# AQA

# AS
# FURTHER MATHEMATICS
# 7366/2M

Paper 2  Mechanics

**Mark scheme**

June 2022

Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

# Mark scheme instructions to examiners

## General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded.  Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution.  This response is one we expect to see frequently.  However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied.  Credit should be given to any valid methods.  Examiners should seek advice from their senior examiner if in any doubt.

**Key to mark types**

| M | mark is for method |
|---|---|
| R | mark is for reasoning |
| A | mark is dependent on M marks and is for accuracy |
| B | mark is independent of M marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

**Key to mark scheme abbreviations**

| CAO | correct answer only |
|---|---|
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| 'their' | indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| sf | significant figure(s) |
| dp | decimal place(s) |

Examiners should consistently apply the following general marking principles:

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

**Work erased or crossed out**

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

**Choice**

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

# AS/A-level Maths/Further Maths assessment objectives

| AO | | Description |
|---|---|---|
| **AO1** | AO1.1a | Select routine procedures |
| | AO1.1b | Correctly carry out routine procedures |
| | AO1.2 | Accurately recall facts, terminology and definitions |
| **AO2** | AO2.1 | Construct rigorous mathematical arguments (including proofs) |
| | AO2.2a | Make deductions |
| | AO2.2b | Make inferences |
| | AO2.3 | Assess the validity of mathematical arguments |
| | AO2.4 | Explain their reasoning |
| | AO2.5 | Use mathematical language and notation correctly |
| **AO3** | AO3.1a | Translate problems in mathematical contexts into mathematical processes |
| | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes |
| | AO3.2a | Interpret solutions to problems in their original context |
| | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
| | AO3.3 | Translate situations in context into mathematical models |
| | AO3.4 | Use mathematical models |
| | AO3.5a | Evaluate the outcomes of modelling in context |
| | AO3.5b | Recognise the limitations of models |
| | AO3.5c | Where appropriate, explain how to refine models |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 1 | Circles correct answer. | 1.1b | B1 | 50 J |
| | **Question total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 2 | Circles correct answer. | 1.1b | B1 | $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ ms$^{-1}$ |
| | **Question total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 3(a) | Recalls the formula for kinetic energy and calculates the initial kinetic energy.<br><br>Condone missing units. | 1.1b | B1 | $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(0.75)(12)^2 = 54\,\text{J}$ |
| | **Subtotal** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 3(b) | Uses conservation of energy to form an equation with PE and their KE from part (a). | 3.3 | M1 | $mgh = 54$<br><br>$h = \frac{54}{(0.75)(9.8)} = 7.34..$ |
| | Solves the equation to obtain $h$ = 7.3<br>AWRT 7.3<br>Must have clearly rearranged the equation to find $h$ **or** obtains correct value (7.3469..) to at least 3 sf.<br>AG | 1.1b | A1 | Jeff has assumed no air resistance to obtain h = 7.3<br><br>Gurjas includes air resistance and so knows the ball will not reach 7. 3 metres |
| | Makes an inference about one or more assumptions for both Jeff and Gurjas.<br>For example:<br>• Jeff has assumed no air resistance, Gurjas has taken this into account<br>• Jeff assumes that all energy is conserved, Gurjas does not. | 2.2b | E1 | |
| | **Subtotal** | | **3** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 4(a) | Substitutes dimensions for length, time and acceleration due to gravity into both sides of any correct version of the formula.<br><br>Do not condone use of units. | 1.1a | M1 | $[t] = T$<br>$[g] = LT^{-2}$<br>$[w] = L$<br><br>$[k] = \dfrac{[gt^2]}{[w]} = \dfrac{LT^{-2}T^2}{L} = 1$<br><br>Therefore, $k$ is a dimensionless constant |
| | Completes a rigorous argument using dimensions to verify that $k$ is a dimensionless constant. | 2.1 | R1 | |
| | **Subtotal** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 4(b) | Uses dimensions to form a correct expression for the dimensions of $[(gd)^{\alpha}t^{\beta}]$<br>Need not be simplified.<br><br>Do not condone use of units. | 1.1b | B1 | $[(gd)^{\alpha}t^{\beta}] = (LT^{-2}L)^{\alpha}(T)^{\beta}$<br><br>$= (L)^{2\alpha}(T)^{-2\alpha+\beta}$<br><br>$2\alpha = 1$<br>$-2\alpha + \beta = 0$<br><br>$\alpha = \dfrac{1}{2}$<br>$\beta = 1$ |
| | Forms two simultaneous equations in $\alpha$ and $\beta$ consistent with their simplified $[(gd)^{\alpha}t^{\beta}]$<br>PI by a correct value of $\alpha$ or $\beta$ | 1.1a | M1 | |
| | Obtains correct values for $\alpha$ and $\beta$ | 1.1b | A1 | |
| | **Subtotal** | | **3** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **5** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **5** | Explains that at maximum speed the driving force equals the resistance or that the acceleration is zero. OE | 2.4 | E1 | For maximum power the car travels at maximum speed when the driving force equals the resistance $R = F$ |
| | Converts km h$^{-1}$ to m s$^{-1}$ to obtain 20 m s$^{-1}$ | 1.1b | B1 | $v = \frac{72 \times 1000}{60 \times 60} = 20$ ms$^{-1}$ $R = 25v = 25(20) = 500$ |
| | Obtains their correct driving force or resistive force using their value for speed. | 3.4 | M1 | $P = Fv$ |
| | Forms an equation by modelling power as $Fv$ with both their $F$ and $v$ substituted. | 3.3 | M1 | $P = 500 \times 20 = 10000$ Max $P = 10$ $kW$ |
| | Finds the correct maximum power of 10 kW OE Must state units. | 3.2a | A1 | |
| | **Question total** | | **5** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **6(a)** | Forms a correct definite integral for impulse. | 3.4 | B1 | Impulse $= \int_0^T (2t + 3)dt$ |
| | Integrates with at least one term correct. | 1.1a | M1 | $= [t^2 + 3t]_0^T$ <br><br> $= T^2 + 3T$ |
| | Obtains $T^2 + 3T$ <br> or $a$ = 1 and $b$ = 3 | 1.1b | A1 | $a$ = 1 and $b$ = 3 |
| | **Subtotal** | | **3** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **6(b)** | Uses $mv - mu$ | 1.1a | M1 | $I = mv - mu$ |
| | Obtains 0.6 | 1.1b | A1 | $I = 0.2(4) - 0.2(1) = 0.6$ <br><br> $T^2 + 3T = 0.6$ |
| | Equates their answer to part **(a)** to their change in momentum and solves their quadratic equation. | 1.1a | M1 | $T$ = 0.188  or  -3.19 <br><br> As $0 \le t \le T$, $T$ = 0.188 |
| | Obtains $T$ = 0.188 and clearly rejects the negative value. | 3.2a | A1 | |
| | **Subtotal** | | **4** | |

