



GCE

Further Mathematics A

Y535/01: Additional pure mathematics

AS Level

Mark Scheme for June 2024

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Question		Answer	Marks	AO	Guidance
1		$28A3B_{12} = 11 + (3 \times 12) + (10 \times 12^2) + (8 \times 12^3) + (2 \times 12^4)$ $= 56783$	M1 A1 [2]	1.1 1.1	DR Use of powers of 12 with $A = 10$ and $B = 11$. Condone one error in the expansion of powers of 12. Values of A and B can be implied by correct answer below.

Question		Answer	Marks	AO	Guidance
2	(a)	Attempt at vector product of their a and b : $\begin{pmatrix} 2 \\ 4 \\ 9 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$ $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 60 \\ 15 \\ -20 \end{pmatrix}$ $ \mathbf{a} \times \mathbf{b} = \sqrt{60^2 + 15^2 + (-20)^2}$ $\text{Area } \triangle OAB = \frac{1}{2} \mathbf{a} \times \mathbf{b} = \frac{65}{2} \text{ (square units)}$	M1* A1 M1dep* A1	1.1 1.1 1.1 1.1	Must be clear that it is the vector product being attempted. Can be implied by two correct components. If using Area $\triangle OAB = \frac{1}{2} \mathbf{a} \mathbf{b} \sin \theta$, attempt at $ \mathbf{a} $, $ \mathbf{b} $ and θ via the scalar product. Or $\mathbf{b} \times \mathbf{a}$ If using Area $\triangle OAB = \frac{1}{2} \mathbf{a} \mathbf{b} \sin \theta$, $ \mathbf{a} = \sqrt{101}$, $ \mathbf{b} = \sqrt{61}$, $\theta = 55.9 \dots^{\circ}$ Attempt at the magnitude of their $\mathbf{a} \times \mathbf{b}$ so Or Area $\triangle OAB = \frac{1}{2} \sqrt{101} \times \sqrt{61} \sin 55.9 \dots^{\circ}$ Triangle formula used CAO
			[4]		

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Question			Answer	Marks	AO	Guidance
2	(b)		$\begin{pmatrix} 2 \\ 4 \\ 3\lambda \end{pmatrix} \times \begin{pmatrix} \lambda \\ -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 24 + 12\lambda \\ 3\lambda^2 - 12 \\ -8 - 4\lambda \end{pmatrix} = \mathbf{0}$	M1	1.1	Vector product attempted and result equated to 0 . Allow sign errors. Condone calculation of only one correct component equated to 0 leading to $\lambda = -2$.
			$\lambda = -2$	A1	1.1	A check that remaining two components are 0 is required. www
			ALT. Want $\mathbf{b} = m\mathbf{a}$ (or $\mathbf{a} // \mathbf{b}$) and attempt at $m = \frac{\lambda}{2} = \frac{-4}{4} = \frac{6}{3\lambda}$	M1		
			$\lambda = -2$	A1		
				[2]		

Question			Answer	Marks	AO	Guidance
3	(a)		$f_x = 8xy - 6y^2 - \frac{1}{3}x^3$	B1	1.1	
			$f_y = 4x^2 - 12xy$	B1	1.1	
			$f_y = 0 \Rightarrow x = 3y$ (Noting $x \neq 0$ not required)	M1	1.1	Establishing a relationship between x and y by setting one first partial derivative to zero
			$f_x = 0 \Rightarrow 8.3y.y - 6y^2 - \frac{1}{3}.27y^3 = 0$	M1	1.1	Substituting correctly for first variable in second partial derivative set to zero and solving attempt of some cubic polynomial
			$\Rightarrow b = 2$	A1	1.1	Second variable value CAO
			$\Rightarrow a = 6$ and $c = 36$	A1	1.1	Both remaining values correct Condone x, y, z for a, b, c throughout

Question			Answer	Marks	AO	Guidance
				[6]		
	(b)	i	At U_1 , $x = 5.9, y = 2$, $f_x = 1.94 (= 8 \times 5.9 \times 2 - 6 \times 2^2 - \frac{1}{3} \times 5.9^3)$ At U_2 , $x = 6.1, y = 2$, $f_x = -2.06 (= 8 \times 6.1 \times 2 - 6 \times 2^2 - \frac{1}{3} \times 6.1^3)$	B1 B1 [2]	1.1 1.1	If B0B0, SCB1 for both correct substitutions
		ii	At V_1 , $x = 6, y = 1.9$, $f_y = 7.2 (= 4 \times 6^2 - 12 \times 6 \times 1.9)$ At V_2 , $x = 6, y = 2.1$, $f_y = -7.2 (= 4 \times 6^2 - 12 \times 6 \times 2.1)$	B1 B1 [2]	1.1 1.1	If B0B0, SCB1 for both correct substitutions
		iii	$\cap$ -shaped parabola on an x - z set of axes labelled or implied by an equation in (x,z) or $(x, f(x,2))$ $\cap$ -shaped parabola on a y - z set of axes labelled or implied by an equation in (y,z) or $(y, f(6,y))$	B1 B1 [2]	1.1 1.1	Condone positional vaguenesses Ditto

