AQA

AS
FURTHER MATHEMATICS
7366/1

Paper 1

Mark scheme

June 2022

Version: 1.0 Final Mark Scheme

226A7366/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the correct answer.	1.1b	B1	$e^x - e^{-x}$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the correct answer.	1.2	B1	q
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks the correct box.	1.1b	B1	Reflection in the plane $y = 0$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box.	1.1b	B1	$6(\cos(\alpha+\beta)+i\sin(\alpha+\beta))$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5	Expands $(2+\mathrm{i})^3$ to produce an expression of four terms with no more than one incorrect term. Or correctly expands $(2+\mathrm{i})^2$ to two, three or four terms equivalent to $4+4\mathrm{i}+\mathrm{i}^2$ **and** then multiplies by $2+\mathrm{i}$ to produce an expression of at least three terms with no more than one incorrect term. The terms may be unsimplified.	1.1a	M1	$(2+i)^3$ $= 1.2^3 i^0 + 3.2^2 i^1 + 3.2^1 i^2 + 1.2^0 i^3$ $= 8 + 12i + 6(-1) + (-i)$ $= 2 + 11i$
	At least one instance of i^2 replaced with -1 or i^3 replaced with $-\mathrm{i}$ PI by $3+4\mathrm{i}$	1.2	B1	
	Completes a reasoned argument to show that $(2+\mathrm{i})^3$ is $2+11\mathrm{i}$	2.1	R1	
	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Obtains the correct determinant.	1.1b	B1	det **A** $= 5 \times 4 - (-3) \times 2$ $= 26$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains the correct inverse matrix ACF FT their determinant	1.1b	B1F	$\mathbf{A}^{-1} = \frac{1}{26}\begin{bmatrix} 4 & -2 \\ 3 & 5 \end{bmatrix}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	Selects a method to find either matrix **B** or matrix **AM** e.g. calculates $\mathbf{A}^{-1}\mathbf{AB}$ with their $\mathbf{A}^{-1}$ or calculates $2\mathbf{A}^2+\mathbf{AB}$ or writes four simultaneous equations in w, x, y, z where $\begin{bmatrix} 9 & 6 \\ 5 & 12 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix}$	3.1a	M1	$\mathbf{B} = \mathbf{A}^{-1}\mathbf{AB}$ $\mathbf{B} = \frac{1}{26}\begin{bmatrix} 4 & -2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 9 & 6 \\ 5 & 12 \end{bmatrix}$ $= \frac{1}{26}\begin{bmatrix} 26 & 0 \\ 52 & 78 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ $\mathbf{M} = 2\begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ $= \begin{bmatrix} 11 & 4 \\ -4 & 11 \end{bmatrix}$
	Obtains a correct matrix for **B** or **AM** i.e. $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ or $\begin{bmatrix} 47 & 42 \\ -49 & 32 \end{bmatrix}$ PI by a correct matrix **M** FT their $\mathbf{A}^{-1}$	1.1b	A1F	
	Obtains matrix **M** FT their $\mathbf{A}^{-1}$	2.2a	A1F	
	Subtotal		**3**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Calculates a value of λ for point *P* Or writes a correct equation for l_1 in Cartesian form (accept one error) and substitutes at least one of $x = -3, \quad y = 9, \quad z = -4$	1.1a	M1	$3 + 3\lambda = -3 \Rightarrow 3\lambda = -6 \Rightarrow \lambda = -2$ $1 - 4\lambda = 9 \Rightarrow -4\lambda = 8 \Rightarrow \lambda = -2$ $-2 + \lambda = -4 \Rightarrow \lambda = -2$ All three values of λ are the same so *P* lies on l_1
	Completes an argument to show that *P* lies on l_1 Accept three correct calculations using $\lambda = -2$ which lead to $\begin{bmatrix} -3 \\ 9 \\ -4 \end{bmatrix}$	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Writes the scalar product of the two direction vectors.	1.1a	M1	$\begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = 3 \times 3 + (-4) \times 2$ $+ 1 \times (-1)$ $= 9 - 8 - 1$ $= 0$ $\therefore$ l_1 and l_2 are perpendicular
	Completes an argument to show that l_1 and l_2 are perpendicular. Accept, for two marks, $3 \times 3 + (-4) \times 2 + 1 \times (-1) = 0$ so they are perpendicular Condone $9 - 8 - 1 = 0$ with a reference to the scalar product.	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Selects a method to find the value of a eg equates the $\boldsymbol{i}$ or $\boldsymbol{k}$ component to form at least one equation in λ and μ	3.1a	M1	$3+3\lambda=-12+3\mu \quad \Rightarrow \quad \lambda=\mu-5$ $-2+\lambda=-3-\mu \quad \Rightarrow \quad \lambda=-1-\mu$
	Forms two correct equations in λ and μ PI by a correct value of a PI by a correct value of λ **or** μ	1.1b	A1	$\mu-5=-1-\mu$ $2\mu=4$ $\mu=2$
	Calculates correct values of λ **and** μ PI by a correct value of a	1.1b	A1	$\lambda=-1-2=-3$
	Obtains a correct value of a FT their λ and μ	1.1b	A1F	$1-4\lambda=a+2\mu$ $1-4\times(-3)=a+2\times2$ $a=9$
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(d)	Obtains the correct coordinates of the point of intersection. Condone an answer of $\begin{bmatrix}-6\\13\\-5\end{bmatrix}$ FT their λ **or** their a and μ	1.1b	B1F	$\begin{bmatrix}3+(-3)\times3\\1+(-3)\times(-4)\\-2+(-3)\times1\end{bmatrix}=\begin{bmatrix}-6\\13\\-5\end{bmatrix}$ Point of intersection $=(-6,13,-5)$
	Subtotal		**1**	

