GCE

Further Mathematics A

Y545/01: Additional Pure Mathematics

A Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

   YOU MUST MARK THE REQUIRED NUMBER OF PRACTICE AND STANDARDISATION RESPONSES BEFORE YOU CAN BE APPROVED TO MARK LIVE SCRIPTS.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.

5. The RM Assessor **comments box** is used by your team leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.**
   If you have any questions or comments for your team leader, use the phone, the RM Assessor messaging system, or e-mail.

6. Assistant Examiners will send a brief report on the performance of candidates to their Team Leader (Supervisor) via email by the end of the marking period. The report should contain notes on particular strengths displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

7. **Annotations**

Annotation	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

8. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
   If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AO	Guidance
1	(a)	Even (or odd)-numbered places sum to $(a + b) \times$ no. of blocks	M1	2.1	Or considering a single block: $a - b + b - a = 0$ (terms must be in this order, may be embedded but must be identifiable, *eg* 'Consider *abba*…')
		Odd (or even)-numbered places sum to $(a + b) \times$ no. of blocks	A1	1.1	Considering all blocks $a - b + b - a + a - b + b - a + \cdots = 0$ (at least two blocks and …) or a clear explanation in words
		Difference is 0, and **11 | 0** $\Rightarrow$ 11 | *N*	A1	2.2a	**M1A0A1** is possible
		SCB1 Each block "*abba*" is of the form $1001a + 110b = 11(91a + 10b)$ is divisible by 11 $\Rightarrow$ *N* is			For fully correct working using method not asked for
			[3]		
	(b)	Each block "*cddc*" = $cn^3 + dn^2 + dn + c(n^0)$ or $cn^k + dn^{k-1} + dn^{k-2} + cn^{k-3}$	M1	2.1	Expressing each block in base *n* (could be seen as part of the expansion of *M*)
		$= (n + 1)(n^2 + \alpha n + 1)c + n(n + 1)d$ or $= n^{k-3}((n + 1)(n^2 + \alpha n + 1)c + n(n + 1)d)$	M1	3.1a	Attempt to factor $(n + 1)$ out of their polynomial in *n*; if using the *Factor theorem*, $c-d + d - c$ must be seen
		$(r)(n + 1)((n^2 - n + 1)c + n\,d)$ (r an integer) $\Rightarrow$ *M* always a multiple of $(n + 1)$	A1	2.4	Correct answer from fully correct working. If using the *Factor theorem*, $c-d + d - c = 0$ must be seen
			[3]		
2		$\frac{\partial z}{\partial x} = 4\sqrt{y} - \frac{y}{2\sqrt{x}}$	M1*	1.1	Attempt at first partial derivative(s) $\frac{\partial z}{\partial x} = k_1\sqrt{y} + k_2\frac{y}{\sqrt{x}}$ and $\frac{\partial z}{\partial y} = k_3\frac{x}{\sqrt{y}} + k_4\sqrt{x} + k_5 y$ $k_r \neq 0$, $r = 1, \ldots 5$

Question		Answer	Marks	AO	Guidance
		$\frac{\partial z}{\partial y} = \frac{4x}{2\sqrt{y}} - \sqrt{x} + 2y$	**A1**	**1.1**	Both correct
		At P, $\frac{\partial z}{\partial x} = 6$ and $\frac{\partial z}{\partial y} = 8$	**A1**	**1.1**	Substituting coordinates correctly in both partial derivatives (can be implied by a correct tangent plane equation)
		Tangent plane eqn. is $z - 20 = 6(x-1) + 8(y-4)$	**M1dep***	**1.1**	Attempt at plane equation using this form and *their* numerical gradients
		$6x + 8y - z = 18$	**A1**	**1.1**	Equation in the form $ax + by + cz = d$ (or $d = ax + by + cz$)
		ALT for the last two marks $\boldsymbol{n} = \begin{pmatrix}1\\0\\6\end{pmatrix} \times \begin{pmatrix}0\\1\\8\end{pmatrix} = \begin{pmatrix}-6\\-8\\1\end{pmatrix}$ and $\boldsymbol{n} \cdot \begin{pmatrix}1\\4\\20\end{pmatrix} = -18$	**M1dep***		Finding the normal vector to the plane and k in $\boldsymbol{n} \cdot \boldsymbol{r} = k$
		$6x + 8y - z = 18$	**A1**		Equation in the form $ax + by + cz = d$ (or $d = ax + by + cz$)
			[5]		

