



Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

AS FURTHER MATHEMATICS

Paper 1

Monday 15 May 2023

Afternoon

Time allowed: 1 hour 30 minutes

Materials

- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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14	
TOTAL	



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Answer **all** questions in the spaces provided.

1 Which expression below is equivalent to $\tanh x$?

Circle your answer.

[1 mark]

$$\sinh x \cosh x$$

$$\frac{\sinh x}{\cosh x}$$

$$\frac{\cosh x}{\sinh x}$$

$$\sinh x + \cosh x$$

2 The two vectors **a** and **b** are such that $\mathbf{a} \cdot \mathbf{b} = 0$

State the angle between the vectors **a** and **b**

Circle your answer.

[1 mark]

$$0^\circ$$

$$45^\circ$$

$$90^\circ$$

$$180^\circ$$



3 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 0 & 5 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 4 \\ 7 & 1 \end{bmatrix}$$

Calculate **AB**

Circle your answer.

[1 mark]

$$\begin{bmatrix} 3 & 5 \\ 7 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 20 \\ 21 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 \\ 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 13 \\ 35 & 5 \end{bmatrix}$$

4 The roots of the equation

$$5x^3 + 2x^2 - 3x + p = 0$$

are α , β and γ

Given that p is a constant, state the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

Circle your answer.

[1 mark]

$$-\frac{3}{5}$$

$$-\frac{2}{5}$$

$$\frac{2}{5}$$

$$\frac{3}{5}$$

Turn over ►



5 The function f is defined by

$$f(x) = 3x^2 \qquad 1 \leq x \leq 5$$

5 (a) Find the mean value of f

[2 marks]

5 (b) The function g is defined by

$$g(x) = f(x) + c \qquad 1 \leq x \leq 5$$

The mean value of g is 40

Calculate the value of the constant c

[2 marks]



6 (a) Find and simplify the first five terms in the Maclaurin series for e^{2x}

[2 marks]

6 (b) Hence, or otherwise, write down the first five terms in the Maclaurin series for e^{-2x}

[1 mark]

6 (c) Hence, or otherwise, show that the Maclaurin series for $\cosh(2x)$ is

$$a + bx^2 + cx^4 + \dots$$

where a , b and c are rational numbers to be determined.

[3 marks]

Turn over ►



- 8** Abdoallah wants to write the complex number $-1 + i\sqrt{3}$ in the form $r(\cos \theta + i \sin \theta)$ where $r \geq 0$ and $-\pi < \theta \leq \pi$

Here is his method:

$$\begin{aligned} r &= \sqrt{(-1)^2 + (\sqrt{3})^2} & \tan \theta &= \frac{\sqrt{3}}{-1} \\ &= \sqrt{1 + 3} & \Rightarrow \tan \theta &= -\sqrt{3} \\ &= \sqrt{4} & \Rightarrow \theta &= \tan^{-1}(-\sqrt{3}) \\ &= 2 & \Rightarrow \theta &= -\frac{\pi}{3} \end{aligned}$$

$$-1 + i\sqrt{3} = 2 \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right)$$

There is an error in Abdoallah's method.

- 8 (a)** Show that Abdoallah's answer is wrong by writing

$$2 \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right)$$

in the form $x + iy$

Simplify your answer.

[1 mark]



8 (b) Explain the error in Abdoallah's method.

[1 mark]

8 (c) Express $-1 + i\sqrt{3}$ in the form $r(\cos \theta + i \sin \theta)$

[1 mark]

8 (d) Write down the complex conjugate of $-1 + i\sqrt{3}$

[1 mark]

Turn over ►



9 The matrix \mathbf{M} represents the transformation T and is given by

$$\mathbf{M} = \begin{bmatrix} 3p + 1 & 12 \\ p + 2 & p^2 - 3 \end{bmatrix}$$

9 (a) In the case when $p = 0$ show that the image of the point $(4, 5)$ under T is the point $(64, -7)$

[2 marks]

9 (b) In the case when $p = -2$ find the gradient of the line of **invariant points** under T

[3 marks]



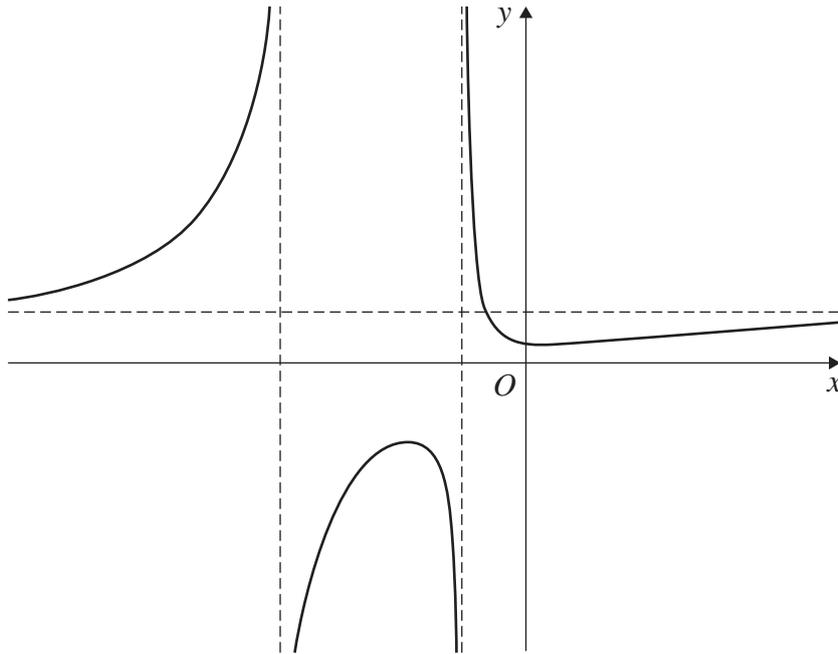
10 The curve C has equation

$$y = \frac{3x^2 + mx + p}{x^2 + px + m}$$

where m and p are integers.

The vertical asymptotes of C are $x = -4$ and $x = -1$

The curve C is shown in the diagram below.



10 (a) Write down the equation of the horizontal asymptote of C

[1 mark]

10 (b) Find the value of m and the value of p

[2 marks]



12 (b) (iii) One of the linear factors of $f(z)$ is

$$(z - (1 + i))$$

Write down another linear factor and hence, or otherwise, find a quadratic factor of $f(z)$ with real coefficients.

[3 marks]

12 (b) (iv) Find another quadratic factor of $f(z)$ with real coefficients.

[2 marks]

12 (b) (v) Hence explain why the graph of $y = f(x)$ does not intersect the x -axis.

[2 marks]

Turn over ►



- 13 (b)** Hence, or otherwise, write down a factorised expression for the sum of the first $2n$ squares

$$1^2 + 2^2 + 3^2 + \dots + (2n)^2$$

[1 mark]

- 13 (c)** Use the formula in part (a) to write down a factorised expression for the sum of the first n even squares

$$2^2 + 4^2 + 6^2 + \dots + (2n)^2$$

[2 marks]

- 13 (d)** Hence, or otherwise, show that the sum of the first n odd squares is

$$an(bn - 1)(bn + 1)$$

where a and b are rational numbers to be determined.

[3 marks]

Turn over ►



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