GCE

Further Mathematics B MEI

Y435/01: Extra pure

A Level

Mark Scheme for June 2023

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.

5. **Annotations**

Annotation	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.
BP	Blank Page
Seen	
Highlighting	

6. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$\frac{\partial z}{\partial x} = 9x^2 + 6y$	**B1**	**1.1a**	Condone poor notation for **B1B1** provided intention clear.	Allow correctly embedded in grad
			$\frac{\partial z}{\partial y} = 6x + 2y$	**B1**	**1.1**		
			SP where $x = 0$ and $y = 0$	**B1**	**1.1**	No justification required. Must be paired. Must follow from their derivatives.	
			SPs where **both** $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$	**M1**	**1.1**	Can be implied by seeing both their derivatives set to 0 and an attempt to solve simultaneously	
			$6x + 2y = 0 \Rightarrow y = -3x$ $\therefore 9x^2 + 6y = 0 \Rightarrow 9x^2 - 18x = 0$	**M1**	**1.1**	Setting both derivatives equal to 0 and eliminating one unknown	or eg $6x + 2y = 0 \Rightarrow 3x = -y$ $\therefore 9x^2 + 6y = 0 \Rightarrow y^2 + 6y = 0$
			$9x(x-2) = 0$ (and $x \neq 0$) $\Rightarrow x = 2 \Rightarrow y = -6$	**A1**	**1.1**	Finding the x and y coordinates of the "non-zero" SP. Must be paired.	Correct answer, no working is **A0**.
			So (0, 0, 0) and (2, –6, –12) and no others	**B1** **[7]**	**1.1**	Do not ISW. Do not condone position vectors. Accept $x =$, $y =$, $z =$.if clearly in triplets.	Correct answer, no working is **B0**.

Question			Answer	Marks	AO	Guidance	
2	(a)		$4t_{n+1} - t_n = 0 \Rightarrow t_n = \alpha \times 4^{-n}$ oe	B1	1.1	Correct form for complementary function (must include arbitrary constant). "+ β" is **B0** unless legitimately recovered.	Could see eg $\alpha \times 0.25^n$ or $\alpha\left(\frac{1}{4}\right)^n$ or $\frac{\alpha}{4^n}$ or $\alpha \times 4^{-(n+1)}$ etc
			Try $t_n = pn + q$	M1	1.1	Correct form for trial function (could be higher polynomial etc if other coefficients found to be 0).	Must have "$t_n = \ldots$" or "trial function is…" oe or see clear evidence of substitution into RR.
			$4(p(n + 1) + q) - (pn + q) = 15n + 17$	M1	1.1	Substituting their form correctly into correct recurrence relation	$4t_n - t_{n-1} = 15n + 2 \Rightarrow$ $4(pn + q) - (p(n-1) + q) = 15n + 2$
			($3p = 15$ and $4p + 3q = 17 \Rightarrow$) $p = 5$, $q = -1$ oe	A1	1.1		
			So GS is ($t_n =$) $5n - 1 + \alpha \times 4^{-n}$	B1FT	1.1	Their numerical PTF plus their CF with one arbitrary constant originally.	Condone missing brackets for $(¼)^n$ for **B1** if used correctly in next step.
			$t_n = 5n - 1 + \alpha \times 4^{-n}$ and $t_1 = 2 \Rightarrow 2 = 4 + \frac{\alpha}{4}$ so $\alpha = \ldots$	M1	1.1	Using initial condition $t_1 = 2$ correctly in their GS (= PTF + CF with one arb const) to find α	Substituting into just CF is **M0**
			… -8 so $t_n = 5n - 1 - 8 \times 4^{-n}$ oe (eg $t_n = 5n - 1 - 2 \times 4^{-n+1}$)	A1	1.1	Full form for solution must be seen (including "$t_n =$") but ISW.	Do not condone $\frac{1}{4}^n$ for **A1**.
				[7]			
2	(b)	(i)	Because it is non-linear	B1	2.4	or because the coefficients are not constant or because it cannot be written in the form $u_{n+1} = cu_n + \mathrm{f}(n)$ where c is a constant oe	Must answer the question or be readable as an answer to the question. Ignore other comments unless egregiously incorrect or directly contradictory
				[1]			