Question		Answer	Marks	AO	Guidance
3		eg $x = 7u + 1 = 37v + 22$	**M1**	**3.1a**	Setting up a relationship between two "bases"
		$\Rightarrow x = 259a + 22$	**M1**	**2.2a**	Solving to parameterise one variable. Condone arithmetic errors in '22'. Must be mod product of the two 'bases'

Question	Answer	Marks	AO	Guidance
	$\Rightarrow$ *their* $259a + 22 \equiv 7 \pmod{67}$	**M1**	**3.1a**	Attempt to relate result to the third "base" oe
	their $a \equiv 24 \pmod{67}$ or $a = 67n + 24$ $\Rightarrow x = 259(67n + 24) + 22$	**M1**	**1.1**	Calculating a in explicit form and substituting back for x
	$x = 17\,353n + 6238$	**A1**	**1.1**	cao **NB** any letter for "n" acceptable
	hcf(7, 37) = 1 or hcf(3, 37) = 1 or …	**B1**	**2.4**	One justification explicitly stated
	ALT I - 1st 2-marks (first relationship sorted)			
	$x \equiv 1 \pmod 7 \Leftrightarrow x \equiv 22 \pmod 7$	**M1**		Setting up a relationship between two "bases"
	$x \equiv 22 \pmod{7 \times 37 = 259}$	**M1**		Attempt to relate result to the third "base"
	ALT II - Using the Chinese Remainder Theorem			
	$x = 2479\,m + 259\,l + 469\,k \pmod{7\times67\times37}$	**M1**		
	$2479\,m \equiv 1 \pmod 7 \Rightarrow m \equiv 1 \pmod 7$	**M1**		Calculating m in explicit form
	$259\,l \equiv 7 \pmod{67} \Rightarrow l \equiv 29 \pmod{67}$	**M1**		Calculating l in explicit form
	$469\,k \equiv 22 \pmod{37} \Rightarrow k \equiv 29 \pmod{37}$	**M1**		Calculating k in explicit form
	$x = 2479 \times 1 + 259 \times 29 + 469 \times 29 = 23591$ $\equiv 6238 \pmod{17353}$	**A1**		Substituting m, l, and k into x and stating equivalence (mod 17353)
	A solution exists because 7, 67, 37 are co-prime or hcf(7, 67) = 1 or hcf(7, 37) = 1 or …	**B1**		One justification explicitly stated
		[6]		

Question		Answer	Marks	AO	Guidance
4	(a)	Considering **a** = **0** and/or **b** = **0**	**M1**	**1.1**	Soi by subsequent working. Allow |**a**|=0 and |**b**|=0. Not $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$ or $\boldsymbol{a} \cdot \boldsymbol{b} = 0$
		$(p, q, r) = (1, -2, \frac{3}{2})$ when **a** = **0**	**A1**	**1.1**	Or when |**a**|=0
		$(p, q, r) = (-2, \frac{5}{2}, -3)$ when **b** = **0**			Or when |**b**|=0 And no others.
			A1	**1.1**	If **A0A0**, **SCB1** if values for (p, q, r) are 'ungrouped' but the two sets $(1, -2, \frac{3}{2})$ and $(-2, \frac{5}{2}, -3)$ can be distinguished www
			[3]		
	(b)	$\lambda = \frac{2p+4}{p-1} = \frac{2q-5}{q+2} = \frac{r+3}{2r-3}$	**M1**	**2.1**	Noted with an attempt to rearrange for p or q or r
		$p = \frac{\lambda+4}{\lambda-2}$, $q = -\frac{2\lambda+5}{\lambda-2}$ and $r = \frac{3\lambda+3}{2\lambda-1}$	**A1**	**1.1**	For any one correct expression
		E.g. $p = 1 + \frac{6}{\lambda-2}$ and p an integer $\Rightarrow (\lambda-2) \mid 6$	**M1**	**3.1a**	Method for determining values of λ in any one case

