

| Please write clearly ir | า block capitals.              |
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| Centre number           | Candidate number               |
| Surname                 |                                |
| Forename(s)             |                                |
| Candidate signature     | I declare this is my own work. |

# AS **MATHEMATICS**

Paper 1

Thursday 18 May 2023

Afternoon

Time allowed: 1 hour 30 minutes

### **Materials**

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Exam | iner's Use |
|----------|------------|
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# Section A

Answer all questions in the spaces provided.

1 At a point *P* on a curve, the gradient of the tangent to the curve is 10

State the gradient of the normal to the curve at P

Circle your answer.

[1 mark]

-10

-0.1

0.1

10

2 Identify the expression below which is equivalent to  $\left(\frac{2x}{5}\right)^{-3}$ 

Circle your answer.

$$\frac{8x^3}{125}$$

$$\frac{125x^3}{8}$$

$$\frac{125}{8x^3}$$

$$\frac{8}{125x^3}$$

| 3 | The coefficient of $x^2$ in the binomial expansion of $(1 + ax)^6$ is $\frac{20}{3}$ |           |
|---|--|-----------|
|   | Find the two possible values of $a$  | [3 marks] |
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Turn over for the next question



| 4     | It is given that $5\cos^2\theta - 4\sin^2\theta = 0$  |           |
|-------|---|-----------|
| 4 (a) | Find the possible values of $\tan\theta$ , giving your answers in exact form.                             | [3 marks] |
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| 4 (b) | Hence, or otherwise, solve the equation   |           |
|       | $5\cos^2\theta - 4\sin^2\theta = 0$   |           |
|       |   |           |
|       | giving all solutions of $\theta$ to the nearest 0.1° in the interval $0^\circ \leq \theta \leq 360^\circ$ | [2 marks] |
|       |   | [2 marks] |



| 5 (a) | Given that $y = x\sqrt{x}$ , find $\frac{\mathrm{d}y}{\mathrm{d}x}$ | [2 marks] |
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| 5 (b) | The line, L, has equation $6x - 2y + 5 = 0$                         |           |
|       | L is a tangent to the curve with equation $y = x\sqrt{x} + k$       |           |
|       | Find the value of $k$   | [5 marks] |
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| 6 (a) | The curve $C_1$ has equation $y = 2x^2 - 20x + 42$                                      |             |
|-------|---|-------------|
|       | Express the equation of $C_1$ in the form   |             |
|       | $y = a(x - b)^2 + c$  |             |
|       | where $a$ , $b$ and $c$ are integers.   | [2 marks]   |
|       |   | [3 marks]   |
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| 6 (b) | Write down the coordinates of the minimum point of $C_1$                                |             |
| , ,   |   | [1 mark]    |
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| 6 (c) | The curve $C_1$ is mapped onto the curve $C_2$ by a stretch in the <i>y</i> -direction. |             |
| · (·) | The minimum point of $C_2$ is at $(5, -4)$  |             |
|       | Find the equation of $C_2$  |             |
|       | Tilla the equation of O2  | [2 marks]   |
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| 7     | Points $P$ and $Q$ lie on the curve with equation $y = x^4$  |
|-------|--|
|       | The $x$ -coordinate of $P$ is $x$<br>The $x$ -coordinate of $Q$ is $x + h$                         |
| 7 (a) | Expand $(x+h)^4$ [2 marks]   |
| 7 (b) | Hence, find an expression, in terms of $x$ and $h$ , for the gradient of the line $PQ$             |
| , ,   | [1 mark]   |
| 7 (c) | Explain how to use the answer from part (b) to obtain the gradient function of $y = x^4$ [2 marks] |
|       |  |



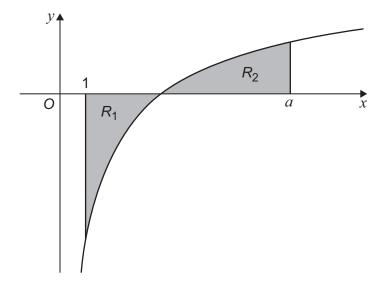
| 8 (a) | Show that |   |               |
|-------|-----------|---|---------------|
|       |           | $\int_{1}^{a} \left( 6 - \frac{12}{\sqrt{x}} \right) dx = 6a - 24\sqrt{a} + 18$ |               |
|       |           | J1 \ \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\   | [3 marks]     |
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8 (b) The curve  $y = 6 - \frac{12}{\sqrt{x}}$ , the line x = 1 and the line x = a are shown in the diagram below.

The shaded region  $R_1$  is bounded by the curve, the line x = 1 and the x-axis.

The shaded region  $R_2$  is bounded by the curve, the line x = a and the x-axis.



It is given that the areas of  $R_1$  and  $R_2$  are equal.