Question			Answer	Marks	AO	Guidance	
2	(b)	(ii)	$n = 1 \Rightarrow a + b = 2$	B1	1.1	Substituting n = 1 into the given formula for u_n.	This could be done as verification that $u_1 = 2$ at the end, after $a = b = 1$ has been established.
			$(n+1)u_{n+1} - u_n^2$ $= (n+1)\left(a(n+1) + \frac{b}{n+1}\right) - \left(an + \frac{b}{n}\right)^2$	*M1	1.1	Correctly substituting for both u_{n+1} and u_n in (LHS of) RR	If $b = 2 - a$ used then: $(n+1)\left(a(n+1) + \frac{2-a}{n+1}\right) - \left(an + \frac{2-a}{n}\right)^2$
			$= an^2 + 2an + a + b - a^2n^2 - 2ab - \frac{b^2}{n^2}$ $= (a - a^2)n^2 + 2an + a + b - 2ab - \frac{b^2}{n^2}$ $= 2n - \frac{1}{n^2}$	dep*M1	1.1	Simplifying LHS correctly with n^2 and constant terms collected (can be implied by correct equations: $2a = 2$, $a - a^2 = 0$, $a + b - 2ab = 0$, $(-)b^2 = (-)1$)	$= an^2 + 2an + a + 2 - a - a^2n^2 - 4a + 2a^2 - \frac{(2-a)^2}{n^2}$ $= (a - a^2)n^2 + 2an + 2 - 4a + 2a^2 - \frac{(2-a)^2}{n^2}$ If **M1M0** then **SCB1** for a = 1, b = 1 found without justification
			eg n term $\Rightarrow a = 1$ and const term $\Rightarrow b = 1$	A1	1.1	(Equating to RHS and) comparing coefficients twice, or once and using $a + b = 2$, to derive values of a and b www.	Could see eg $a + b = 2$ and $a + b - 2ab = 0$ solved simultaneously
			hence $b^2 = 1$(so n^{-2} term is correct) and coefficient of n^2 is zero, as required	A1	2.2a	Verifying properly that all the other coefficients equate/cancel and that $u_1 = 2$. Could also be done by substituting solution with $a = b = 1$ back into the recurrence relation. $u_1 = 2$ must be properly established (but after **B1** this could be done by stating $a + b = 2$).	IF **B0M1M1A1** then **SCA1** can be awarded without establishing $u_1 = 2$.

Question			Answer	Marks	AO	Guidance	
			Alternative method: $n = 1 \Rightarrow a + b = 2$	**B1**	1.1	Substituting $n = 1$ into the given formula for u_n.	
			$n=1,\ u_1=2 \Rightarrow 2u_2-u_1^2=2\times1-\frac{1}{1^2}=1$ $\Rightarrow 2u_2=1+2^2=5 \Rightarrow u_2=\frac{5}{2}$	***B1**	1.1	Substituting $n = 1$ and $u_1 = 2$ into the RR correctly to derive u_2	
			$\therefore 2a+\frac{b}{2}=\frac{5}{2}$ $\therefore 4a+b=4a+2-a=3a+2=5$ $\therefore a=1,\ b=1$	**dep* M1***	1.1	Using the suggested solution on u_2 to derive a second equation in a and b and solving simultaneously	Award for $a = b = 1$ conjecture based on $u_1 = 2/1$, $u_2 = 5/2$, $u_3 = 10/3$, $u_4 = 17/4$, ..., $u_n = (n^2 + 1)/n$
			$\text{LHS}=(n+1)u_{n+1}-u_n^2$ $=(n+1)\left(n+1+\frac{1}{n+1}\right)-\left(n+\frac{1}{n}\right)^2$	**dep** M1**	1.1	Correctly substituting for u_{n+1} and u_n in (LHS of) RR with their values for a and b	
			$=n^2+2n+1+1-n^2-2-\frac{1}{n^2}$ $=2n-\frac{1}{n^2}=\text{RHS as required}$	**A1**	1.1	Simplifying LHS and, after opening all brackets, showing it equals given RHS with $a = b = 1$. Some observation or conclusion is required	
				[5]			
2	**(c)**		$\lim_{n\to\infty} v_n = \lim_{n\to\infty}\frac{5n-1-8\times4^{-n}}{n+\frac{1}{n}} = \lim_{n\to\infty}\frac{5-\frac{1}{n}-\frac{8\times4^{-n}}{n}}{1+\frac{1}{n^2}}$	**B1**	3.1a	Expressing v_n in a way in which the top and bottom both have finite, non-zero limits. Or clear explanation based on behaviour of each term.	or for large n, $v_n \approx \frac{5n}{n}$. Do not condone use of ∞ as a number.
			$= 5$	**B1FT**	2.2a	FT their p/a from a solution for t_n of the form $pn + q + \gamma\delta^{-n}$, $\delta > 1$. Do not accept lim $\to$ 5 or just $v_n = 5$ or similar.	Condone $\frac{"5"}{a}$ if a never found. A solution must be declared for t_n. The limit must follow from this.

