully correct ie $0.\dot{1}$, $0.\dot{7}$ and $1.\dot{6}$ for 1/9 etc.

Question			Answer	Marks	AO	Guidance	
				[2]			
4	(c)	(ii)	$\therefore \mathbf{E}^{-1} = \frac{1}{9}\left(\begin{pmatrix}12&11&1\\3&4&-7\\6&6&3\end{pmatrix} - \begin{pmatrix}21&14&7\\7&14&-14\\7&7&14\end{pmatrix} + \begin{pmatrix}15&0&0\\0&15&0\\0&0&15\end{pmatrix}\right)$			Must see this expression oe. eg could see $\begin{pmatrix}3&2&1\\1&2&-2\\1&1&2\end{pmatrix}^2$ for $\begin{pmatrix}12&11&1\\3&4&-7\\6&6&3\end{pmatrix}$.	
			$= \frac{1}{9}\begin{pmatrix}6&-3&-6\\-4&5&7\\-1&-1&4\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{3}&-\frac{1}{3}&-\frac{2}{3}\\-\frac{4}{9}&\frac{5}{9}&\frac{7}{9}\\-\frac{1}{9}&-\frac{1}{9}&\frac{4}{9}\end{pmatrix}$	B1	1.1	Condone uncancelled fractions.	
				[1]			
4	(c)	(iii)	$\mathbf{D} = \begin{pmatrix}6&0&0\\0&12&0\\0&0&3\end{pmatrix}$	B1	1.2	Order of e-vals must be correct. **D** must be explicitly seen.	or $3\begin{pmatrix}2&0&0\\0&4&0\\0&0&1\end{pmatrix}$
			$\mathbf{D}^4 = \begin{pmatrix}6^4&0&0\\0&12^4&0\\0&0&3^4\end{pmatrix}$ $\left(= \begin{pmatrix}1296&0&0\\0&20736&0\\0&0&81\end{pmatrix}\right)$	M1FT	1.1	Correctly finding $\mathbf{D}^4$ from their **D**. FT their 6, 12, 3 in order given, provided diagonal entries are distinct. If correct can be awarded if **D** not seen but can't be inconsistent with **D**.	or $3^4\begin{pmatrix}2^4&0&0\\0&4^4&0\\0&0&1\end{pmatrix}$ or $81\begin{pmatrix}16&0&0\\0&256&0\\0&0&1\end{pmatrix}$
			$\mathbf{P}^4 = \begin{pmatrix}3&2&1\\1&2&-2\\1&1&2\end{pmatrix}\begin{pmatrix}6^4&0&0\\0&12^4&0\\0&0&3^4\end{pmatrix}\begin{pmatrix}3&2&1\\1&2&-2\\1&1&2\end{pmatrix}^{-1}$	M1	1.1	For $\mathbf{P}^4 = \mathbf{E}\mathbf{D}^4\mathbf{E}^{-1}$ which can be implied, if required, by forming their $\mathbf{E}\mathbf{D}^4\mathbf{E}^{-1}$ provided that **D** or $\mathbf{D}^4$ is diagonal.	Must relate to 4, not n.
			$= \begin{pmatrix}3888&41472&81\\1296&41472&-162\\1296&20736&162\end{pmatrix} \times \frac{1}{9}\begin{pmatrix}6&-3&-6\\-4&5&7\\-1&-1&4\end{pmatrix}$ $= \begin{pmatrix}3&2&1\\1&2&-2\\1&1&2\end{pmatrix} \times \begin{pmatrix}864&-432&-864\\-9216&11520&16128\\-9&-9&36\end{pmatrix}$				$3^4\begin{pmatrix}48&512&1\\16&512&-2\\16&256&2\end{pmatrix} \times \frac{1}{9}\begin{pmatrix}6&-3&-6\\-4&5&7\\-1&-1&4\end{pmatrix}$ $= 27\begin{pmatrix}-587&805&1100\\-650&838&1160\\-310&410&568\end{pmatrix}$
			$= \begin{pmatrix}-15849&21735&29700\\-17550&22626&31320\\-8370&11070&15336\end{pmatrix}$	A1	1.1		

Question			Answer	Marks	AO	Guidance	
				[4]			

Question			Answer	Marks	AO	Guidance	
5	(a)		Assume that $3 = 2^{\frac{a}{b}}$ for some $a, b \in \mathbb{Z}$ $(b \neq 0)$. So $2^a = 3^b$ which gives $a = b = 0$ from the given statement which is not a valid solution to the original equation (or which cannot be so as $b \neq 0$) and hence is a contradiction. Hence our original assumption must be wrong.	**M1** **A1** **[2]**	**3.1a** **3.2a**	Making initial assumption. Raising both sides to the power of b and expressing in a form from which the given statement can be used to show the contradiction. Argument must be complete. Could use different argument (eg LHS is even for $a > 0$ while RHS is not).	$a, b \in \mathbb{Z}$ must be seen *before* $a = 0$ & $b = 0$ claimed from given statement. If **M0** then **SCB1** if correct argument made except $a, b \in \mathbb{Z}$ missed or late. Argument must not be based on eg $\log_2 3$ being irrational.
5	(b)		If $\log_2 3 \in \mathbb{Q}$ then $\log_2 3 = \frac{a}{b}$ for some integers a and b. $\Rightarrow 3 = 2^{\frac{a}{b}}$ for $a, b \in \mathbb{Z}$ which, (from **(a)**), is a contradiction and so $\log_2 3 \notin \mathbb{Q}$.	**M1** **A1** **[2]**	**3.1a** **3.2a**	Making initial assumption and using definition of rationality	If not by contradiction, argument could run: (From (a)) It is not possible to find a pair $a, b \in \mathbb{Z}$ such that $3 = 2^{\frac{a}{b}}$ and $3 = 2^{\frac{a}{b}}$ is equivalent to $\log_2 3 = \frac{a}{b}$ (**M1**). Completing argument **A1**. But it must be clear that no such integers a and b can exist, not simply that $3 \neq 2^{\frac{a}{b}}$ for some a and b. Argument must **not** depend on $a, b \notin \mathbb{Z} \Rightarrow \frac{a}{b} \notin \mathbb{Q}$. Argument must be rigorous and contain proper conclusion. If **M0** then **SCB1** if correct argument made except $a, b \in \mathbb{Z}$ missed.

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