AQA

AS
FURTHER MATHEMATICS
7366/2D

Paper 2 Discrete

Mark scheme

June 2023

Version: 1.0 Final

236A7366/2D/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

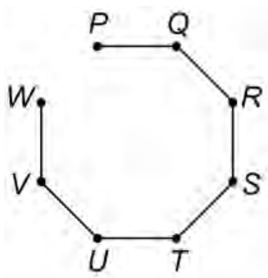
Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

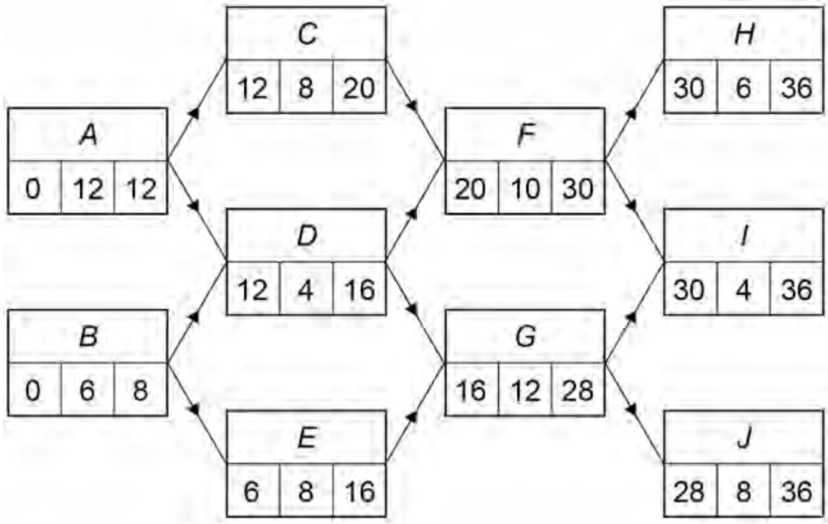
AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks correct box	1.1b	B1	
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks correct box	1.1b	B1	*A* and *G*
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Defines two variables to represent the **number of** town boxes and the **number of** country boxes	3.1b	B1	x = number of town boxes y = number of country boxes Maximise $P = x + y$ subject to $10x + 4y \leq 253$ $2x + 8y \leq 151$ $x \geq 0$, $y \geq 0$ x and y are integers
	Obtains a correct non-trivial constraint for chicken eggs or duck eggs (condone strict inequality)	1.1a	M1	
	Obtains both correct non-trivial constraints for chicken eggs and duck eggs	1.1b	A1	
	Formulates the linear programming problem with statement of maximising $x + y$ and all constraints fully correct	2.5	A1	
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)(i)	Finds correctly the earliest start time for activity *H* or activity *J*	1.1a	M1	(see diagram below)
	Finds correctly the earliest start time for each activity on the network.	1.1b	A1	
	Finds correctly the latest finish time for each activity on the network. Condone inclusion of 'END' activity with zero duration	1.1b	B1	
				
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)(ii)	States correct minimum completion time **FT** their values in activity network Condone missing units	1.1b	B1F	36 days
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	States all correct critical activities and no others	1.1b	B1	*A*, *C*, *D*, *F*, *G*, *H* and *J*
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(c)(i)	Finds that 60 days is the sum of the durations for each correct critical activity, and compares this with 36 days (the minimum completion time)	2.3	E1	$12 + 8 + 4 + 10 + 12 + 6 + 8 = 60$ $36 \neq 60$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(c)(ii)	Explains that Glyn's statement is true for an activity network with all critical activities on a single critical path.	2.4	E1	Glyn's statement is true for an activity network with all critical activities on a single critical path.
	Subtotal		**1**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(i)	States correct identity element	1.1b	B1	1
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(ii)	Uses identity element to search for self-inverse elements **PI** by at least one correct self-inverse element	3.1a	M1	$0 \times_6 0 = 0$, $1 \times_6 1 = 1$, $2 \times_6 2 = 4$, $3 \times_6 3 = 3$, $4 \times_6 4 = 4$, $5 \times_6 5 = 1$ $1 \times_6 1$ and $5 \times_6 5$ produce the identity element, 1 Therefore, self-inverse elements of S are 1 and 5
	Correctly identifies both self-inverse elements of S and no others	1.1b	A1	
	Subtotal		**2**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td>5(b)(i)</td><td>Correctly completes Cayley table</td><td>1.1b</td><td>B1</td><td>
<table>
<tr><td>♦</td><td>a</td><td>b</td><td>c</td></tr>
<tr><td>a</td><td>a</td><td>c</td><td>b</td></tr>
<tr><td>b</td><td>c</td><td>b</td><td>a</td></tr>
<tr><td>c</td><td>b</td><td>a</td><td>c</td></tr>
</table>
</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>1</td><td></td></tr>
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Q	Marking instructions	AO	Marks	Typical solution
5(b)(ii)	Sets up a test for associativity by stating 2 appropriate expressions	1.1a	M1	$(a \blacklozenge b) \blacklozenge c = c \blacklozenge c = c$ $a \blacklozenge (b \blacklozenge c) = a \blacklozenge a = a$ As $(a \blacklozenge b) \blacklozenge c = c \neq a = a \blacklozenge (b \blacklozenge c)$ then ♦ is not associative
	Correctly evaluates their 2 appropriate expressions	1.1b	A1	
	Constructs a complete mathematical argument to justify non-associativity	2.1	R1	
	Subtotal		**3**	

