Oxford Cambridge and RSA

Friday 14 June 2024 – Afternoon

A Level Further Mathematics A

Y543/01 Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 A particle P of mass 12.5 kg is moving on a smooth horizontal plane when it collides obliquely with a fixed vertical wall.

At the instant before the collision, the velocity of P is $-5\mathbf{i}+12\mathbf{j}\ \mathrm{m\,s^{-1}}$.

At the instant after the collision, the velocity of P is $\mathbf{i}+4\mathbf{j}\ \mathrm{m\,s^{-1}}$.

(a) Find the magnitude of the momentum of P **before** the collision. **[2]**

(b) Find, in vector form, the impulse that the wall exerts on P. **[2]**

(c) State, in vector form, the impulse that P exerts on the wall. **[1]**

(d) Find in either order.

- The magnitude of the impulse that the wall exerts on P.
- The angle between $\mathbf{i}$ and the impulse that the wall exerts on P. **[3]**

2 One end of a light elastic string of natural length 1.4 m and modulus of elasticity 20 N is attached to a small object B of mass 2.5 kg. The other end of the string is attached to a fixed point O.

Object B is projected vertically upwards from O with a speed of $u\ \mathrm{m\,s^{-1}}$.

(a) State **one** assumption required to model the motion of B. **[1]**

The greatest height above O achieved by B is 8.1 m.

(b) Determine the value of u. **[4]**

3 The mass of a truck is 6000 kg and the maximum power that its engine can generate is 90 kW. In a model of the motion of the truck it is assumed that while it is moving the total resistance to its motion is constant.

At first the truck is driven along a straight horizontal road. The greatest constant speed that it can be driven at when it is using maximum power is $25\ \mathrm{m\,s^{-1}}$.

(a) Find the value of the resistance to motion. **[2]**

The truck is being driven along the horizontal road with the engine working at 60 kW.

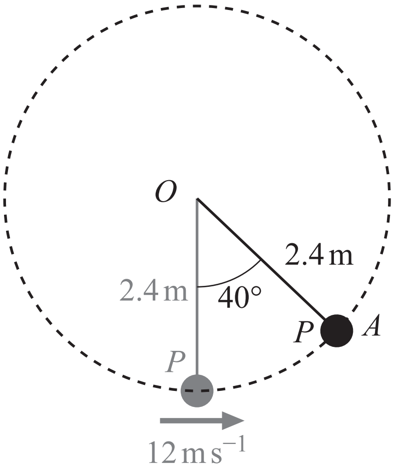
(b) Find the acceleration of the truck at the instant when its speed is $10\ \mathrm{m\,s^{-1}}$. **[2]**

The truck is now driven **down** a straight road which is inclined at an angle θ below the horizontal. The greatest constant speed that the truck can be driven at maximum power is $40\ \mathrm{m\,s^{-1}}$.

(c) Determine the value of θ. **[3]**

4 A particle, P, of mass 6 kg is attached to one end of a light inextensible rod of length 2.4 m. The other end of the rod is smoothly hinged at a fixed point O and the rod is free to rotate in any direction.

Initially, P is at rest, vertically below O, when it is projected horizontally with a speed of $12\,\text{m s}^{-1}$. It subsequently describes complete vertical circles with O as the centre.



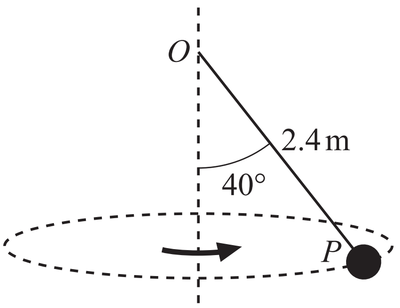
The angle that the rod makes with the downward vertical through O at each instant is denoted by θ and A is the point which P passes through where $\theta = 40°$ (see diagram).

(a) Find the tangential acceleration of P at A, stating its direction. **[2]**

(b) Determine the radial acceleration of P at A, stating its direction. **[6]**

(c) Find the magnitude of the force in the rod when P is at A, stating whether the rod is in tension or compression. **[2]**

The motion is now stopped when P is at A, and P is then projected in such a way that it now describes horizontal circles at a constant speed with $\theta = 40°$ (see diagram).



(d) Find the speed of P. **[4]**

(e) Explain why, wherever P's motion is initiated from and whatever its initial velocity, it is **not** possible for P to describe horizontal circles at constant speed with $\theta = 90°$. **[1]**

5 In this question you may assume that if x and y are any physical quantities then $\left[\frac{dy}{dx}\right] = \left[\frac{y}{x}\right]$.

A machine drives a piston of mass m into a vertical cylinder. The equation below is suggested to model the power developed by the machine, P, while it is not doing any other external work.

$$P = k_1 mv\frac{dv}{dt} + k_2 mgv + k_3 E$$

in which

- v is the velocity of the piston at a given time,
- g is the acceleration due to gravity,
- E is the **rate** at which heat energy is lost to the surroundings,
- k_1, k_2 and k_3 are dimensionless constants.

