Oxford Cambridge and RSA

Friday 16 June 2023 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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1 The table shows some spreadsheet output. The values displayed in cells K21 and K22 are exact, but the values displayed in rows 21 and 22 in columns L, M and N have been found using appropriate cell formulae.

	K	L	M	N
20	x	e^x	10^x	$e^x - 10^x$
21	0.000000001	1.000000001	1.000000002	1.30258E-09
22	1E-10	1	1	1.30258E-10

(a) **(i)** Write the value displayed in cell N21 in standard mathematical notation. **[1]**

The formula in cell N22 is $\boxed{= \text{L22} - \text{M22}}$.

(ii) Explain why the value displayed in cell N22 is not zero. **[2]**

(b) **(i)** Determine the absolute error when $\sqrt{e^{0.1}}$ is **chopped** to **3** decimal places. **[2]**

(ii) **Without** doing any further calculation, explain whether the absolute error found in part **(b)(i)** will be the same as the absolute error when $e^{\sqrt{0.1}}$ is **chopped** to **3** decimal places. **[1]**

2 The table gives three values of x and the associated values of y.

x	3	5	8
y	9.26	19.3	37.96

Use Lagrange's method to construct the interpolating polynomial of degree 2 for the values in the table, giving your answer in the form

$$y = ax^2 + bx + c,$$

where a, b and c are constants to be determined. **[4]**

3 You are given that

$P = \dfrac{43.2}{x+y}$ and $Q = \dfrac{43.2}{x-y}$

and that $x = 2.17$ and $y = 2.14$.

The values of x and y have been **rounded** to **2** decimal places.

(a) Calculate the range of possible values of P. **[2]**

(b) Calculate the range of possible values of Q. **[2]**

(c) Explain why the answer to part **(b)** is much larger than the answer to part **(a)**. **[1]**

4 It is given that the function $g(x)$ is continuous and differentiable and that $g'(1.8) = 0.576$, correct to **3** decimal places.

(a) Use this result to determine the error when $g(1.8)$ is used to approximate $g(1.802)$, giving your answer correct to **3** significant figures. **[2]**

The largest root, α, of the equation $x^3 - x^2 - 2x + 1 = 0$ is approximately 1.8. The iterative formula $x_{n+1} = g(x_n)$ where $g(x_n) = \sqrt[3]{x_n^2 + 2x_n - 1}$ is to be used to find this root.

(b) Use the iterative formula with $x_0 = 1.8$ to find α correct to **5** decimal places. **[3]**

The other positive root of the equation, β, is approximately 0.445. It is given that $g'(0.445) = 4.87$, correct to **2** decimal places.

(c) Explain why the iterative formula may not be used to find β. **[1]**

(d) Use the relaxed iteration $x_{n+1} = (1-\lambda)x_n + \lambda g(x_n)$, with $\lambda = -0.258$ and $x_0 = 0.445$, to determine β correct to **8** decimal places. **[3]**

5 The table shows some approximations to $\int_0^1 \sqrt[5]{x^2+3x+2}\,dx$, correct to **8** decimal places, which have been found using the midpoint rule and the trapezium rule. The number of strips, n, used in each application of the two rules is also given.

n	M_n	T_n
1	1.30258554	1.28983372
2	1.29948881	

(a) Determine the value of T_2, giving your answer correct to **8** decimal places. **[2]**

(b) Use your answer to part **(a)** and the values in the table to determine **two** Simpson's rule approximations to $\int_0^1 \sqrt[5]{x^2+3x+2}\,dx$. **[3]**

(c) Hence state the value of $\int_0^1 \sqrt[5]{x^2+3x+2}\,dx$ as accurately as you can, justifying the precision quoted. **[1]**

(d) Use the fact that Simpson's rule is generally a fourth order method to obtain the value of $\int_0^1 \sqrt[5]{x^2+3x+2}\,dx$ as accurately as you can, justifying the precision quoted. **[3]**

6 After taking his pet dog to the vet for a check-up, a dog owner is advised that the dog is too heavy and must therefore go on a special diet. The owner records the dog's mass, W, in kg, at the end of each week. The time, t weeks, from the appointment and the corresponding mass of the dog are shown in the difference table below.

t	W	ΔW	$\Delta^2 W$	$\Delta^3 W$
0	35.90			
1	33.91			
2	32.38			
3	31.25			
4	30.46			

(a) Complete the copy of the difference table in the **Printed Answer Booklet**. **[2]**

The dog owner believes that the relationship between W and t may be modelled by a cubic polynomial.

(b) Explain how the values in the difference table support this belief. **[1]**

(c) Use Newton's forward difference interpolation method to construct a cubic polynomial to model the data, giving your answer in the form

$$W = at^3 + bt^2 + ct + d,$$

where a, b, c and d are constants to be determined. **[4]**

When $t = 6$ it is found that $W = 29.8$.

(d) Determine whether the model is a good fit for this value of t. **[1]**

(e) Explain why the model will not be suitable in the long term. **[1]**

7 The equation $5.39x^3 - 2.17x^2 - 1.11x + 0.45 = 0$ has a root, α, close to -0.5.

(a) Use the Newton-Raphson method with $x_0 = -0.5$ to determine the value of α correct to **7** decimal places. **[4]**

The equation has another root, β, which is close to 0.5. The Newton-Raphson method is implemented using a spreadsheet to find a sequence of approximations to β. These approximations, together with some further analysis, are shown in the output below.

A	B	C	D
r	x_r	difference	ratio
0	0.5		
1	0.46557	−0.03443	
2	0.44744	−0.01814	0.52682
3	0.4381	−0.00934	0.51472
4	0.43336	−0.00474	0.50775
5	0.43097	−0.00239	0.50398
6	0.42977	−0.0012	0.50202
7	0.42917	−0.0006	0.50102
8	0.42887	−0.0003	0.50051
9	0.42872	−0.00015	0.50026
10	0.42865	−7.5E-05	0.50013
11	0.42861	−3.8E-05	0.50006
12	0.42859	−1.9E-05	0.50003
13	0.42858	−9.4E-06	0.50002
14	0.42858	−4.7E-06	0.50001

(b) Write down a suitable cell formula for

(i) cell C10, **[1]**

(ii) cell D12. **[1]**

(c) With reference to the values in column D, explain

- what may be inferred about the order of convergence of this sequence of approximations,
- whether this order of convergence is unusual. **[3]**

8 **Table 8.1** shows some values of x and the associated values of a function $f(x)$.

Table 8.1

x	1.2	2	2.8
$f(x)$	0.9638087	1.3258177	1.4439304

(a) Use the forward difference method to determine an approximation to $f'(2)$. **[2]**

(b) Use the central difference method to determine an approximation to $f'(2)$. **[2]**

(c) Explain whether your answer to part **(a)** or your answer to part **(b)** is likely to be closest to the true value of $f'(2)$. **[1]**

Some additional approximations to $f'(2)$, found using the central difference method, together with some further analysis, are shown in **Table 8.2**. The additional values of $f(x)$ have **not** been given.

Table 8.2

h	approximation	difference	ratio
0.4	0.2506259		
0.2	0.2390539	−0.011572	
0.1	0.2362293	−0.0028246	0.2440877
0.05	0.2355276	−0.0007017	0.2484204

(d) Use extrapolation to determine the value of $f'(2)$ as accurately as you can, justifying the precision quoted. **[4]**

END OF QUESTION PAPER