Find the value of a

Fully justify your answer.

| [4 marks] |
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**9** A continuous curve has equation y = f(x)

The curve passes through the points A(2, 1), B(4, 5) and C(6, 1)

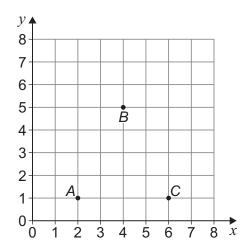
It is given that f'(4) = 0

Jasmin made two statements about the nature of the curve y = f(x) at the point B:

Statement 1: There is a turning point at B

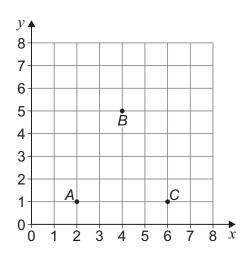
Statement 2: There is a maximum point at B

9 (a) Draw a sketch of the curve y = f(x) such that Statement 1 is correct and Statement 2 is correct.

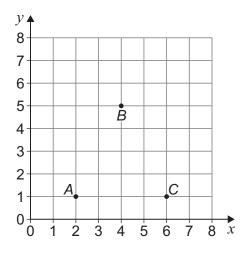


9 (b) Draw a sketch of the curve y = f(x) such that Statement 1 is correct and Statement 2 is **not** correct.

[1 mark]



9 (c) Draw a sketch of the curve y = f(x) such that Statement 1 is **not** correct and Statement 2 is **not** correct.



| 10     | Charlie buys a car for £18 000 on 1 January 2016.  |   |
|--------|--|---|
|        | The value of the car decreases exponentially.  |   |
|        | The car has a value of £12 000 on 1 January 2018.  |   |
| 10 (a) | <ul> <li>Charlie says:</li> <li>because the car has lost £6000 after two years, after another two years it will be worth £6000.</li> </ul> |   |
|        | Charlie's friend Kaya says:  |   |
|        | <ul> <li>because the car has lost one third of its value after two years, after another<br/>two years it will be worth £8000.</li> </ul>   |   |
|        | Explain whose statement is correct, justifying the value they have stated.  [2 marks]  | ] |
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| 10 (b) | The value of Charlie's car, $\pounds V$ , $t$ years after 1 January 2016 may be modelled the equation        | d by        |
|--------|--|-------------|
|        | $V = Ae^{-kt}$   |             |
|        | where $A$ and $k$ are positive constants.  |             |
|        | Find the value of $t$ when the car has a value of £10 000, giving your answer t two significant figures.     | o           |
|        |  | [5 marks]   |
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| 10 (c) | Give a reason why the model, in this context, will not be suitable to calculate value of the car when $t=30$ | the         |
|        |  | [1 mark]    |
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11 (a) A circle has equation

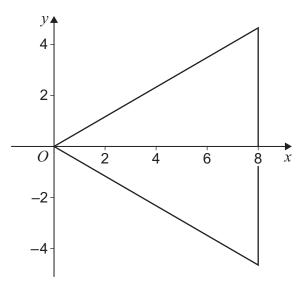
$$x^2 + y^2 - 10x - 6 = 0$$

Find the centre and the radius of the circle.

[2 marks]

\_\_\_\_\_

11 (b) An equilateral triangle has one vertex at the origin, and one side along the line x = 8, as shown in the diagram below.



**11 (b) (i)** Show that the vertex at the origin lies inside the circle  $x^2 + y^2 - 10x - 6 = 0$ 

| 11 (b) (ii) | Prove that the triangle lies completely within the circle $x^2 + y^2 - 10x - 6 = 0$ [4 marks] |
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# END OF SECTION A TURN OVER FOR SECTION B

## Section B

Answer all questions in the spaces provided.

A particle, initially at rest, starts to move forward in a straight line with constant acceleration,  $a\,\mathrm{m}\,\mathrm{s}^{-2}$ 

After 6 seconds the particle has a velocity of  $3\,\mathrm{m\,s^{-1}}$ 

Find the value of a

Circle your answer.

[1 mark]

-2

-0.5

0.5

2

13 A resultant force of  $\begin{bmatrix} -2 \\ 6 \end{bmatrix}$  N acts on a particle.

The acceleration of the particle is  $\left[ \begin{smallmatrix} -6 \\ \mathcal{Y} \end{smallmatrix} \right] \text{m}\,\text{s}^{-2}$ 

Find the value of y

Circle your answer.

[1 mark]

2

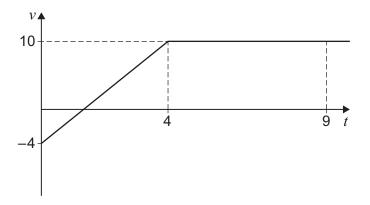
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18

| 14     | A ball, initially at rest, is dropped from a vertical height of $h$ metres above the Earth's surface.               |           |
|--------|---|-----------|
|        | After 4 seconds the ball's height above the Earth's surface is $0.2h$ metres.                                       |           |
| 14 (a) | Assuming air resistance can be ignored, show that   |           |
|        | h = 10g   | [3 marks] |
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| 14 (b) | Assuming air resistance cannot be ignored, explain the effect that this would the value of $h$ in part <b>(a)</b> . | have on   |
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A particle is moving in a straight line such that its velocity,  $v \, \text{m} \, \text{s}^{-1}$ , changes with respect to time, t seconds, as shown in the graph below.



