



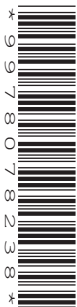
Oxford Cambridge and RSA

Thursday 22 June 2023 – Afternoon

A Level Further Mathematics A

Y544/01 Discrete Mathematics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

1 The table below shows the activities involved in a project together with the immediate predecessors and the duration of each activity.

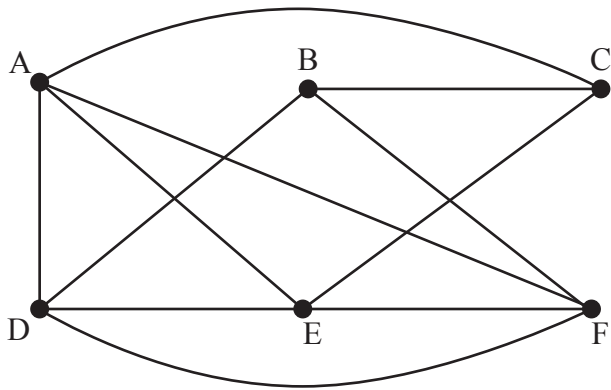
Activity	Immediate predecessors	Duration (hours)
A	–	2
B	A	3
C	–	4
D	C	2
E	B, C	2
F	D, E	3
G	E	2
H	F, G	1

- (a) Model the project using an activity network. [3]
- (b) Determine the minimum project completion time. [2]

The start of activity C is delayed by 2 hours.

- (c) Determine the minimum project completion time with this delay. [2]

2 A graph is shown below.



- (a) Write down a cycle through all six vertices. [1]
- (b) Write down a continuous route that uses every arc exactly once. [2]
- (c) Use Kuratowski’s theorem to show that the graph is **not** planar. [2]
- (d) Show that the graph has thickness 2. [3]

3 An initial simplex tableau is given below.

P	x	y	z	s	t	RHS
1	-2	3	-1	0	0	0
0	5	-4	1	1	0	20
0	2	-1	0	0	1	6

(a) Carry out **two** iterations of the simplex algorithm, choosing the first pivot from the x column. [4]

After three iterations the resulting tableau is as follows.

P	x	y	z	s	t	RHS
1	3	-1	0	1	0	20
0	5	-4	1	1	0	20
0	2	-1	0	0	1	6

- (b) State the values of P , x , y , z , s and t that result from these three iterations. [2]
- (c) Explain why no further iterations are possible. [2]

The initial simplex tableau is changed to the following, where k is a positive real value.

P	x	y	z	s	t	RHS
1	2	-3	1	0	0	0
0	5	k	1	1	0	20
0	2	-1	0	0	1	6

After one iteration of the simplex algorithm the value of P is 500.

(d) Deduce the value of k . [4]

- 4 The first 20 consecutive positive integers include the 8 prime numbers 2, 3, 5, 7, 11, 13, 17 and 19.

Emma randomly chooses 5 distinct numbers from the first 20 consecutive positive integers. The order in which Emma chooses the numbers does **not** matter.

- (a) Calculate the number of possibilities in which Emma's 5 numbers include exactly 2 prime numbers and 3 non-prime numbers. [2]
- (b) Calculate the number of possibilities in which Emma's 5 numbers include at least 2 prime numbers. [3]

The pairs $\{3, 13\}$ and $\{7, 17\}$ each consist of numbers with a difference of exactly 10.

- (c) Calculate the number of possibilities in which Emma's 5 numbers include at least one pair of prime numbers in which the difference between them is exactly 10. [3]

A new set of 20 consecutive positive integers, each with at least two digits, is chosen. This set of 20 numbers contains 5 prime numbers.

- (d) Use the pigeonhole principle to show that there is at least one pair of these prime numbers for which the difference between them is exactly 10. [2]

5 A list of 8 values is given below.

3 24 8 1 4 20 30 18

The list is to be sorted into increasing order using quick sort, as given in the Formulae Booklet.

(a) Carry out the first **two** passes of the sort. [4]

A different list of 8 values is to be sorted into increasing order using quick sort, as given in the Formulae Booklet.

(b) (i) State the maximum number of passes that could be required. [1]

(ii) Find the minimum number of passes that could be required. [2]

The run-time for quick sort could be measured by counting the number of comparisons used.