Question		Answer	Marks	AO	Guidance
		E. g. $q = -2 - \dfrac{9}{\lambda - 2}$ and q an integer $\Rightarrow$ $(\lambda - 2) \mid 9$	**M1**	**1.1**	Method for determining values of λ in a second case
		$\Rightarrow$ $(\lambda - 2 = 3, 1, -1, -3 \Rightarrow)$ one of $\lambda = 5,\ 3,\ 1,\ -1$	**A1**	**2.2a**	At least one of the possible values of λ; candidates may note that λ must be odd when considering $q \in \mathbb{Z}$ www
		$\lambda = 5$ gives $(p, q, r) = (3, -5, 2)$	**A1**	**1.1**	Selecting $\lambda = 5$
		Other cases visibly rejected: since $\lvert\lambda\rvert > 1$ and $\lambda = 3$ gives $r = \frac{12}{5}$ (and $p = 5,\ q = -11$)	**A1**	**3.2a**	Must be correct cases rejected for correct reasons If (**M1A1**)**M0M0**, **SCB1** $\lambda = 5$ www **SCB1** $(p, q, r) = (3, -5, 2)$ www
			[7]		

Question			Answer	Marks	AO	Guidance
5	**(a)**		Solving $10\,000 = 20\,000(1 - 10\,000k)$ $\Rightarrow$ $k = \frac{1}{20000}$ or $0.000\,05$	**M1** **A1** **[2]**	**1.1** **1.1**	**AG** If **M0**, **SCB1** for verifying that $k = 0.000\,05$ with $10\,000$ substituted
	(b)		P_n converges to $10\,049$ (as there are no part animals)	**B1** **[1]**	**2.2a**	Condone $10\,050$, if $10\,049.75\ldots$ or $10\,049.8$ is seen, either as the value of any P_k , $k > 1$, or obtained by equating P_n and P_{n+1}
	(c)	**i**	$P_{n+1} = 2P_n(1 - 0.00005P_n) - 2400$ (for $n \geq 0$)	**B1** **[1]**	**3.3**	

Question			Answer	Marks	AO	Guidance
		ii	P_n **decreases** (monotonically) (each year)	**B1** **[1]**	**2.5**	Accept 'Population will decrease'. Not 'Population will decline'. Ignore any statement on long-term behaviour, such as 'P_n approaches a limiting value' **or** '$P_n \to 6000$'
		iii	$x = 2x(1-0.00005x) - 2400$ $\Rightarrow 0.0001x^2 - x + 2400 = 0 \Rightarrow x = 6000, 4000$ Reject 4000, so 6000 since when first equilibrium point is reached, P_n does not continue to decrease	**M1** **A1** **A1** **[3]**	**3.4** **1.1** **2.3**	Setting up "fixed-term" equation and attempt to solve Correct answer chosen and other rejected (the word 'decrease' may be seen in part **(c) (ii)**). Condone selecting 6000 with justification and not explicitly rejecting 4000. '$P_0 = 10\,000$ and P_n decreases, so in the long term $P_n \to 6000$' Accept '$4000 < 6000$' as justification only if P_n is stated as decreasing in part **(c) (ii)**. NB **M0** for non-algebraic method
		iv	Solving $10\,000 = 20\,000(1 - 10\,000k) - 2400$ $\Rightarrow k = 0.000\,038$ oe	**M1** **A1** **[2]**	**3.5c** **1.1**	NB **M0A0** for answer with no working