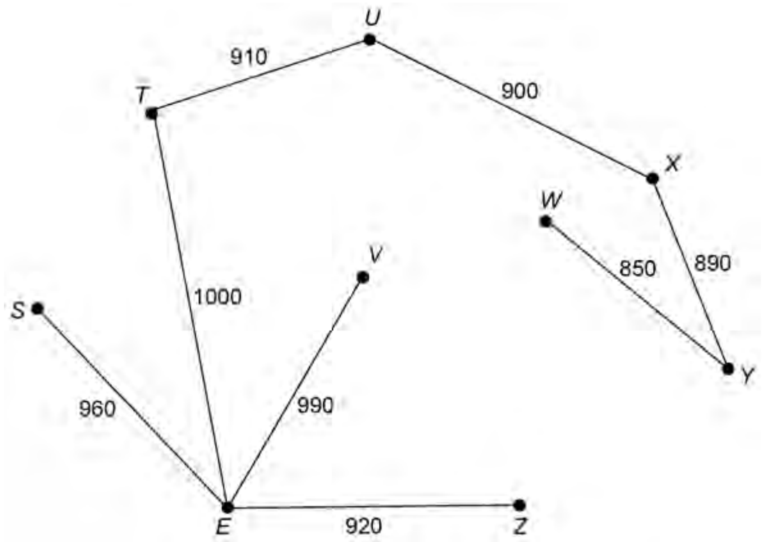
	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Identifies correctly the row minima or column maxima OR Identifies at least one dominated strategy	1.1a	M1	row minima: (–4, –3, –2) col maxima: (4, 1, –2) max(row minima) = –2 min(col maxima) = –2 As max(row minima) = –2 = min(col maxima), a stable solution exists.
	Finds correctly max(row minima) and min(col maxima) OR Finds a 1 × 1 pay-off matrix by removing all dominated strategies	1.1b	A1	
	Completes a reasoned argument to show that the game has a stable solution	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	States the correct play-safe strategy for each player	1.1b	B1	Play-safe strategy for Xander is $\mathbf{X}_3$ Play-safe strategy for Yvonne is $\mathbf{Y}_3$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	Explains that the maximum outcome for a game for either player from the pay-off matrix	3.1b	E1	The maximum amount of marbles that can be won or lost by either player in a game is 4. Because the game is a zero-sum game, when one player wins 4, the other player loses 4, so the difference is 8. 24 ÷ 8 = 3 If the players repeated the same strategies for each game, it would take 3 games to reach a difference of 24 marbles
	Explains clearly how a value from the pay-off matrix relates to the minimum number of games needed to complete the challenge	2.4	E1	
	Subtotal		**2**	

	Question total		**6**	

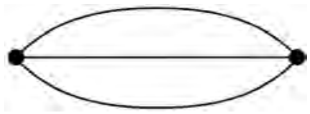
Q	Marking instructions	AO	Marks	Typical solution
7(a)(i)	Sets up a model of finding a maximum spanning tree by listing 8 arcs with at least 4 correct	3.3	M1	*ET*: 1000 *EV*: 990 *ES*: 960 *EZ*: 920 *TU*: 910 *UX*: 900 *XY*: 890 *YW*: 850
	Uses their model to find the 8 correct arcs of the maximum spanning tree	3.4	A1	
				
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(ii)	Finds the total of the weights of the 8 arcs from their model	3.1b	M1	1000 + 990 + 960 + 920 + 910 + 900 + 890 + 850 = 7420 metres
	Finds the correct length with units from their model	3.2a	A1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Calculates an estimate of cost by using an average cost	3.1a	M1	Average cost of track removal is 87600 ÷ 14.6 = £6000 per km Need to remove (14.6 – 7.42) km of track (14.6 – 7.42) × 6000 = £43,080
	Calculates the correct estimate for the total cost, with units, for their total length	3.2a	A1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Comments on the limitation of modelling the removal cost per unit length with an average cost	3.5b	B1	The cost of track removal may not be proportional to track length.
	Subtotal		**1**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Draws a graph with 2 vertices and 3 edges	1.1b	B1	
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Recalls Euler's formula for connected planar graphs **PI**	1.2	B1	Euler's formula for connected planar graphs: $v - e + f = 2$ $18(x - 1) - x(5x + 1) + 4x(x - 2) = 2$ $18x - 18 - 5x^2 - x + 4x^2 - 8x = 2$ $-x^2 + 9x - 20 = 0$ $x^2 - 9x + 20 = 0$ $(x - 5)(x - 4) = 0$ $x = 5$ or $x = 4$ For an Eulerian graph, all vertices have even degree. Therefore $x = 4$ When $x = 4$, $e = x(5x + 1) = 4 \times 21 = 84$ Each edge has 2 ends, so each edge contributes 2 to the total vertex degree. Therefore, total vertex degree = $2e = 2 \times 84 = 168$
	Forms an equation in x using Euler's formula for connected planar graphs Allow one sign error	3.1a	M1	
	Solves the quadratic equation, giving both correct solutions	1.1b	A1	
	States that all vertices of an Eulerian graph have even degree	1.1b	E1	
	Deduces that $x = 4$	2.2a	M1	
	Finds the correct sum of degrees for P	3.2a	A1	
	Subtotal		**6**	

	Question total		**7**	

	Question Paper total		**40**	