

Please write clearly in	n block capitals.	
Centre number	Candidate number	
Surname		
Forename(s)		
Candidate signature	I declare this is my own work.	ノ

# AS **MATHEMATICS**

Paper 2

Thursday 25 May 2023

Afternoon

Time allowed: 1 hour 30 minutes

## **Materials**

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Question         Mark           1         2           3         4           5         6           7         8           9         9	
2 3 4 5 6 7 8	
6 7 8	
6 7 8	
6 7 8	
6 7 8	
8	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
TOTAL	



# Section A

Answer all questions in the spaces provided.

1 Simplify  $\log_2 8^a$ 

Circle your answer.

[1 mark]

 $a^3$ 

2*a* 

3*a* 

8*a* 

2 It is given that  $\sin\theta = \frac{4}{5}$  and  $90^{\circ} < \theta < 180^{\circ}$ 

Find the value of  $\cos\theta$ 

Circle your answer.

[1 mark]

$$-\frac{3}{4}$$

$$-\frac{3}{5}$$

$$\frac{3}{4}$$

3 (a)	Find $\int \left(2x^3 + \frac{8}{x^2}\right) dx$	[3 marks]
3 (b)	A curve has gradient function $\frac{dy}{dx} = 2x^3 + \frac{8}{x^2}$	
	The <i>x</i> -intercept of the curve is at the point (2, 0)	
	Find the equation of the curve.	[2 marks]

4	Find the exact solution of the equation $\ln(x+1) + \ln(x-1) = \ln 15 - 2 \ln 7$	
	Fully justify your answer.	
		[5 marks]
	-	



5	It is given that $\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$ and $\cos 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$	
	Use these two expressions to show that $ tan 15^\circ = 2 - \sqrt{3}$	
	Fully justify your answer.  [4 mag	arks]

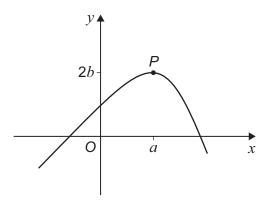
Turn over for the next question

6	A curve has equation
	$y = 2x^2 + px + 1$
	A line has equation
	y = 5x - 2
	Find the set of values of $p$ for which the line intersects the curve at two distinct points.
	Give your answer in exact form.  [5 marks]



7 The curve C has equation y = f(x)

C has a maximum point at P with coordinates (a, 2b) as shown in the diagram below.



7 (a) C is mapped by a single transformation onto curve  $C_1$  with equation y = f(x + 2)

State the coordinates of the maximum point on curve  $C_1$ 

[1 mark]


**7 (b)** C is mapped by a single transformation onto curve  $C_2$  with equation y = 4f(x)

State the coordinates of the maximum point on curve $C_2$
---

[1 mark]