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4	(a)	$\begin{aligned} F_{n+1} &= F_n + F_{n-1} & F_{n+4} &= F_{n+3} + F_{n+2} \\ F_{n+2} &= 2F_n + F_{n-1} & &= 2F_{n+2} + F_{n+1} \\ F_{n+3} &= 3F_n + 2F_{n-1} & &= 3F_{n+1} + 2F_n \end{aligned}$ or $F_{n+4} = 5F_n + 3F_{n-1}$	M1 A1 [2]	2.1 2.2a	Fibonacci recurrence relation used correctly at least twice CAO If M0 , SCB1 for trial and error with F_r and F_s checked for $r, s \geq 6$
	(b)	When $n = 4$, $F_4 = 3$ which is a multiple of 3 so result is true (or $n = 8$, $F_8 = 21$) Assume the result is true for $n = k$, where k is a multiple of 4 (or $n=4k$) Therefore $F_k = 3N$ (or $F_{4k} = 3N$) where N is a positive integer or F_k (or F_{4k}) is a multiple of 3 Using part (a), $F_{k+4}(F_{4k+4}) = (their\ 5)\ F_r + (their\ 3)\ F_s$, r, s in terms of k, $4k - 4 < r, s < 4k + 4$, $r \neq s$ $F_{k+4} = 3(5N + F_{k-1}) \text{ or } F_{4k+4} = 3(5N + F_{4k-1})$ which is also a multiple of 3 Since $F_4 = 3$ is a multiple of 3 and F_k a multiple of 3 when k is a multiple of 4 $\Rightarrow F_{k+4}$ a multiple of 3, it follows by induction that F_n is always a multiple of 3 if n is a multiple of 4	B1 M1 A1	1.1 2.1 2.4	Basic case Clearly reasoned inductive step (no need to note that all terms are integers) Fully correct conclusion with consistent use of subscripts www

Question		Answer	Marks	AO	Guidance
			[3]		

Question		Answer	Marks	AO	Guidance
5	(a)	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1 [1]	1.2	No need to observe that $\det(\mathbf{I}) = 1$ Accept I
	(b)	G consists of the powers of P , which has order 6 $(n =)6$	B1 B1 [2]	2.4 2.2a	oe such as $\mathbf{P}^6 = \mathbf{I}$. Can be implied by explicit calculation of $\mathbf{P}^6$. Or recognising that P is a clockwise rotation through 60° . Allow unsupported statement and without mention that no lower power of P gives I .
	(c) i	$\mathbf{Q}^2 = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix} (= \mathbf{RQR})$ $\mathbf{QR} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$ or $\mathbf{RQ} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$ $\mathbf{QR} = \mathbf{RQ}^2$ and $\mathbf{RQ} = \mathbf{Q}^2\mathbf{R}$ $\mathbf{Q} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \mathbf{I}$ and $m = 6$	B1 B1 B1 B1	1.1 1.1 1.1 2.2a	All six elements must be seen (explicitly as matrices or in terms of Q and R) and no others for this B1 . (Condone failure to note that $\mathbf{Q}^3 = \mathbf{R}^2 = \mathbf{I}$.)

Question		Answer	Marks	AO	Guidance
			[4]		
	(c) ii	One of $\{\mathbf{R}, \mathbf{I}\}$ (or $\{\mathbf{R}, \mathbf{R}^2\}$), $\{\mathbf{Q}, \mathbf{Q}^2, \mathbf{I}\}$ (or $\{\mathbf{Q}, \mathbf{Q}^2, \mathbf{Q}^3\}$), $\{\mathbf{QR}, \mathbf{I}\}$, $\{\mathbf{RQ}, \mathbf{I}\}$ Another correct subgroup. All correct and no others.	B1 B1 B1 [3]	2.2a 1.2 1.1	One correct subgroup Ignore $\{\mathbf{I}\}$ and/or H if mentioned. Alternative forms may be used or matrices given instead. Condone absence of brackets.
5	(d)	True: Cyclic $\Rightarrow$ Abelian, so G is Abelian or All powers of a single element commute $\Rightarrow$ Abelian True: G is cyclic, generated by $\mathbf{P}$ False: H is not Abelian, since (e.g.) $\mathbf{QR} \neq \mathbf{RQ}$ False: Not Abelian $\Rightarrow$ not cyclic, so H is not cyclic or There is no generator in H	B1 B1 B1 B1 [4]	2.4 2.5 2.2a 2.4	Each answer must have a valid reason supporting it