In the worst case, the run time for quick sort is $O(n^2)$.

A computer takes at most 0.03 seconds to sort a list of 100 values into increasing order using quick sort.

(c) Calculate an estimate for the time taken, in the worst case, to sort a list of 500 values using quick sort. [2]

A list of n values (where $n > 10$) is to be sorted into increasing order using quick sort, as given in the Formulae Booklet.

(d) Explain why, in the **best** case, $n - 3$ comparisons are used in the second pass. [3]

- 6 A graph is shown in **Fig. 1.1**.
The graph is weighted to form the network represented by the weighted matrix in **Fig. 1.2**.
The weights represent distances in km.
A dash (-) means that there is no direct arc between that pair of vertices.

Fig. 1.1

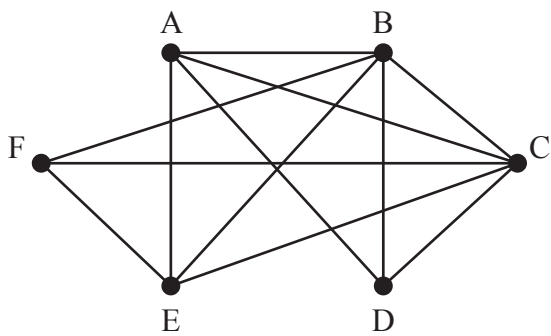


Fig. 1.2

	A	B	C	D	E	F
A	-	5	3	2	8	-
B	5	-	3	4	7	6
C	3	3	-	1	6	5
D	2	4	1	-	-	-
E	8	7	6	-	-	6
F	-	6	5	-	6	-

The shortest path from D to F has total weight 6.

- (a) Write down a path from D to F of total weight 6. [1]

The total weight of the 12 arcs in the network is 56.

- (b) Use the route inspection algorithm to calculate the total weight of the least weight route that covers every arc at least once, starting at vertex A. [3]
- (c) Determine the total weight of the least weight route that covers every arc at least once, starting at vertex B but finishing at any vertex. [2]

Sasha wants to find a continuous route through every vertex, starting and finishing at vertex A, with the least total weight.

- (d) (i) Use an appropriate algorithm to find a lower bound for the total weight of Sasha's route. [4]
- (ii) Use the Nearest Neighbour Algorithm, starting at vertex A, to find an upper bound for the total weight of Sasha's route. [2]

Sasha decides to use the route A – B – F – E – C – D – A.

- (e) Comment on the suitability of this route as a solution to Sasha's problem. [2]

- 7 Player 1 and player 2 are playing a two-person zero-sum game.

In each round of the game the players each choose a strategy and simultaneously reveal their choice.

The number of points won in each round by player 1 for each combination of strategies is shown in the table below.

Each player is trying to maximise the number of points that they win.

		Player 2		
		A	B	C
Player 1	X	2	-3	-4
	Y	0	1	3
	Z	-2	2	4

- (a) (i) Determine play-safe strategies for each player. [3]
- (ii) Show that the game is **not** stable. [1]
- (b) Show that the number of strategies available to player 1 **cannot** be reduced by dominance. You must make it clear which values are being compared. [2]

Player 1 intends to make a random choice between strategies X, Y, Z, choosing strategy X with probability x , strategy Y with probability y and strategy Z with probability z .

Player 1 formulates the following LP problem so they can find the optimal values of x , y and z using the simplex algorithm.

Maximise $M = m - 4$
 subject to $m \leq 6x + 4y + 2z$
 $m \leq x + 5y + 6z$
 $m \leq 7y + 8z$
 $x + y + z \leq 1$
 and $m \geq 0, x \geq 0, y \geq 0, z \geq 0$

- (c) Explain how the inequality $m \leq 6x + 4y + 2z$ was formed. [2]

The problem is solved by running the simplex algorithm on a computer.

The printout gives a solution in which $x + y = 1$.

This means that the LP problem can be reduced to the following formulation.

Maximise $M = m - 4$
 subject to $m \leq 4 + 2x$
 $m \leq 5 - 4x$
 $m \leq 7 - 7x$
 and $m \geq 0, x \geq 0$

- (d) Solve this problem to find the optimal values of x , y and z and the corresponding value of the game to player 1. [4]

END OF QUESTION PAPER



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