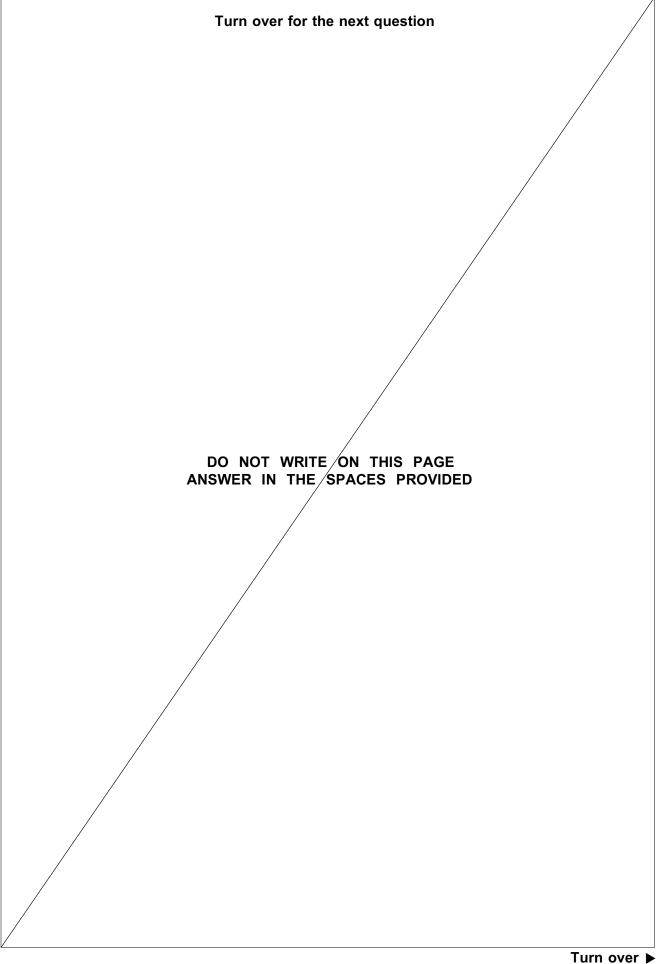

7 (c) C is mapped by a stretch in the x-direction onto curve  $C_3$  with equation y = f(3x)State the scale factor of the stretch.

[1 mark]



of 4.		a r 
		[5
	·	







9	A craft artist is producing items and selling them in a local market.
	The selling price, $\pounds P$ , of an item is inversely proportional to the number of items produced, $n$
9 (a)	When $n = 10$ , $P = 24$
	Show that $P = \frac{240}{n}$
	n [1 mark]
9 (b)	The production cost, $\pounds C$ , of an item is inversely proportional to the square of the number of items produced, $n$
	When $n=10$ , $C=60$
	Find the set of values of $n$ for which $P>C$ [4 marks]

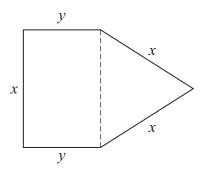
9 (c)	Explain the significance to the craft artist of the range of values found in part (b). [1 r	nark]
	Turn over for the next question	



A piece of wire of length 66 cm is bent to form the five sides of a pentagon.

The pentagon consists of three sides of a rectangle and two sides of an equilateral triangle.

The sides of the rectangle measure  $x \, \text{cm}$  and  $y \, \text{cm}$  and the sides of the triangle measure  $x \, \text{cm}$ , as shown in the diagram below.



**10 (a) (i)** You are given that  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ 

Explain why the area of the triangle is  $\frac{\sqrt{3}}{4}x^2$ 

[1 mark]

10 (a) (ii) Show that the area enclosed by the wire,  $A \text{ cm}^2$ , can be expressed by the formula

$$A = 33x - \frac{1}{4}(6 - \sqrt{3})x^2$$

[3 marks]

 	 <del> </del>	

10 (b)	Use calculus to find the value of $x$ for which the wire encloses the maximum area.			
	Give your answer in the form $p+q\sqrt{3}$ , where $p$ and $q$ are integers.			
	Fully justify your answer.	[7 marks]		

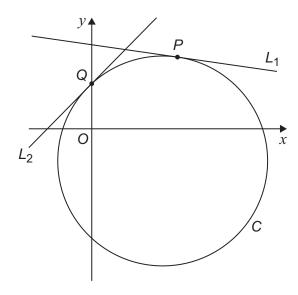
11 The line  $L_1$  has equation x + 7y - 41 = 0

 $L_1$  is a tangent to the circle C at the point P(6, 5)

The line  $L_2$  has equation y = x + 3

 $L_2$  is a tangent to the circle C at the point Q(0, 3)

The lines  $L_1$  and  $L_2$  and the circle  ${\it C}$  are shown in the diagram below.



11 (a)	Show that the equation of the radius of the circle through $P$ is $y = 7x - 37$	
		[3 marks]



11 (b)	Find the equation of C	[4 marks]

# END OF SECTION A TURN OVER FOR SECTION B



_			_
<u>د</u> د	~tı	on	ĸ
95	CLI	vii	

	Section B		
	Answer all questions in the spaces provided.		
12	The mass of a bag of nuts produced by a company is known to have a mean of 40 grams and a standard deviation of 3 grams.		
	The company produces five different flavours of nuts.		
	The bags of nuts are packed in large boxes.		
	Given the information above, identify the continuous variable from	om the options below.	
	Tick (✓) one box.	[1 mark]	
	The flavours of the bags of nuts		
	The known standard deviation of the mass of a bag of nuts		
	The mass of an individual bag of nuts		
	The number of bags of nuts in a large box		



13 The table below shows the frequencies for a set of data from a continuous variable X

x	Frequency
11 < <i>x</i> ≤ 21	7
21 < <i>x</i> ≤ 24	9
24 < <i>x</i> ≤ 42	36
42 < <i>x</i> ≤ 50	18

A histogram is drawn to represent this data.

