



Oxford Cambridge and RSA

Wednesday 14 June 2023 – Afternoon

AS Level Further Mathematics B (MEI)

Y412/01 Statistics a

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 Ryan has 6 one-pound coins and 4 two-pound coins. Ryan decides to select 3 of these coins at random to donate to a charity. The total value, in pounds, of these 3 coins is denoted by the random variable X .

(a) Show that $P(X = 3) = \frac{1}{6}$. [2]

The table below shows the probability distribution of X .

r	3	4	5	6
$P(X = r)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

(b) Draw a graph to illustrate the distribution. [2]

(c) **In this question you must show detailed reasoning.**

Find each of the following.

- $E(X)$
 - $\text{Var}(X)$
- [5]

Ryan’s friend Sasha decides to give the same amount as Ryan does to the charity plus an extra three pounds. The random variable Y represents the **total** amount of money, in pounds, given by Ryan and Sasha.

(d) Determine each of the following.

- $E(Y)$
 - $\text{Var}(Y)$
- [3]

- 2 A group of friends live by the sea. Each day they look out to sea in the hope of seeing a dolphin. The probability that they see a dolphin on any day is 0.15. You should assume that this probability is not affected by whether or not they see a dolphin on any other day.
- (a) Explain why you can use a geometric distribution to model the number of days that it takes for them to first see a dolphin. [1]
 - (b) Find the probability that they see a dolphin for the first time on the fifth day. [1]
 - (c) Find the probability that they do not see a dolphin for at least 10 days. [1]
 - (d) Determine the mean and the variance of the number of days that it takes for them to see a dolphin. [3]
- 3 At a pottery which manufactures mugs, it is known that 5% of mugs are faulty. The mugs are produced in batches of 20. Faults are modelled as occurring randomly and independently. The number of faulty mugs in a batch is denoted by the random variable X .
- (a) Determine $P(X \geq 2)$. [2]
 - (b) Find $\text{Var}(X)$. [1]

Independently of the mugs, the pottery also manufactures cups, and it is known that 7% of cups are faulty. The cups are produced in batches of 30. Faults are modelled as occurring randomly and independently. The number of faulty cups in a batch is denoted by the random variable Y .

- (c) Determine the standard deviation of $X + Y$. [3]

When 10 batches of cups have been produced, a sample of 15 cups is tested to ensure that the handles of the cups are properly attached.

- (d) Explain why it might not be sensible to select a sample of 15 cups from the same batch. [1]

- 4 At a parcel delivery company it is known that the probability that a parcel is delivered to the wrong address is 0.0005. On a particular day, 15 000 parcels are delivered. The number of parcels delivered to the wrong address is denoted by the random variable X .
- (a) Explain why the binomial distribution and the Poisson distribution could both be suitable models for the distribution of X . [3]
- (b) Use a Poisson distribution to find each of the following.
- $P(X = 5)$
 - $P(X \geq 8)$
- [3]

You are given that 15 000 parcels are delivered each day in a 5-day working week.

- (c) (i) Determine the probability that at least 40 parcels are delivered to the wrong address during the week. [2]
- (ii) Determine the probability that at least 8 parcels are delivered to the wrong address on each of the 5 days in the week. [2]

- 5 Two practice GCSE examinations in mathematics are given to all of the students in a large year group. A teacher wants to check whether there is a positive relationship between the marks obtained by the students in the two examinations. She selects a random sample of 20 students. Summary data for the marks obtained in the first and second practice examinations, x and y respectively, are as follows.

$$\Sigma x = 565 \quad \Sigma y = 724 \quad \Sigma x^2 = 17103 \quad \Sigma y^2 = 29286 \quad \Sigma xy = 21635$$

The teacher decides to carry out a hypothesis test based on Pearson's product moment correlation coefficient.

- (a) **In this question you must show detailed reasoning.**
- Calculate the value of Pearson's product moment correlation coefficient. [4]
- (b) Carry out the test at the 5% significance level. [5]
- (c) Given that the teacher did not draw a scatter diagram before carrying out the test, comment on the validity of the test. [1]

- 6 An eight-sided dice has its faces numbered 1, 2, ..., 8.
- (a) In this part of the question you should assume that the dice is fair.
- (i) State the probability that, when the dice is rolled once, the score is at least 6. [1]
- (ii) Show that the probability that the score is within 2 standard deviations of its mean is 1. [4]
- (b) A student thinks that the dice may be biased. To investigate this, the student decides to roll the dice 80 times and then carry out a χ^2 goodness of fit test of a uniform distribution. The spreadsheet below shows the data for the test, where some of the values have been deliberately omitted.

	A	B	C	D
1	Score	Observed frequency	Expected frequency	Chi-squared contribution
2	1	14	10	1.6
3	2	4	10	3.6
4	3	10	10	0
5	4	15	10	
6	5	6	10	1.6
7	6	11	10	0.1
8	7	7	10	0.9
9	8		10	0.9

- (i) Explain why all of the expected frequencies are equal to 10. [1]
- (ii) Determine the missing values in each of the following cells.
- B9
 - D5
- [3]
- (iii) In this question you must show detailed reasoning.
- Carry out the χ^2 test at the 5% significance level. [6]

END OF QUESTION PAPER

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