



Oxford Cambridge and RSA

Wednesday 22 June 2022 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 $C = 3.7622$ and $S = 3.6269$ are used to approximate $\cosh 2$ and $\sinh 2$ respectively.
- (a) Determine whether these approximations are the result of chopping or rounding the values of $\cosh 2$ and $\sinh 2$. [1]
- (b) Calculate the relative error when $C^2 - S^2$ is used to approximate $\cosh^2 2 - \sinh^2 2$, giving your answer correct to 3 significant figures. [2]
- (c) **Without** doing any further calculations, explain whether the same value for the relative error is obtained when $(C - S)^2$ is used to approximate $(\cosh 2 - \sinh 2)^2$. [1]

- 2 The table shows some values of x and the associated values of $y = f(x)$.

x	2.75	3	3.25
$f(x)$	0.920799	1	1.072858

- (a) Calculate an estimate of $\frac{dy}{dx}$ at $x = 3$ using the forward difference method, giving your answer correct to 5 decimal places. [2]
- (b) Calculate an estimate of $\frac{dy}{dx}$ at $x = 3$ using the central difference method, giving your answer correct to 5 decimal places. [2]
- (c) Explain why your answer to part (b) is likely to be closer than your answer to part (a) to the true value of $\frac{dy}{dx}$ at $x = 3$. [1]

When $x = 5$ it is given that $y = 1.4645$ and $\frac{dy}{dx} = 0.1820$, correct to 4 decimal places.

- (d) Determine an estimate of the error when $f(5)$ is used to estimate $f(5.024)$. [2]

3 The equation $f(x) = \sin^{-1}(x) - x + 0.1 = 0$ has a root α such that $-1 < \alpha < 0$.

Alex uses an iterative method to find a sequence of approximations to α . Some of the associated spreadsheet output is shown in the table.

	C	D	E
4	r	x_r	$f(x_r)$
5	0	-1	-0.4707963
6	1	-0.8	-0.0272952
7	2	-0.787691	-0.0193610
8	3	-0.7576546	-0.0020574
9	4	-0.7540834	-0.0001740
10	5		
11	6		

The formula in cell D7 is

$$=(D5 * E6 - D6 * E5) / (E6 - E5)$$

and equivalent formulae are in cells D8 and D9.

- (a) State the method being used. [1]
- (b) Use the values in the spreadsheet to calculate x_5 and x_6 , giving your answers correct to 7 decimal places. [3]
- (c) State the value of α as accurately as you can, justifying the precision quoted. [1]

Alex uses a calculator to check the value in cell D9, his result is -0.7540832686 .

- (d) Explain why this is different to the value displayed in cell D9. [1]

The value displayed in cell E11 in Alex’s spreadsheet is $-1.4629\text{E-}09$.

- (e) Write this value in standard mathematical notation. [1]

4 Fig. 4.1 shows part of the graph of $y = e^x - x^2 - x - 1.1$.

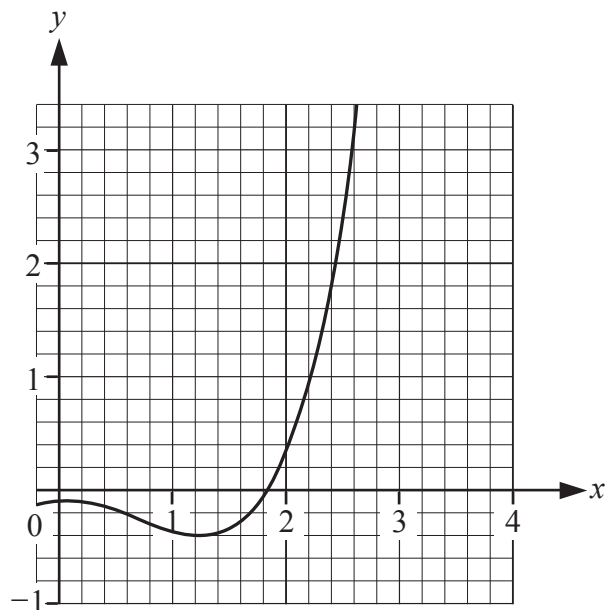


Fig. 4.1

The equation $e^x - x^2 - x - 1.1 = 0$ has a root α such that $1 < \alpha < 2$.

Ali is considering using the Newton-Raphson method to find α . Ali could use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$.

- (a) **Without** doing any calculations, explain whether Ali should use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$, or whether using either starting value would work equally well. [2]

Ali is also considering using the method of fixed point iteration to find α . Ali could use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$.

Fig. 4.2 shows parts of the graphs of $y = x$ and $y = \ln(x^2 + x + 1.1)$.

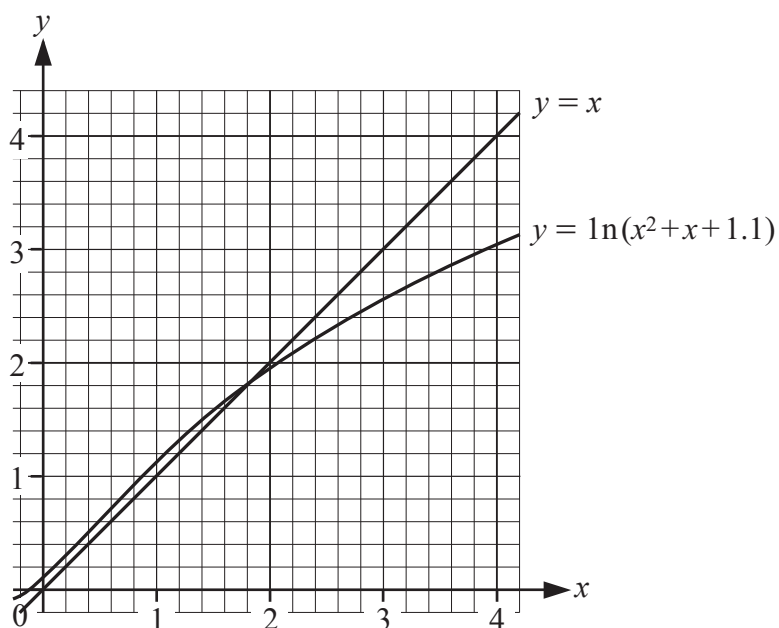


Fig. 4.2

- (b) **Without** doing any calculations, explain whether Ali should use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$ or whether either starting value would work equally well. [2]

Ali used one of the above methods to find a sequence of approximations to α . These are shown, together with some further analysis in the associated spreadsheet output in **Fig. 4.3**.