Find the frequency density of the bar in the histogram representing the class  $24 < x \le 42$ 

Circle your answer.

[1 mark]

2

18

36

70

Turn over for the next question

14	The manager of a factory wants to introduce a bonus scheme.
	The factory has 65 employees who work in production and 28 employees who work in the office.
	The manager decides to collect the opinions of a sample of these 93 employees.
14 (a)	Explain how the manager could collect a simple random sample of 20 employees.  [3 marks]
14 (b)	The manager collected a simple random sample of 20 employees.
	The manager noticed that all 20 of the employees in the sample worked in production and therefore the sample was not representative.
	State a different method of sampling that would give a representative sample.  [1 mark]



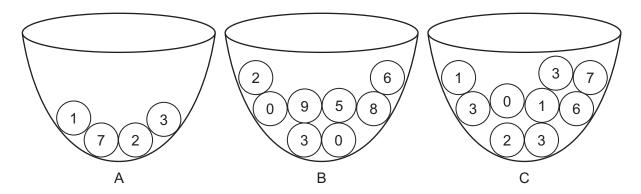
Turn over for the next question DO NOT WRITE ON THIS PAGE ANSWER IN THE SPACES PROVIDED

1 9

15	Numbered balls are placed in bowls A, B and C
	In bowl A there are four balls numbered 1, 2, 3 and 7
	In bowl B there are eight balls numbered 0, 0, 2, 3, 5, 6, 8 and 9

In bowl C there are nine balls numbered 0, 1, 1, 2, 3, 3, 3, 6 and 7

This information is shown in the diagram below.



A three-digit number is generated using the following method:

- a ball is selected at random from each bowl
- the first digit of the number is the ball drawn from bowl A
- the second digit of the number is the ball drawn from bowl B
- the third digit of the number is the ball drawn from bowl C

15 (a)	Find the probability that the number generated is even.	[1 mark]
15 (b)	Find the probability that the number generated is 703	[2 marks]



15 (c)	Find the probability that the number generated is divisible by 111	[2 marks]
	Turn over for the next question	



16	The discrete	random	variable 2	Y has	probability	distribution

x	1	2	3	4	5	6
P(X = x)	0.3	0.1	0.2	0.1	0.1	0.2

The discrete random variable Y has probability distribution

у	2	3	4	5	6	7
P(Y = y)	0.3	р	0.2	0.1	р	3p + 0.05

It is claimed that  $P(X \ge 3)$  is greater than  $P(Y \le 4)$ 

Determine if this claim is correct.

Fully Justify your answer.	[4 marks]



Turn over for the next question DO NOT WRITE ON THIS PAGE ANSWER IN THE SPACES PROVIDED

2 3

17	An archer is training for the Olympics.
	Each of the archer's training sessions consists of 30 attempts to hit the centre of a target.
	The archer consistently hits the centre of the target with 79% of their attempts.
	It can be assumed that the number of times the centre of the target is hit in any training session can be modelled by a binomial distribution.
17 (a)	Find the mean of the number of times that the archer hits the centre of the target during a training session.  [1 mark]
17 (b)	Find the probability that the archer hits the centre of the target exactly 22 times during a particular training session.  [1 mark]
17 (c)	Find the probability that the archer hits the centre of the target 18 times or less during a particular training session.  [1 mark]



Turn over for the next question  Turn over for the next question		
a training session.  [2 marks	17 (d)	Find the probability that the archer hits the centre of the target more than 26 times in
		a training session.
Turn over for the next question		[2 marks
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
Turn over for the next question		
		Turn over for the next question