Question			Answer	Marks	AO	Guidance	
				[2]			

Question			Answer	Marks	AO	Guidance	
3			$\frac{\partial g}{\partial x}=6x^2-2xy+2y^2$	B1*	3.1a	Can be embedded in ∇g.	Could be rearranged to $z = f(x, y)$: $\frac{\partial f}{\partial x}=\frac{-6x^2+2xy-2y^2}{27}$
			$\frac{\partial g}{\partial y}=-x^2+4xy$	B1*	3.1a	Can be embedded in ∇g.	$\frac{\partial f}{\partial y}=\frac{x^2-4xy}{27}$
			$\nabla g=\begin{pmatrix}6x^2-2xy+2y^2\\ -x^2+4xy\\ 27\end{pmatrix}$	M1	1.2	Attempt to form ∇g using their partial derivatives, either in general form or using numerical values for either of the given points, with a consistent z component. This mark can be awarded for the 3 components clearly and consistently shown, even if not put into vector form.	$\nabla g=\begin{pmatrix}\frac{-6x^2+2xy-2y^2}{27}\\ \frac{x^2-4xy}{27}\\ -1\end{pmatrix}$
			$x=1,\ y=1\Rightarrow$ $\nabla g=\begin{pmatrix}6\times1^2-2\times1\times1+2\times1^2\\ -1^2+4\times1\times1\\ 27\end{pmatrix}\left(=\begin{pmatrix}6\\3\\27\end{pmatrix}\right)$	M1	1.1	Using their ∇g correctly to find a direction vector for normal line to S at $\left(1,1,-\frac{1}{9}\right)$. Condone "$=\begin{pmatrix}2\\1\\9\end{pmatrix}$"	$x=1,\ y=1\Rightarrow\nabla g=\begin{pmatrix}-\frac{2}{9}\\ -\frac{1}{9}\\ -1\end{pmatrix}$
			So equation of normal is $(\mathbf{r}=)\begin{pmatrix}1\\1\\-\frac{1}{9}\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\9\end{pmatrix}$	dep*A1	1.1	or with any equivalent direction vector.	eg $\lambda\begin{pmatrix}-\frac{2}{9}\\ -\frac{1}{9}\\ -1\end{pmatrix}$ or $\lambda\begin{pmatrix}6\\3\\27\end{pmatrix}$