		M	N	O
	r	x_r		
4	0	2		
5	1	1.879008	-0.121	
6	2	1.858143	-0.021	0.172
7	3	1.857565	-6E-04	0.028
8	4	1.857564	-4E-07	8E-04
9	5	1.857564	-2E-13	6E-07

Fig. 4.3

The formula in cell N5 is

=M5 – M4

and the formula in cell O6 is

=N6/N5

equivalent formulae are in cells N6 to N9 and O7 to O9 respectively.

- (c) State what is being calculated in the following columns of the spreadsheet.
- (i) Column N [1]
- (ii) Column O [1]
- (d) Explain whether the values in column O suggest that Ali used the Newton-Raphson method or the iterative formula $x_{n+1} = \ln(x_n^2 + x_n + 1.1)$ to find this sequence of approximations to α . [2]

- 5 Kai uses the midpoint rule, trapezium rule and Simpson's rule to find approximations to $\int_a^b f(x)dx$, where a and b are constants. The associated spreadsheet output is shown in the table. Some of the values are missing.

	F	G	H	I
3	n	M_n	T_n	S_{2n}
4	1	0.2436699	0.1479020	
5	2	0.2306967		

- (a) Write down a suitable spreadsheet formula for cell H5. [2]
- (b) Complete the copy of the table in the Printed Answer Booklet, giving the values correct to 7 decimal places. [4]
- (c) Use your answers to part (b) to determine the value of $\int_a^b f(x)dx$ as accurately as you can, justifying the precision quoted. [3]
- 6 Charlie uses fixed point iteration to find a sequence of approximations to the root of the equation $\sin^{-1}(x) - x^2 + 1 = 0$.

Charlie uses the iterative formula $x_{n+1} = g(x_n)$, where $g(x_n) = \sin(x_n^2 - 1)$.

Two sections of the associated spreadsheet output, showing x_0 to x_6 and x_{102} to x_{108} , are shown in **Fig. 6.1**.

r	x_r	difference	ratio
0	0		
1	-0.841471	-0.84147	
2	-0.287798	0.553673	-0.65798
3	-0.793885	-0.50609	-0.91405
4	-0.361379	0.432507	-0.85461
5	-0.763945	-0.40257	-0.93078
6	-0.404459	0.359486	-0.89299

102	-0.596302	0.004626	-0.95886
103	-0.600738	-0.00444	-0.95911
104	-0.596484	0.004254	-0.95887
105	-0.600564	-0.00408	-0.95910
106	-0.596652	0.003912	-0.95888
107	-0.600404	-0.00375	-0.95909
108	-0.596806	0.003598	-0.95889

Fig. 6.1

- (a) Use the information in **Fig. 6.1** to find the value of the root as accurately as you can, justifying the precision quoted. [4]

The relaxed iteration $x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n)$, with $\lambda = 0.51$ and $x_0 = 0$, is to be used to find the root of the equation $\sin^{-1}(x) - x^2 + 1 = 0$.

- (b) Complete the copy of **Fig. 6.2** in the Printed Answer Booklet, giving the values of x_r correct to 7 decimal places and the values in the difference column and ratio column correct to 3 significant figures.

r	x_r	difference	ratio
0	0		
1			
2			
3			
4		-0.000192	
5		-1.99×10^{-7}	0.00103
6		-1.82×10^{-10}	0.000914

Fig. 6.2

- [4]
- (c) Write down the value of the root correct to 7 decimal places. [1]
- (d) Explain why extrapolation could not be used in this case to find an improved approximation using this sequence of iterates. [1]

In this case the method of relaxation has been used to speed up the convergence of an iterative scheme.

- (e) Name another application of the method of relaxation. [1]

- 7 Sam decided to go on a high-protein diet. Sam’s mass in kg, M , after t days of following the diet is recorded in **Fig. 7.1**.

t	0	10	20	30
M	88.3	80.05	78.7	78.85

Fig. 7.1

A difference table for the data is shown in **Fig. 7.2**.

t	M	ΔM	$\Delta^2 M$	$\Delta^3 M$
0	88.3			
10	80.05			
20	78.7			
30	78.85			

Fig. 7.2

- (a) Complete the copy of the difference table in the Printed Answer Booklet. [1]

Sam’s doctor uses these data to construct a cubic interpolating polynomial to model Sam’s mass at time t days after starting the diet.

- (b) Find the model in the form $M = at^3 + bt^2 + ct + d$, where a , b , c and d are constants to be determined. [4]

Subsequently it is found that when $t = 40$, $M = 78.7$ and when $t = 50$, $M = 80.05$.

- (c) Determine whether the model is a good fit for these data. [2]
- (d) By completing the extended copy of **Fig. 7.2** in the Printed Answer Booklet, explain why a quartic model may be more appropriate for the data. [2]
- (e) Refine the doctor’s model to include a quartic term. [3]
- (f) Explain whether the new model for Sam’s mass is likely to be appropriate over a longer period of time. [2]

END OF QUESTION PAPER

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