Show that the acceleration of the particle over the first 4 seconds is  $3.5 \,\mathrm{m\,s^{-2}}$  [1 mark]

| <br> |  |  |
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**15 (b)** The particle is initially at a fixed point P

| Show that the displacement of the particle from $P$ , when $t=9$ , is 62 metres. | 3 marks] |
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| 16     | A toy remote control speed boat is launched from one edge of a small pond and moves in a straight line across the pond's surface.           |  |  |
|--------|---|--|--|
|        | The boat's velocity, $v\mathrm{m}\mathrm{s}^{-1}$ , is modelled in terms of time, $t$ seconds after the boat is launched, by the expression |  |  |
|        | $v = 0.9 + 0.16t - 0.06t^2$   |  |  |
| 16 (a) | Find the acceleration of the boat when $t=2$ [3 marks]  |  |  |
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| 16 (b) | Find the displacement of the boat, from the point where it was launched, when $t=2$ <b>[4 marks]</b>  |  |  |
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17 A particle, *P*, is initially at rest on a smooth horizontal surface.

A resultant force of  $\begin{bmatrix} 12 \\ 9 \end{bmatrix}$  N is then applied to P, so that it moves in a straight line.

17 (a) Find the magnitude of the resultant force.

[1 mark]

\_\_\_\_\_

**17 (b)** Two fixed points A and B have position vectors

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$
 metres and  $\overrightarrow{OB} = \begin{bmatrix} k \\ k-1 \end{bmatrix}$  metres

with respect to a fixed origin, O

P moves in a straight line parallel to  $\overrightarrow{AB}$ 

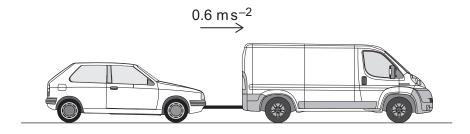
**17 (b) (i)** Find  $\overrightarrow{AB}$  in terms of k

| 17 (b) (ii) | Find the value of $k$           | [2 marks] |
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18 A rescue van is towing a broken-down car by using a tow bar.

The van and the car are moving with a constant acceleration of  $0.6\,\mathrm{m\,s^{-2}}$  along a straight horizontal road as shown in the diagram below.



The van has a total mass of 2780 kg

The car has a total mass of 1620 kg

The van experiences a driving force of D newtons.

The van experiences a total resistance force of R newtons.

The car experiences a total resistance force of 0.6*R* newtons.

**18 (a)** The tension in the tow bar, T newtons, may be modelled by

$$T = kD - 18$$

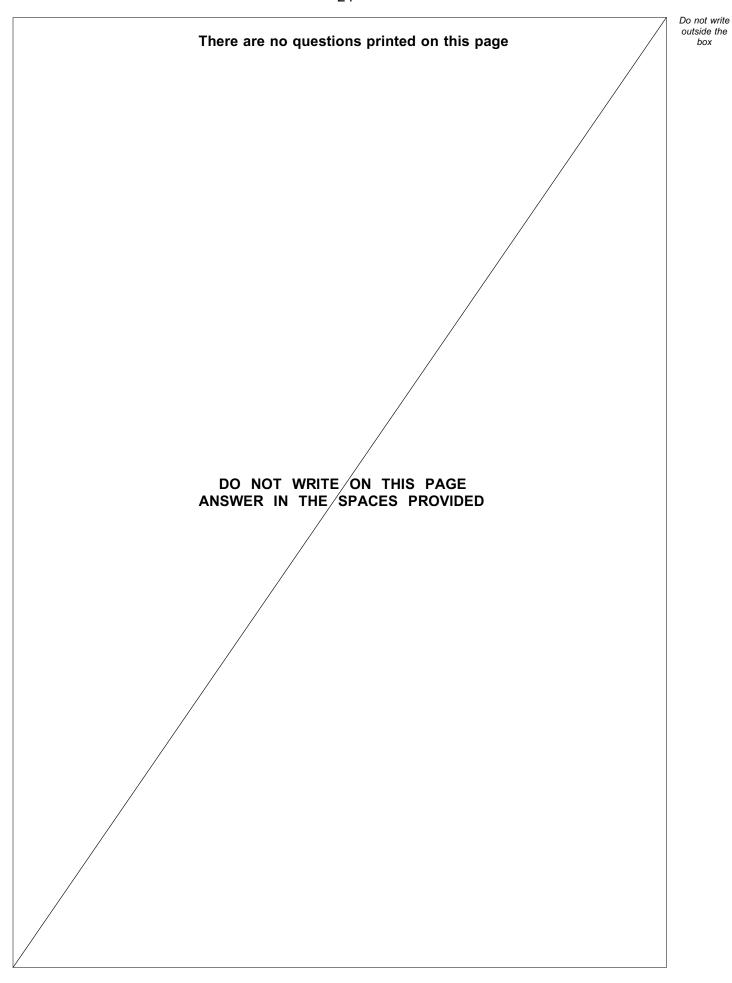
where k is a constant.

| Find k |      | [5 marks] |
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| 18 (b) | State one assumption that must be made in answering part (a). | [1 mark] |
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