Question			Answer	Marks	AO	Guidance	
			$x=3,\ y=3$ $\Rightarrow \nabla g = \begin{pmatrix} 6\times3^2-2\times3\times3+2\times3^2 \\ -3^2+4\times3\times3 \\ 27 \end{pmatrix} \left(= \begin{pmatrix} 54 \\ 27 \\ 27 \end{pmatrix}\right)$	M1	1.1	Using their ∇g correctly to find a normal vector to the tangent plane to *S* at (3, 3, –3) (or using its components in the tangent plane formula). Condone $"= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}"$	$x=3,\ y=3 \Rightarrow \nabla g = \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}$
			So equation of plane is $\mathbf{r}.\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}.\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} (=6)$	dep*A1	1.1	oe eg $2x+y+z=6$ or $54x+27y+27z=162$	$\mathbf{r}.\begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}.\begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} = -6$
			$\left(\begin{pmatrix} 1 \\ 1 \\ -\frac{1}{9} \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix}\right).\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \frac{26}{9} + 14\lambda = 6 \Rightarrow \lambda = \frac{2}{9}$ so position vector of P is $\begin{pmatrix} 1 \\ 1 \\ -\frac{1}{9} \end{pmatrix} + \frac{2}{9}\begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} = \begin{pmatrix} \frac{13}{9} \\ \frac{11}{9} \\ \frac{17}{9} \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 13 \\ 11 \\ 17 \end{pmatrix}$	A1 **[8]**	2.2a	Or from subbing $x = 1 + 2\lambda$ etc into $2x + y + z = 6$. Condone presentation of final answer as coordinates but not with 1/9 outside. Do not ISW if $\begin{pmatrix} 13 \\ 11 \\ 17 \end{pmatrix}$ seen as final answer.	$\left(\begin{pmatrix} 1 \\ 1 \\ -\frac{1}{9} \end{pmatrix} + \lambda\begin{pmatrix} -\frac{2}{9} \\ -\frac{1}{9} \\ -1 \end{pmatrix}\right).\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \frac{26}{9} - \frac{14}{9}\lambda$ $= 6 \Rightarrow \lambda = -2$ so position vector of P is $\begin{pmatrix} 1 \\ 1 \\ -\frac{1}{9} \end{pmatrix} - 2\begin{pmatrix} -\frac{2}{9} \\ -\frac{1}{9} \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{13}{9} \\ \frac{11}{9} \\ \frac{17}{9} \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 13 \\ 11 \\ 17 \end{pmatrix}$

Question			Answer	Marks	AO	Guidance	
4	(a)		Identity: $\mathbf{I}=\begin{pmatrix}1&0\\0&1\end{pmatrix}\in G$ since $\det\mathbf{I}=1$ and $\mathbf{IA}=\mathbf{AI}=\mathbf{A},\ \forall\mathbf{A}\in G$ So identity property is satisfied.	B1	3.1a	Identity stated as **I**. det**I** = 1 must be stated but condone omission of necessary properties. A statement such as "The identity is $\begin{pmatrix}1&0\\0&1\end{pmatrix}$" alone is insufficient.	There is no need to assert the 'realness' of the relevant matrices in any of the justifications in this part.
			Closure: Two (real) 2×2 matrices multiplied together give a (real) 2×2 matrix. Also if $\mathbf{A},\mathbf{B}\in G$ then det**A** = det**B** = 1. $\therefore\det(\mathbf{AB})=\det\mathbf{A}\times\det\mathbf{B}=1\times1=1$ $\therefore\mathbf{AB}\in G$ and so it's closed.	B1	2.2a	No justification required for multiplication statement. Can be implied by example given in the closure context. Condone non-independence/generality of matrices in any example. Must see some justification for det(**AB**) = 1 (could be based on area scale factors) and must explicitly state that **AB** is in G to claim closure.	
			Inverse: $\forall\mathbf{A}\in G$ since detA = 1 ($\neq 0$), $\mathbf{A}^{-1}$exists.	B1	2.1	or $\begin{pmatrix}a&b\\c&d\end{pmatrix}^{-1}=\begin{pmatrix}d&-b\\-c&a\end{pmatrix}$ stated	
			and since $\mathbf{AA}^{-1}=\mathbf{I}$ and $\det(\mathbf{AA}^{-1})=\det\mathbf{A}\det\mathbf{A}^{-1}$, $\therefore\det\mathbf{A}^{-1}=1$ (so $\mathbf{A}^{-1}\in G$) so the inverse property is satisfied.	B1	2.1	or $\det\mathbf{A}^{-1}=ad-bc=\det\mathbf{A}=1$ (so $\mathbf{A}^{-1}\in G$) so the inverse property is satisfied. Some justification for $\det\mathbf{A}^{-1}=1$ needed.	
			Associativity: Given. So all 4 properties (axioms) are satisfied and so $(G,\times)$ is a group	B1	3.2a	Must be a conclusion which demonstrates understanding that precisely these 4 properties are required. Can be awarded even if not all other marks awarded.	Ignore reference to commutativity except if it is considered to be a required property. Do not award this mark if it is clear that the candidate has confused associativity with commutativity (eg stating "**AB** = **BA**, given")