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6	(a)	i	$n = 9k + 3$ and $n = 9k + 6$ or $n \equiv 3 \pmod{9}$ and $n \equiv 6 \pmod{9}$	B1 [1]	1.2	Allow $n = 9k \pm 3$ Condone $9n + 3$ and $9n + 6$ (or $9n - 3$), not $n = 9k + r$
		ii	For $n = 9k + 3$, $f(n) = 1 + 2^{9k+3} + 4^{9k+3} = 1 + 8.(2^9)^k + 64.(2^9)^{2k}$ $2^9 = 512 \equiv 1 \pmod{73}$ so that $f(n) \equiv 1 + 8.(1)^k + 64.(1)^{2k} \pmod{73}$ $= 1 + 8 + 64 = 73 \equiv 0 \pmod{73}$ and $f(n)$ is a multiple of 73 Similarly, for $n = 9k + 6$, $f(n) = 1 + 64.(2^9)^k + 4096.(2^9)^{2k}$ so that $f(n) \equiv 1 + 64.(1)^k + 8.(1)^{2k} = 1 + 64 + 8 = 73 \equiv 0 \pmod{73}$ and $f(n)$ is a multiple of 73	M1 B1 M1 A1 M1 A1 [6]	3.1a 1.1 3.1a 3.2a 1.1 3.2a	Writing 2^{r+3} as $2^r \times 2^3$ and 4^{r+3} as $4^r \times 4^3$ Noted or used at any stage NB $262144 = 512^2$ Considering $f(n) \pmod{73}$ by substituting $(2^9)^k$ and $(2^9)^{2k}$ with 1 All correctly concluded Writing 2^{r+6} as $2^r \times 2^6$ and 4^{r+6} as $4^r \times 4^6$ (or suitably modified for $n = 9k - 3$) From fully correct working, including noting that $4096 \equiv 8 \pmod{73}$ NB $n = 9k + 6$ could be considered first.
	(b)		If $n = 9k$, then $f(n) = 1 + 512^k + (512^2)^k$ or $1 + 512^k + (262144)^k$ $\equiv 1 + 1^k + (1^2)^k = 3 \pmod{73}$	M1 A1 [2]	1.1 2.2a	Correct set up using indices CAO It must be clear that this is always the case If M0 , SC1 substituting $n=9$ and evaluating mod 73 to find 3: $f(9) = 1 + 2^9 + 4^9 (=262657) \equiv 3 \pmod{73}$

Question		Answer	Marks	AO	Guidance
7	(a)	If $E_0 = H_0 = 1200$ and $H_{n+1} = \left(1 - \frac{1}{8}\right) H_n$, then $H_{n+6} = \left(\frac{7}{8}\right)^6 H_n$	B1	3.3	H_n being the hourly amount of enzyme Adequately explained in words and/or symbols.
		and $E_{n+1} = \left(\frac{7}{8}\right)^6 E_n$ plus a ‘boost’ of 500	B1	2.4	
			[1]		
	(b)	Gen. Soln. is given by $(E_n =)$ CS (+) PS	M1	1.1	Including good attempts at $CS = a \times r^n$ (with 1 arbitrary constant) and $PS = \text{constant}$ Must be $E_n = CS + PS$ PS must be numerical Allow approximate numerical equivalents; e.g. $292.9 \times 0.4488^n + 907.1$
		$CS = a \left[\left(\frac{7}{8}\right)^6\right]^n$	A1	1.1	
		$PS = \frac{500}{1 - \left(\frac{7}{8}\right)^6}$	A1	1.1	
		Use of $E_0 = 1200$ to evaluate their “a”	M1	3.4	
		$E_n = \left(1200 - \frac{500}{1 - \left(\frac{7}{8}\right)^6}\right) \times \left(\frac{7}{8}\right)^{6n} + \frac{500}{1 - \left(\frac{7}{8}\right)^6}$	A1	1.1	
			[5]		
	(c)	$\left(\frac{7}{8}\right)^n$ or $\left(\frac{7}{8}\right)^{6n} \rightarrow 0$ as $n \rightarrow \infty$	B1	2.4	Condone explanation that $\left(\frac{7}{8}\right)^n$ or $\left(\frac{7}{8}\right)^{6n}$ becomes small (negligible) as $n \rightarrow \infty$. Correct to 3sf
		$\Rightarrow E_n \rightarrow 907$	B1	2.2b	
			[2]		

Question		Answer	Marks	AO	Guidance
7	(d)	From $H_0 = 1200$, the hourly sequence H_n runs 1050, 918.75, 803.91, 703.42, ... Attaining $H_6 = 538.55 \rightarrow E_1 = \mathbf{1038(.55)}$	M1	3.1a	DR Breaking the problem down into single 1-hour steps, multiplying by 0.875 each time. Making the jump from the end of the first six-hour period to the start of the second. Sight of $1038(.55)$ or $1200 \times \left(\frac{7}{8}\right)^6 + 500$ FT their solution from part (b)
		Continuing one hour at a time; so $H_7 = 908.7$, $H_8 = 795.14$, $H_9 = 695.75$, $H_{10} = 608.78$, $H_{11} = 532.68$, $H_{12} = \mathbf{466.10}$	A1	3.5a	Continue as before to H_{12} . Sight of $466(.10)$ with (numerical) justification. FT their solution from part (b) if their $H_{12} < 500$
		Explaining that, just before the second 500 (mg) is added, the amount of enzyme has fallen below the required minimum amount, so the experiment's validity has failed	A1	3.5b	' $466 < 500$ so requirement not met'
			[3]		

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