	Question total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Verifies that $r = 3$ and $\theta = \frac{\pi}{3}$ satisfies the polar equation. Condone missing conclusion. Accept $4 - 2 \times \frac{1}{2} = 3$ as sufficient verification.	1.1b	B1	$4 - 2\cos\left(\frac{\pi}{3}\right) = 4 - 2 \times \frac{1}{2} = 3$ $\therefore$ $\left(3, \frac{\pi}{3}\right)$ lies on *C*
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Selects a method to find the required polar coordinates, eg substitutes $\cos\theta =$ -1 to find r **or** solves $\cos\theta =$ -1 to find θ PI by a correct value for r **or** θ	3.1a	M1	$r = 4 - 2 \times (-1) = 6$ $\cos\theta = -1 \quad \Rightarrow \quad \theta = \pi$ furthest from *O* is $(6, \pi)$
	Obtains a correct value for r **or** θ	1.1a	A1	
	Obtains the correct polar coordinates.	3.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Substitutes $\theta = \frac{\pi}{6}$ to find r PI by 2.27 or 1.96 or 1.13 or better	1.1a	M1	$r = 4 - 2\cos\left(\frac{\pi}{6}\right)$ $= 4 - 2 \times \frac{\sqrt{3}}{2}$ $= 4 - \sqrt{3}$ $x = (4 - \sqrt{3})\cos\left(\frac{\pi}{6}\right) = 2\sqrt{3} - \frac{3}{2}$ $y = (4 - \sqrt{3})\sin\left(\frac{\pi}{6}\right) = 2 - \frac{\sqrt{3}}{2}$ $\left(2\sqrt{3} - \frac{3}{2}, 2 - \frac{\sqrt{3}}{2}\right)$
	Obtains an expression for the x or the y-coordinate. PI by 1.96 or 1.13 or better	2.2a	M1	
	Obtains the correct exact Cartesian coordinates. ACF	1.1b	A1	
	Subtotal		**3**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Completes a rigorous argument to show that $\ln(r+2) - \ln r = \ln\left(1 + \frac{2}{r}\right)$	2.1	R1	$\ln(r+2) - \ln r = \ln\left(\frac{r+2}{r}\right)$ $= \ln\left(1 + \frac{2}{r}\right)$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Writes at least two pairs of logs in the form $\ln(r+2) - \ln r$	1.1a	M1	$\sum_{r=1}^{n}\left(1 + \frac{2}{r}\right) = \sum_{r=1}^{n}(\ln(r+2) - \ln r)$ $= \ln 3 - \ln 1$ $+ \ln 4 - \ln 2$ $+ \ln 5 - \ln 3$ $+ \ldots\ldots\ldots\ldots\ldots$ $+ \ln n - \ln(n-2)$ $+ \ln(n+1) - \ln(n-1)$ $+ \ln(n+2) - \ln n$ $= \ln(n+2) + \ln(n+1) - \ln 2 - \ln 1$ $= \ln\left(\frac{1}{2}(n+1)(n+2)\right)$
	Writes at least three pairs of logs in the form $\ln(r+2) - \ln r$ including the first pair, the last pair, and at least one other pair.	1.1a	M1	
	Correctly reduces the expression to three or four log terms. Condone missing brackets.	1.1b	A1	
	Completes a fully correct proof to reach the required result. This mark is only available if all previous marks have been awarded. Must include at least one pair of cancelling terms. Must include correct use of brackets throughout.	2.1	R1	
	Subtotal		**4**	