Question			Answer	Marks	AO	Guidance	
				[5]			
4	(b)	(i)	$\mathbf{A}_m \times \mathbf{A}_n = \begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} \quad (m, n \geq 0)$ $= \begin{pmatrix} 1 & 0 \\ m+n & 1 \end{pmatrix} \in S$ since two non-negative integers add to give a non-negative integer so S is closed (under $\times$)	M1 A1 [2]	2.4 2.2a	Considering 2 different, general, independent elements of S multiplied together. Some justification must be given	
4	(b)	(ii)	$\mathbf{A}_n^{-1} = \begin{pmatrix} 1 & 0 \\ -n & 1 \end{pmatrix}$ (where $n > 0$) $\notin S$ since $-n$ is a negative integer so S is not a subgroup since the inverse axiom is not met	M1 A1 [2]	2.4 2.2a	Forming the correct inverse of any non-identity element of S. Can be general or specific. Some justification must be given. Must be from correct reasoning.	
4	(c)	(i)	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\left(\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}, \times \right)$ cao	M1 A1 [2]	3.1a 2.5	Identifying $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ in any form Must be notationally correct (ie elements separated by comma in curly braces) but subset alone is sufficient. Either order. Could be defined indirectly; eg $\{\mathbf{A}, \mathbf{A}^2\}$ where $\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.	

Question			Answer	Marks	AO	Guidance	
4	(c)	(ii)	(Subgroup of order 2 must be isomorphic to the (cyclic) group of order 2 so) if **M** is the non-identity element, $\mathbf{M} = \mathbf{M}^{-1}$ (or $\mathbf{M}^2 = \mathbf{I}$ or equivalent).	B1	2.4	Or $\mathbf{M}^{-1}$ is in the subgroup (closure) and can't be **I** so must be **M**. Or $\mathbf{M}^2$ is in the subgroup and can't be **M** so must be **I**.	
			This, together with $\det \mathbf{M} = 1$ means that $\mathbf{M}^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ $\Rightarrow a = d$ and $b = c = 0$. But $a^2 = 1$ since $ad - bc = 1 \Rightarrow a = d = -1$ is only possibility for g as $a \neq 1$ (**M** is not the identity)	B1 [2]	2.4	Must see form for inverse. Or, instead, complete argument involving square of matrix (must see form for square and correct conclusions based on this equalling identity and det equalling 1)	$ad - bc = 1$ (so $bc = ad - 1$) and $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^2 = \begin{pmatrix} a^2 + bc & b(a+d) \\ c(a+d) & d^2 + bc \end{pmatrix}$ $= \begin{pmatrix} a(a+d)-1 & b(a+d) \\ c(a+d) & d(a+d)-1 \end{pmatrix}$ $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$ either $(a + d = 0$ $\Rightarrow a(a+d) - 1 = -1 = 1\#)$ or $(a + d \neq 0 \Rightarrow b = c = 0 \Rightarrow ad = 1$ and $a^2 = 1$ and $d^2 = 1 \Rightarrow a = d = -1$ since $a = 1 \Rightarrow d = 1$ gives the identity)
4	(d)		The inverse property is not satisfied.	B1	2.4		
			eg $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ is in the set but as $\det \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = 0$ it does not have an inverse in the set.	B1 [2]	2.4	Any singular real 2 by 2 matrix. Stating only that it has no inverse is insufficient; some justification is required.	