18	It is believed that 25% of the customers at a bakery buy a loaf of bread.
	In an attempt to increase this proportion, the manager of the bakery provided free samples for the customers to taste.
	To decide whether providing free samples had been effective, a large random sample of customers leaving the bakery were asked whether they had purchased a loaf of bread.
	A hypothesis test at the 5% significance level was carried out on the data collected.
	The test statistic calculated was found to be in the critical region.
18 (a)	State the Null and Alternative hypotheses for this test.  [1 mark]
18 (b)	State, in context, the conclusion to this test.  [2 marks]



Turn over for the next question DO NOT WRITE ON THIS PAGE ANSWER IN THE SPACES PROVIDED

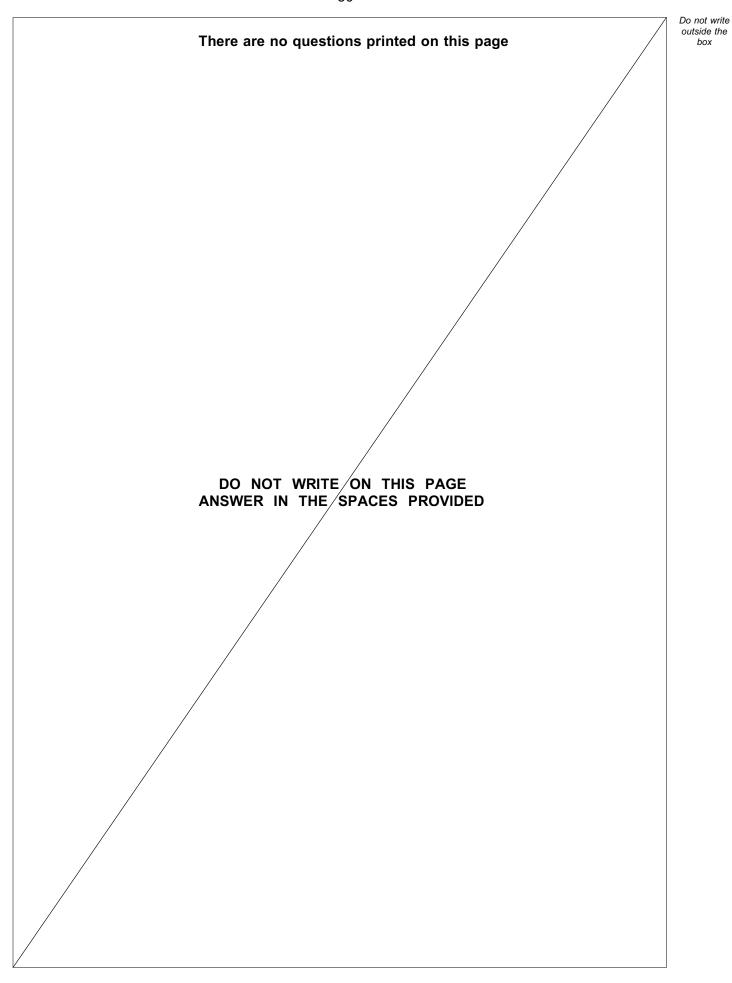
2 7

19	A comparison of the ma Large Data Set.	sses (in kg) of convertible	cars was made using the	ie
	A sample of 20 masses	was chosen from both the	e 2002 data and the 201	6 data.
	The masses of the 20 c each year.	ars in each sample were ι	used to create a box plo	t for
	The box plots were labe diagram below.	lled Box Plot A and Box F	Plot B as shown in the	
		Box Plot A		
		Box Plot B		
0	500	1000	1500	2000
Ü	000	Mass (kg)	1000	2000
19 (a)	Estimate the median of	the masses from Box Plot	Α	[1 mark]



It is claimed that Box Plot B <b>must be</b> incorrectly drawn.
Give a reason why this claim was made.  [1 mark]
Comment on the validity of this claim.  [1 mark]
It is claimed that Box Plot B <b>must be</b> from the 2002 data. Give a reason why this claim is correct. [1 mark]
END OF QUESTIONS







Question number	Additional page, if required. Write the question numbers in the left-hand margin.



Question number	Additional page, if required. Write the question numbers in the left-hand margin.
	Copyright information
	For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from www.aqa.org.uk.
	Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team.
	Copyright © 2023 AQA and its licensors. All rights reserved.