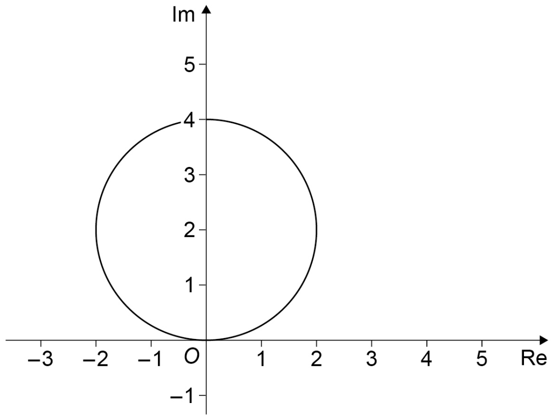
	Question 9 total		**5**	

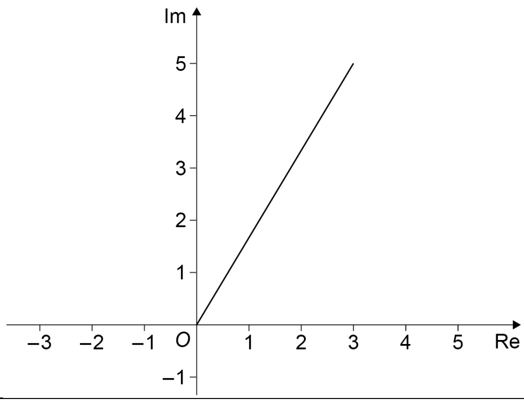
Q	Marking instructions	AO	Marks	Typical solution
10(a)	Writes any equation of a non-circular ellipse.	1.1a	M1	$\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ $\frac{x^2}{9}+\frac{y^2}{4}=1$
	Obtains a correct equation of E. Accept 3^2 for 9 and 2^2 for 4	1.1b	A1	
	Subtotal		**2**	

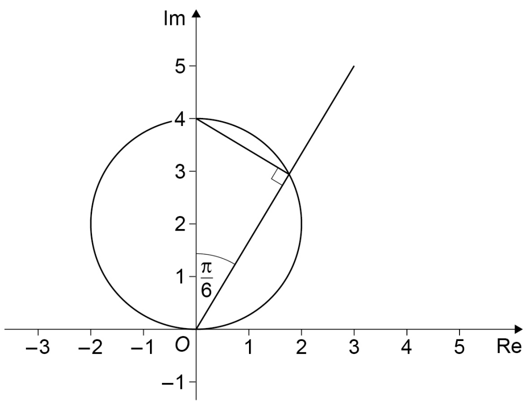
Q	Marking instructions	AO	Marks	Typical solution
10(b)	Obtains a correct expression for y^2 (or y) in terms of x FT their ellipse equation.	1.1b	B1F	$\frac{y^2}{4}=1-\frac{x^2}{9}$ $y^2=4-\frac{4x^2}{9}$ $\text{Volume}=\pi\int_{-3}^{3}\left(4-\frac{4x^2}{9}\right)dx$ $=\pi\left[4x-\frac{4x^3}{27}\right]_{-3}^{3}$ $=\pi\left(4\times3-\frac{4\times3^3}{27}\right)$ $-\pi\left(4\times(-3)-\frac{4\times(-3)^3}{27}\right)$ $=\pi(12-4)-\pi(-12+4)$ $=16\pi$
	Uses the formula for volume of revolution to write any expression of the form $\int(mx^2+c)$ for any non-zero c Condone missing $\pi,\ dx$ and missing or incorrect limits. Condone $\int(my^2+c)$	3.1a	M1	
	Writes a fully correct expression for the volume, eg $\pi\int_{-3}^{3}\left(4-\frac{4x^2}{9}\right)dx$ Condone missing brackets.	2.1	R1	
	Obtains the correct volume in exact form.	1.1b	A1	
	Subtotal		**4**	