Question		Answer	Marks	AO	Guidance
6	(a)	$\frac{\partial z}{\partial x} = 2x + ay \ldots$	M1*	1.1	Both first partial derivatives attempted $k_1x + ay$ and $k_2y + ax,\ k_1, k_2 \neq 0$
		… and $\frac{\partial z}{\partial y} = 3y^2 + ax$	A1	1.1	Both correct
		$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0 \Rightarrow y = -\frac{2}{a}x \Rightarrow 0 = 3\left(-\frac{2}{a}x\right)^2 + ax$	M1dep*	3.1a	Both set to zero and substituting for one variable
		$\Rightarrow x = 0$ or $x = -\frac{a^3}{12}$	M1	1.1	Solving for the second variable. Condone sign errors.
		$\Rightarrow (x, y, z) = (0, 0, 0)$ or	A1	1.1	**AG** for SP at *O* shown. Allow verification that (0,0,0) is a SP. Condone (0,0) or 'the origin'
		$\left(-\frac{a^3}{12}, \frac{a^2}{6}, -\frac{a^6}{432}\right)$	A1	1.1	Second SP correct
			[6]		
	(b)	$\frac{\partial^2 z}{\partial x^2} = 2,\ \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = a,\ \frac{\partial^2 z}{\partial y^2} = 6y$	M1	1.1	All second partial derivatives attempted $\frac{\partial^2 z}{\partial x^2} = k_1$, $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = a$, $\frac{\partial^2 z}{\partial y^2} = 2k_2y$. Can be implied by sight of $\mathbf{H} = \begin{pmatrix} 2 & a \\ a & 6y \end{pmatrix}$ or $\lvert \mathbf{H} \rvert = \begin{vmatrix} 2 & a \\ a & 6y \end{vmatrix}$ or by the formula $\lvert \mathbf{H} \rvert = f_{xx}f_{yy} - f_{xy}^2$

Question		Answer	Marks	AO	Guidance
		$\|\mathbf{H}\| = \begin{vmatrix} 2 & a \\ a & 6y \end{vmatrix} =$	**M1FT**	**2.1**	Determinant of Hessian matrix attempted (with non-zero $\frac{\partial^2 z}{\partial x \partial y}$ *and* $\frac{\partial^2 z}{\partial y \partial x}$) Can be implied by sight of $\begin{vmatrix} 2 & a \\ a & a^2 \end{vmatrix}$ below **FT** only $\left(\frac{a^3}{12}, -\frac{a^2}{6}, \ldots\right)$ in part **(a)**
		$= \begin{vmatrix} 2 & a \\ a & 0 \end{vmatrix} = -a^2$ at O **OR** $\begin{vmatrix} 2 & a \\ a & a^2 \end{vmatrix} = a^2$ at other SP	**M1FT**	**1.1**	Attempt to evaluate at least one case (with non-zero $\frac{\partial^2 z}{\partial x \partial y}$ *and* $\frac{\partial^2 z}{\partial y \partial x}$) **FT** only $\left(\frac{a^3}{12}, -\frac{a^2}{6}, \ldots\right)$ in part **(a)**
		At O, $\|\mathbf{H}\| < 0 \Rightarrow$ a saddle point	**A1**	**2.2a**	Correct conclusion
		At other SP, $\|\mathbf{H}\| > 0$ *and* $f_{xx} > 0 \Rightarrow$ a (local) Minimum	**A1**	**2.2a**	Correct conclusion Condone missing coordinates of SP if y-coordinate is given **FT** only $\left(\frac{a^3}{12}, -\frac{a^2}{6}, \ldots\right)$ in part **(a)** $\|\mathbf{H}\| < 0 \Rightarrow$ a saddle point
			[5]		
6	**(c)**	There is **only** one stationary point (0,0,0)	**B1**	**2.2a**	Accept 'Only one stationary point at the origin' 'Saddle-point at the origin only'