| | **Question total** | | **7** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **7(a)** | Forms a term using *mv* for either particle. | 1.1a | M1 | Momentum for *A* = 4(0.4) = 1.6<br>Momentum for *B* = -2(0.2) = - 0.4 |
| | Obtains 1.2 kg ms⁻¹ or 1.2 Ns Condone missing units. | 1.1b | A1 | Total momentum = 1.2 Ns |
| | **Subtotal** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **7(b)(i)** | Forms a conservation of momentum equation using their answer from **(a)**. | 3.1b | M1 | Speed of *A* = u<br>Speed of *B* = v<br>C of M<br>$1.2 = 0.4u + 0.2v$<br>$12 = 4u + 2v$ |
| | Forms a correct equation using Newton's law of restitution. | 1.1b | B1 | NLR<br>$v - u = 6e$ |
| | Completes a rigorous argument using both conservation of momentum and the Newton's law of restitution to verify the correct speed of *B* | 2.1 | R1 | $12 = 4(v - 6e) + 2v$<br>$12 = 6v - 24e$<br>$2 = v - 4e$<br>$v = 4e + 2$ |
| | **Subtotal** | | **3** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **7(b)(ii)** | Substitutes the speed of *B* back into either of their equations. | 1.1a | M1 | $4e + 2 - u = 6e$ |
| | Rearranges their equation to obtain the correct speed of *A* | 1.1b | A1 | $u = 2 - 2e$ |
| | **Subtotal** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **7(c)** | States that $e$ = 1 | 1.1a | M1 | Perfectly elastic collision so $e$ = 1 |
| | Deduces that *A* comes to rest. | 2.2a | R1 | u = 2 – 2(1) = 0 <br><br> Hence particle *A* comes to rest |
| | **Subtotal** | | **2** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **9** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 8(a) | Demonstrates the use of Hooke's law to obtain the result stated. | 3.3 | B1 | $T = \frac{\lambda x}{l} = \frac{200x}{1} = 200x$ |
| | **Subtotal** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 8(b) | Uses $a = \frac{v^2}{r}$ or $T = \frac{mv^2}{r}$ | 3.4 | M1 | $T = \frac{mv^2}{r}$ |
| | Identifies $r = 1 + x$ | 1.1b | B1 | $T = \frac{3(4)^2}{1+x}$ |
| | Forms an equation in $x$ using their expression for $r$ Must not use $r = 1$ | 1.1a | M1 | $\frac{48}{1+x} = 200x$ $48 = 200x(1+x)$ |
| | Completes a reasoned argument to obtain $25x^2 + 25x - 6 = 0$ | 2.1 | R1 | $200x^2 + 200x - 48 = 0$ $25x^2 + 25x - 6 = 0$ |
| | **Subtotal** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 8(c) | Solves $25x^2 + 25x - 6 = 0$ to obtain either $x = 0.2$ or $x = -1.2$ | 1.1a | M1 | $25x^2 + 25x - 6 = 0$ As $x > 0$, $x = 0.2$ |
| | Deduces the correct radius. Condone omission of units. | 2.2a | A1 | Radius = 1 + 0.2 = 1.2 m |
| | **Subtotal** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 8(d) | States any valid limitation eg String may not be light. There may be air resistance / resistance. | 3.5b | E1 | String may not be light |
| | **Subtotal** | | **1** | |

| | **Question total** | | **8** | |

| | **Question Paper total** | | **40** | |