	Question 10 total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
11	Shows that $(\mathbf{ABA}^{-1})^n = \mathbf{AB}^n\mathbf{A}^{-1}$ is true for $n = 1$	2.1	B1	Let $n = 1$: $(\mathbf{ABA}^{-1})^1 = \mathbf{ABA}^{-1} = \mathbf{AB}^1\mathbf{A}^{-1}$ $\therefore$ it is true for $n = 1$ If it is true for $n = k$, then $(\mathbf{ABA}^{-1})^k = \mathbf{AB}^k\mathbf{A}^{-1}$ $\Rightarrow (\mathbf{ABA}^{-1})^k\mathbf{ABA}^{-1} = \mathbf{AB}^k\mathbf{A}^{-1}\mathbf{ABA}^{-1}$ $\Rightarrow (\mathbf{ABA}^{-1})^{k+1} = \mathbf{AB}^k\mathbf{IBA}^{-1}$ $\Rightarrow (\mathbf{ABA}^{-1})^{k+1} = \mathbf{AB}^k\mathbf{BA}^{-1}$ $\Rightarrow (\mathbf{ABA}^{-1})^{k+1} = \mathbf{AB}^{k+1}\mathbf{A}^{-1}$ $\Rightarrow$ it is also true for $n = k + 1$ Therefore, by induction, $(\mathbf{ABA}^{-1})^n = \mathbf{AB}^n\mathbf{A}^{-1}$ is true for all integers $n \geq 1$
	Assumes $(\mathbf{ABA}^{-1})^k = \mathbf{AB}^k\mathbf{A}^{-1}$ and multiplies by $\mathbf{ABA}^{-1}$	2.4	M1	
	Completes rigorous working to show $(\mathbf{ABA}^{-1})^{k+1} = \mathbf{AB}^{k+1}\mathbf{A}^{-1}$ Condone $\mathbf{A}^{-1}\mathbf{A}$ removed without reference to $\mathbf{I}$	2.2a	A1	
	Concludes a reasoned argument by stating that $(\mathbf{ABA}^{-1})^n = \mathbf{AB}^n\mathbf{A}^{-1}$ is true for $n = 1$, and that $(\mathbf{ABA}^{-1})^k = \mathbf{AB}^k\mathbf{A}^{-1}$ implies $(\mathbf{ABA}^{-1})^{k+1} = \mathbf{AB}^{k+1}\mathbf{A}^{-1}$ and hence, by induction, that $(\mathbf{ABA}^{-1})^n = \mathbf{AB}^n\mathbf{A}^{-1}$ is true for all integers $n \geq 1$ Condone $\mathbf{A}^{-1}\mathbf{A}$ removed without reference to $\mathbf{I}$	2.1	R1	
	Question total		**4**	