Question		Answer	Marks	AO	Guidance
		| **H** | = 0 so the nature of the SP cannot be determined (by this method)	**B1** **[2]**	**2.4**	Or origin is a saddle point **with justification**, eg 'When x=0, $z = y^3$ has a point of inflection at the origin When y=0, $z = x^2$ has a minimum point at the origin So the origin is a saddle point' **Or** diagrams of $z = y^3$ and $z = x^2$ seen

Question		Answer	Marks	AO	Guidance
7	**(a)**	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3+1\right)^{\frac{1}{2}} = \frac{1}{2}\left(x^3+1\right)^{-\frac{1}{2}}.3x^2$	**M1**	**1.1**	Attempt at differentiation by the chain rule $k\,x^2(x^3+1)^{-\frac{1}{2}}$ soi
		So $(I_2 =) \left[\frac{2}{3}\sqrt{x^3+1}\right]_0^2 = \frac{4}{3}$	**A1** **[2]**	**1.1**	Needs to be via $\frac{2}{3}\int f'(x)\,dx$ (not via substitution) cao (Intermediate step must be seen)

Question		Answer	Marks	AO	Guidance
	(b)	$I_n = \int x^{n-2}.\frac{x^2}{\sqrt{x^3+1}}\,dx = \left[a\,x^{n-2}.\sqrt{x^3+1}\right] - b\int(n-2).x^{n-3}\sqrt{x^3+1}\,dx$	**M1**	**3.1a**	Use of integration by parts with appropriate splitting so that result of part **(a)** can be used
		$a = b = \frac{2}{3}$	**A1**	**1.2**	First-stage of integration by parts correct
		$\sqrt{x^3+1} = \frac{x^3+1}{\sqrt{x^3+1}}$	**M1***	**1.1**	Valid preparation for second integral in I_k forms $\frac{2}{3}\left(2^{n-2}\times 3-0\right)-\frac{2}{3}(n-2)\int x^{n-3}.\frac{x^3+1}{\sqrt{x^3+1}}\,dx$
		$3I_n = 3\times 2^{n-1} - 2(n-2)(I_n + I_{n-3})$	**M1dep***	**1.1**	Writing in terms of I_n and I_{n-3}
		$(2n-1)I_n = 3\times 2^{n-1} - 2(n-2)I_{n-3}$	**A1**	**1.1**	**AG** fully shown (showing correct substitution of limits at some point) www
			[5]		
	(c)	$(I = \int x^5.\frac{x^3+1}{\sqrt{x^3+1}}\,dx =)\ I_8 + I_5$	**B1**	**3.1a**	
		$n=5 \Rightarrow 9I_5 = a - 6I_2$ oe $n=8 \Rightarrow 15I_8 = b - 12I_5$ oe	**M1**	**1.1**	Must use given reduction formula for $n = 5$ **and** $n = 8$ ($a, b \neq 0$). I_2 can only be $\frac{4}{3}$ or their numerical value from part **(a)**. I_5 can be numerical in the expression for I_8.
		$9I_5 = 3\times 16 - 6I_2 \Rightarrow I_5 = \frac{40}{9}$ $15I_8 = 3\times 128 - 12I_5 \Rightarrow I_8 = \frac{992}{45} \Rightarrow I = \frac{1192}{45}$ or $26\frac{22}{45}$	**A1**	**1.1**	cao from full working
			[3]		