Question			Answer	Marks	AO	Guidance	
5	(a)		$\begin{vmatrix} a-\lambda & 0 \\ 2 & 3-\lambda \end{vmatrix} = (a-\lambda)(3-\lambda) = 0$	M1	3.1a	Forming characteristic equation.	
			e-vals are 3 and a and no others.	A1	1.1		
			$\lambda = 3: \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ 2x+3y \end{pmatrix} = \begin{pmatrix} 3x \\ 3y \end{pmatrix}$	M1	1.1	Consideration of $\mathbf{Ae} = \lambda\mathbf{e}$ (or $(\mathbf{A} - \lambda\mathbf{I})\mathbf{e} = \mathbf{0}$) for one correct e-value. (ie $ax = 3x$ or $2x + 3y = 3y$)	or $\begin{pmatrix} a-3 & 0 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (a-3)x \\ 2x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
			$\lambda = 3: \mathbf{e}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ www	A1	1.1	Or any non-zero (possibly algebraic) multiple.	If eg $\begin{pmatrix} 0 \\ s \end{pmatrix}$ then condone omission of condition on s (ie $s \neq 0$).
			$\lambda = a: \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ 2x+3y \end{pmatrix} = \begin{pmatrix} ax \\ ay \end{pmatrix}$	M1	1.1	Consideration of $\mathbf{Ae} = \lambda\mathbf{e}$ (or $(\mathbf{A} - \lambda\mathbf{I})\mathbf{e} = \mathbf{0}$) for other correct e-value. (ie ($ax = ax$ and) $2x + 3y = ay$)	or $\begin{pmatrix} 0 & 0 \\ 2 & 3-a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2x+(3-a)y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
			$\lambda = a: \mathbf{e}_2 = \begin{pmatrix} a-3 \\ 2 \end{pmatrix}$ www	A1	1.1	Or any non-zero (possibly algebraic) multiple.	See above note on multiplier. $\begin{pmatrix} \frac{a-3}{2} \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ \frac{2}{a-3} \end{pmatrix}$
			$\mathbf{e}_1.\mathbf{e}_2 = 2 = 1 \times \sqrt{(a-3)^2 + 2^2} \cos\frac{1}{4}\pi$	M1	2.2a	Forming numerical equation in a by using the dot product with reasonable attempt at $\lvert\mathbf{e}_2\rvert$. **M0** if $\mathbf{e}_1.\mathbf{e}_2 = 0$. Ignore minor inconsistency in angle between their e-vecs (ie acute/obtuse).	Could argue via gradients; direction of $\mathbf{e}_1$ is along y-axis so 'gradient' of $\mathbf{e}_2$ must be ± 1; $\frac{2}{a-3} = \pm 1$ (= 1 insufficient for **M1**).
			($a^2 - 6a + 13 = 8$ so) $a = 1$ or $a = 5$ www	A1	1.1	For both and no others.	Not from wrong e-vecs, eg $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ a-3 \end{pmatrix}$ or $\begin{pmatrix} 3-a \\ 2 \end{pmatrix}$

Question			Answer	Marks	AO	Guidance	
			Alternative method for final M1A1M1A1:				
			$\mathbf{e}_2 = \begin{pmatrix} p \\ q \end{pmatrix} \Rightarrow \mathbf{e}_1.\mathbf{e}_2 = q = 1 \times \sqrt{p^2+q^2}\cos\frac{1}{4}\pi$	M1	1.1	Forming dot product of their e-vecs with general form for $\mathbf{e}_2$ to derive numerical equation in two unknowns, the components of $\mathbf{e}_2$.	
			=> $p^2 = q^2$ => $p = \pm q$ => $\mathbf{e}_2 = \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$ oe	A1	1.1	Must be two possible e-vecs.	
			$\therefore \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} a \\ 2 \pm 3 \end{pmatrix} = a\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}$	M1	2.2a	Using e-vec property with $\mathbf{e}_2$ and a as the e-val.	or $\begin{pmatrix} 0 & 0 \\ 2 & 3-a \end{pmatrix}\begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \pm (3-a) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
			$a = 1$ or $a = 5$ www	A1	1.1	For both and no others.	Not from $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
				[8]			
5	(b)	(i)	$\lambda^2 - (a+3)\lambda + 3a = 0$	M1	3.1a	Correctly expanding characteristic equation.	Must be an equation but can be recovered.
			$\Rightarrow \mathbf{P}^2 - (a+3)\mathbf{P} + 3a\mathbf{I} = \mathbf{O}$ $\Rightarrow \mathbf{P}^2 - (a+3)\mathbf{P} = -3a\mathbf{I}$	M1	3.1a	Using C-H theorem and expressing either equation in form with which direct comparison can be made (either can be implied by correct equations in a and r but not only by correct answers).	$\mathbf{P}^2 + r\mathbf{P} - \mathbf{I} = \mathbf{O}$
			(So need $-3a = 1 \Rightarrow) a = -\frac{1}{3}$	A1	2.2a	C-H must be used.	
			So $r = (-(a+3) =) -\frac{8}{3}$ oe	A1	2.2a	$r = -2\frac{2}{3}$. C-H must be used	If **M1M0** and C-H apparently used in derivation of a and r then **SCB1** if a and r correctly found.