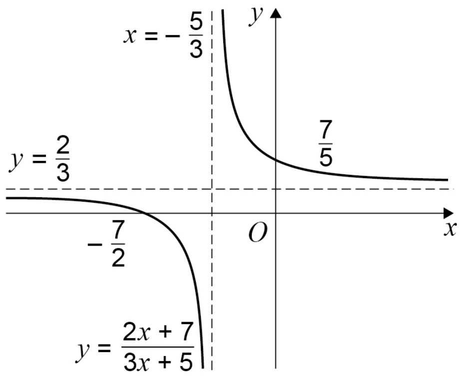
Q	Marking instructions	AO	Marks	Typical solution
12(a)	Draws a circle with radius 2 **or** centre $2i$ Condone a freehand circle if intention is clear.	1.1a	M1	
	Draws a circle with radius 2 **and** centre $2i$ **and** no other curves seen. Condone a freehand circle if intention is clear.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Draws a half-line from *O* into the 1st quadrant at an angle of more than 45° to the real axis. **and** no other straight lines seen.	1.1b	B1	
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Selects a method to find the maximum value of $\|w\|$ eg identifies a triangle with the diameter (or radius) as a side and the intersections of their loci as two of the vertices. eg forms a suitable equation in max $\|w\|$ or $x_{\max\|w\|}$ or $y_{\max\|w\|}$ eg substitutes $y = x\tan\left(\frac{\pi}{3}\right)$ into $(x-0)^2 + (y-2)^2 = 2^2$	3.1a	M1	 max $\|w\| = 4\cos\frac{\pi}{6}$ $= 2\sqrt{3}$
	Obtains a correct expression for max$\|w\|$ Or obtains a correct value for $x_{\max\|w\|}$ **and** $y_{\max\|w\|}$ May be unsimplified. PI by 3.46 or 3 or 1.73 or better Note: $x_{\max\|w\|} = \sqrt{3}$, $y_{\max\|w\|} = 3$	2.2a	A1	
	Obtains the correct maximum value of $\|w\|$ Accept 3.46 or better.	1.1b	A1	
	Subtotal		**3**	

	Question 12 total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Obtains at least one correct asymptote.	1.1a	M1	$y = \frac{2}{3}$ and $x = -\frac{5}{3}$
	Obtains two correct asymptotes and no incorrect asymptotes.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Sketches a curve asymptotic to either a horizontal asymptote or a vertical asymptote. Only one branch required for this mark.	1.1b	B1	
	Sketches a curve with two branches asymptotic to their horizontal asymptote and vertical asymptote.	1.1a	M1	
	Deduces the shape of the curve and sketches it correctly with the correct axis intercepts. Must draw the asymptotes – condone solid lines.	2.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)	Identifies both critical values and no others. Or identifies one correct set of x-values. FT their asymptote or x-intercept.	3.1a	M1	$x \leq -\frac{7}{2}$, $x > -\frac{5}{3}$
	Obtains the correct set of x-values.	3.2a	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(d)	Selects a method to find the equation of the reflected curve. Swaps $-x$ for y **and** $-y$ for x **or** any two of: $x = -\frac{2}{3}$ $y = \frac{5}{3}$ x-intercept $= -1.4$ y-intercept $= 3.5$ Follow through their asymptotes and/or intercepts from parts (a) and (b)	3.1a	M1	$-x = \dfrac{2(-y)+7}{3(-y)+5}$ $-x(-3y+5) = -2y+7$ $3xy - 5x = -2y + 7$ $3xy + 2y = 5x + 7$ $y(3x+2) = 5x + 7$ $y = \dfrac{5x+7}{3x+2}$
	Multiplies an equation of the form $\pm x = \dfrac{\pm 2y + 7}{\pm 3y + 5}$ throughout by the denominator and attempts to isolate y **or** any three of: $x = -\frac{2}{3}$ $y = \frac{5}{3}$ x-intercept $= -1.4$ y-intercept $= 3.5$ Follow through their asymptotes and/or intercepts from parts (a) and (b)	1.1a	M1	
	Obtains a correct equation in the correct format.	3.2a	A1	
	Subtotal		**3**	