Question			Answer	Marks	AO	Guidance
8	(a)	i	(By *Lagrange's theorem)* $o(H) \mid o(G)$	B1	2.4	Allow 'By *Lagrange's Theorem'* only if their orders below are correct
			so $(o(H) =)$ 2, 3, 4 or 6	B1 [2]	2.2a	Ignore inclusion of 1 and/or 12
		ii	(g is the generator, so) $g^{12} = e$	B1	3.1a	Generator element and identity element must be (implicitly) defined Allow $G = \{e, g, g^2, g^3, \ldots, g^{11}\}$
			$(o(H) = 2)$ $\{e, g^6\}$	B1	1.1	
			$(o(H) = 3)$ $\{e, g^4, g^8\}$	B1	1.1	
			$(o(H) = 4)$ $\{e, g^3, g^6, g^9\}$	B1	1.1	
						Ignore $\{e\}$, H
			$(o(H) = 6)$ $\{e, g^2, g^4, g^6, g^8, g^{10}\}$	B1 [5]	1.1	**SCB1** For an example of a cyclic group of order 12, completely defined and subgroups all correct (eg $(\mathbb{Z}_{12}, +)$)
		iii	All powers of g commute	B1 [1]	2.4	Commutative because $g^k g^l = g^l g^k$ $(1 \leq k, l \leq 11, k \neq l)$ (an example is sufficient) "Cyclic $\Rightarrow$ abelian" is insufficient
	(b)	i	(a, b) where $a \in \{0, 1, 2\}$ and $b \in \{0, 1, 2, 3\}$	B1 [1]	3.1a	All 12 elements may be listed (and no extras)

Question			Answer	Marks	AO	Guidance
		ii	(1, 1) generates *J* so **mapping** this to *g* creates the required isomorphism	**B1** **[1]**	**2.4**	*J* is a cyclic group generated by **(1,1)**. *J* and *G* are both cyclic groups of order 12, so isomorphic. **Or** an explicit bijective mapping of each element of *G* onto an element of *J*. Condone one error but not in mapping one generator onto the other generator.

Question			Answer	Marks	AO	Guidance
8	**(c)**	**i**	$m = 2$ and $n = 6$	**B1** **[1]**	**3.1a**	And no extras. Accept 2, 6 (but not 6, 2)
		ii	(a, b) where $a \in \mathbb{Z}_2$ and $b \in \mathbb{Z}_6$	**B1** **[1]**	**1.1**	All 12 elements may be listed (and no extras)
		iii	$(x_1, y_1) \oplus (x_2, y_2) = (x_1 +_2 x_2, y_1 +_6 y_2)$ $= (x_2 +_2 x_1, y_2 +_6 y_1) = (x_2, y_2) \oplus (x_1, y_1)$ $x_1, x_2 \in \mathbb{Z}_2$ and $y_1, y_2 \in \mathbb{Z}_6$	**B1** **[1]**	**2.5**	**Or** Both 'additions' (in $\mathbb{Z}_2$ and $\mathbb{Z}_6$) are commutative and the two components do not interfere with each other (ie components are independent, separate,…). Condone 'individual'
	(d)		*G* is cyclic (or *K* is not cyclic) *G* has one self-inverse element (or *K* has more than one) *G* has a generator (*g*) (or *K* does not have a generator)	**M1**	**2.1**	Identifying an aspect of the groups which will lead to a mismatch of properties, *ie*: Considering (non-)cyclicity of one group Number of self-inverse elements in one group Considering the (absence of a) generator in one group

Question			Answer	Marks	AO	Guidance
			G is cyclic while ***K*** is not cyclic *eg* (1,2), (1,4) cannot be generated by (1,1) or any other element/max order possible for an element of *K* is (LCM(2,6)=)6 ***G*** has one self-inverse element, g^6, while ***K*** has at least two/three *ie* (1,0), (0,3) and (1, 3) (at least two given) ***G*** has a generator (*g*) while ***K*** does not have a generator i.e. (1,2), (1,4) cannot be generated by (1,1) or any other element/ max order possible for an element of *K* is (LCM(2,6)=)6	**A1** **[2]**	**2.4**	Convincingly concluded. Not just '***G*** and ***K*** have different structure'

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