Determine whether the equation is dimensionally consistent. Show **all** the steps in your argument. **[6]**

6 Two identical spheres, A and B, each of mass m kg, are moving directly towards each other along the same straight line on a smooth horizontal surface until they collide. Just before they collide, the speeds of A and B are $20\,\text{m}\,\text{s}^{-1}$ and $10\,\text{m}\,\text{s}^{-1}$ respectively. The coefficient of restitution between A and B is e.

(a) By finding, in terms of e, an expression for the velocity of B after the collision, show that the direction of motion of B is reversed by the collision. **[5]**

After the collision between A and B, which is **not** perfectly elastic, B goes on to collide directly with a fixed, vertical wall. The coefficient of restitution between B and the wall is $\frac{2}{5}e$. After the collision between B and the wall, there are no further collisions between A and B.

(b) Determine the range of possible values of e. **[7]**

7 A body B of mass 1.5 kg is moving along the x-axis. At the instant that it is at the origin, O, its velocity is $u\,\mathrm{m\,s^{-1}}$ in the positive x-direction.

At any instant, the resistance to the motion of B is modelled as being directly proportional to v^2 where $v\,\mathrm{m\,s^{-1}}$ is the velocity of B at that instant. The resistance to motion is the only horizontal force acting on B.

At an instant when B's velocity is $2\,\mathrm{m\,s^{-1}}$, the resistance to its motion is 24 N.

(a) Show that B's motion can be modelled by the differential equation $\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} = -4$. **[3]**

(b) **(i)** Solve the differential equation in part **(a)** to find the particular solution for v in terms of x and u. **[4]**

(ii) By considering the behaviour of v as $x \to \infty$ describe **one** feature of the model that is not realistic. **[1]**

At the instant when B reaches the point A, where $x = X$, its speed is $V\,\mathrm{m\,s^{-1}}$. The work done by the resistance as B moves from O to A is denoted by W J.

(c) **(i)** Use the formula $W = \int F\,\mathrm{d}x$ to determine an expression for W in terms of X and u. **[3]**

(ii) Explain the relevance of the sign of your answer in part **(c)(i)**. **[1]**

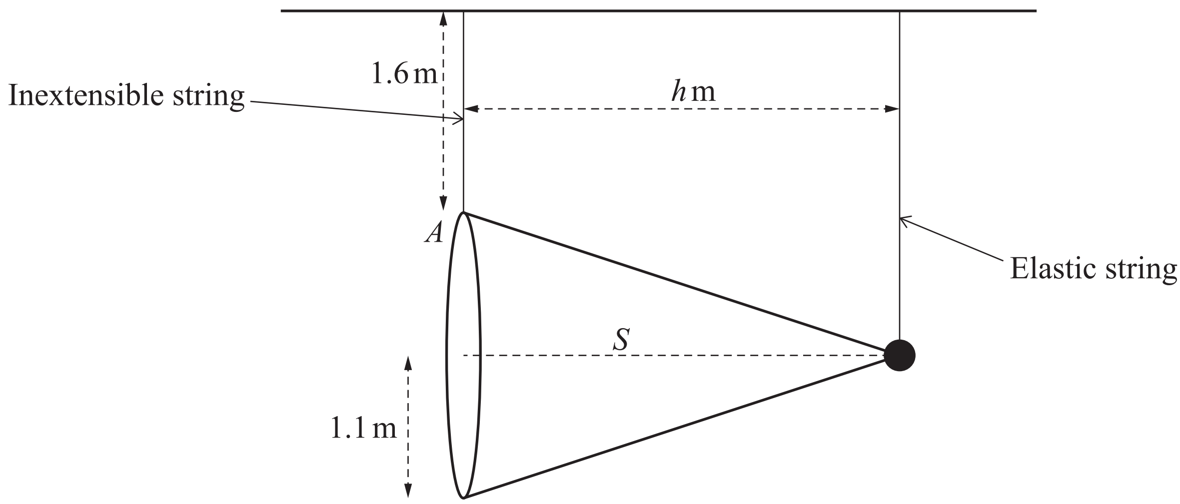
(iii) By writing your answer to part **(c)(i)** in terms of V and u show how the quantity W relates to the energy of B. **[2]**

8 A shape, S, is formed by attaching a particle of mass $2m$ kg to the vertex of a uniform solid cone of mass $8m$ kg. The height of the cone is h m and the radius of the base of the cone is 1.1 m.

(a) Explain why the centre of mass of S must lie on the central axis of the cone. **[1]**

Two strings are attached to S, one at the vertex of the cone and one at A which is a point on the edge of the base of S. The other ends of the strings are attached to a horizontal ceiling in such a way that the strings are both vertical. The string attached to S at A is inextensible and has length 1.6 m. The string attached to S at the vertex is elastic with modulus of elasticity $8mg$ N.

Shape S is in equilibrium with its axis horizontal (see diagram).



(b) Determine the natural length of the elastic string. **[7]**

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