Question			Answer	Marks	AO	Guidance	
			Alternative method:				
			$\mathbf{P}^2 + r\mathbf{P} = \mathbf{I}$ oe (must be the characteristic equation) => $\lambda^2 + r\lambda = 1$ oe	**B1**		Use of C-H. Can be implied by substitution of **both** e-vals into scalar equation	Must be 1 not **I**. Can be recovered
			$3^2 + r \times 3 = 1 \Rightarrow 3r = -8 \Rightarrow r = -\frac{8}{3}$	**M1**		Substituting numerical e-val into char equation to derive value of r.	
			$a^2 + r \times a = 1 \Rightarrow a^2 - \frac{8}{3}a = 1$	**M1**		Substituting algebraic e-val into char equation with their value of r	
			$\therefore 3a^2 - 8a - 3 = 0 \Rightarrow a = -\frac{1}{3}$ or $a = 3$	**A1**		$(3a + 1)(a - 3)$	
			But $a \neq 3$ (given) so $a = -\frac{1}{3}$			$a = 3$ must be explicitly rejected.	
				[4]			
5	(b)	(ii)	$\mathbf{P}^4 = (\mathbf{I} - r\mathbf{P})^2 = \mathbf{I} - 2r\mathbf{P} + r^2\mathbf{P}^2$ or $\mathbf{P}^4 = (\mathbf{I} + \frac{8}{3}\mathbf{P})^2 = \mathbf{I} + \frac{16}{3}\mathbf{P} + \frac{64}{9}\mathbf{P}^2$	**M1**	3.1a	Squaring and expanding expression for $\mathbf{P}^2$ with r algebraic or their value. Need $\mathbf{I}^2 = \mathbf{I}$ and $\mathbf{IP} = \mathbf{P}$ soi.	Or multiplying by **P** to find $\mathbf{P}^3$ and eliminating $\mathbf{P}^2$. $\mathbf{P}^3 = \mathbf{P} - r\mathbf{P}^2 = \mathbf{P} - r(\mathbf{I} - r\mathbf{P})$ or $\mathbf{P}^3 = \mathbf{P} + \frac{8}{3}\mathbf{P}^2 = \mathbf{P} + \frac{8}{3}\left(\mathbf{I} + \frac{8}{3}\mathbf{P}\right)$
			$= \mathbf{I} - 2r\mathbf{P} + r^2(\mathbf{I} - r\mathbf{P}) = (1 + r^2)\mathbf{I} - (2r + r^3)\mathbf{P}$ or $= \mathbf{I} + \frac{16}{3}\mathbf{P} + \frac{64}{9}\left(\mathbf{I} + \frac{8}{3}\mathbf{P}\right)$ $= \left(1 + \frac{64}{9}\right)\mathbf{I} + \left(\frac{16}{3} + \frac{512}{27}\right)\mathbf{P}$	**M1**	2.2a	Substituting for $\mathbf{P}^2$ and collecting **I** and **P** terms and no others. NB Other approaches are possible (eg squaring $\mathbf{I} = \mathbf{P}^2 + r\mathbf{P}$ or multiplying throughout by $\mathbf{P}^2$). 1st **M1** for a useful operation and substitution, 2nd **M1** for further algebra leading to correct form.	Or multiplying by **P** again and eliminating $\mathbf{P}^2$ again. $\mathbf{P}^4 = (1 + r^2)\mathbf{P}^2 - r\mathbf{P}$ $= (1 + r^2)(\mathbf{I} - r\mathbf{P}) - r\mathbf{P} = \text{etc}$
			$\mathbf{P}^4 = \frac{73}{9}\mathbf{I} + \frac{656}{27}\mathbf{P}$ so $s = \frac{73}{9}$, $t = \frac{656}{27}$ cao	**A1**	1.1	Allow embedded answer	If **M1M0** because correct form not reached then **SCB1** for correct values of s and t properly obtained
				[3]			

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