	Question 13 total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Writes the correct equation.	1.1b	B1	$y = 1$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	Indicates that the denominator cannot be equal to zero. PI by use of the discriminant of the denominator.	3.1a	M1	$x^2 + px + 7 \neq 0$ $\therefore \quad p^2 - 4 \times 1 \times 7 < 0$ $p^2 < 28$ $-2\sqrt{7} < p < 2\sqrt{7}$
	Obtains a relevant inequality for p eg use of the discriminant of the denominator.	1.1a	M1	
	Obtains a correct inequality for p	1.1b	A1	
	Obtains the correct set of values for p Accept $-\sqrt{28} < p < \sqrt{28}$ Condone $-5.29 < p < 5.29$ or better.	3.2a	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
14(c)	Obtains the correct y-intercept.	1.1b	B1	$x = 0 \quad \Rightarrow \quad y = \dfrac{0-3}{0+0+7} = -\dfrac{3}{7}$ $y = 0 \quad \Rightarrow \quad x^2 - 3 = 0$ $\Rightarrow \quad x = \pm\sqrt{3}$ $\left(0, -\frac{3}{7}\right), (\sqrt{3}, 0), (-\sqrt{3}, 0)$
	Solves $x^2 - 3 = 0$	1.1a	M1	
	Obtains the correct coordinates for all three intercepts. Must be written as coordinates.	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
14(d)(i)	Multiplies by the denominator and forms a quadratic equation in x	1.1a	M1	$k = \dfrac{x^2 - 3}{x^2 - 3x + 7}$ $k(x^2 - 3x + 7) = x^2 - 3$ $(k-1)x^2 - 3kx + 7k + 3 = 0$ At least one solution, so $b^2 - 4ac \geq 0$ $(-3k)^2 - 4(k-1)(7k+3) \geq 0$ $9k^2 - 4(7k^2 - 4k - 3) \geq 0$ $-19k^2 + 16k + 12 \geq 0$ $19k^2 - 16k - 12 \leq 0$
	Obtains a correct quadratic equation in x in the form $ax^2 + bx + c = 0$ PI by a correct discriminant.	1.1b	A1	
	Selects a method to demonstrate the required inequality. Substitutes k for y and uses the discriminant to form an inequality in k	3.1a	M1	
	Obtains a correct quadratic inequality in k	1.1b	A1	
	Completes a rigorous proof to show that $19k^2 - 16k - 12 \leq 0$	2.1	R1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
14(d)(ii)	Selects a method to find the *y*-coordinate of the minimum point. Obtains at least one correct root of the given quadratic. PI by -0.48 or 1.32 or better	3.1a	M1	$k = \dfrac{8 \pm 2\sqrt{73}}{19}$ $y = \dfrac{8 - 2\sqrt{73}}{19}$
	Obtains the correct *y*-coordinate.	1.1b	A1	
	Subtotal		**2**	

	Question total		**15**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Assesses the validity of Hamzah's work by explaining his error. eg the roots of the quadratic are not the solutions of the equation. eg the roots of the equation are not θ	2.3	E1	The roots of the quadratic are $\sinh\theta_1$ and $\sinh\theta_2$ So $\sinh\theta_1 + \sinh\theta_2 = 1$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Finds the roots of the quadratic. PI by two of 1.44 or -0.88 or 0.56 or better.	1.1a	M1	$(\sinh\theta - 2)(\sinh\theta + 1) = 0$ $\sinh\theta = 2$ or $\sinh\theta = -1$ $\theta_1 = \ln(2+\sqrt{5})$ and $\theta_2 = \ln(-1+\sqrt{2})$ $\theta_1 + \theta_2 = \ln(2+\sqrt{5}) + \ln(-1+\sqrt{2})$ $= \ln\left((2+\sqrt{5})(-1+\sqrt{2})\right)$
	Selects a method to find a value of θ_1 **or** θ_2 PI by 1.44 or -0.88 or 0.56 or better Condone an incorrect base.	3.1a	M1	
	Obtains the correct exact values of θ_1 and θ_2 in log form. PI by correct sum in log form May be unsimplified.	1.1b	A1	
	Correctly changes the sum of two log expressions into one log. Condone an incorrect base.	1.1a	M1	
	Obtains the correct sum as a single log ACF	3.2a	A1	
	Subtotal		**5**	

	Question total		**6**	

	Question Paper total		**80**	