

GCE

Further Mathematics A

Y535/01: Additional pure mathematics

AS Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
۸	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in	Meaning
mark scheme	
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

М

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
 - Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
 - When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
 - NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g Rules for replaced work and multiple attempts:
 - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

	Questi	on	Answer	Marks	AO	Guidance		
1	a		$\mathbf{a} \times \mathbf{b} = -10\mathbf{i} + 7\mathbf{j} + \mathbf{k}$	B1	1.1			
			Area $\triangle OAB = \frac{1}{2} \mathbf{a} \times \mathbf{b} $	M1	1.2	Use of area formula		
			$=\frac{1}{2}\sqrt{150} \text{ or } \frac{5}{2}\sqrt{6}$	A1	1.1	FT their vector product		
				[3]				
	b		$\begin{vmatrix} \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) \\ = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 7 \\ 1 \\ 17 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 17 \\ -6 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} -10 \\ 7 \\ 1 \end{pmatrix}$	M1	1.1	Attempt at any three vector products (FT their a × b from above, and at least one other correct)		
			(14)(-1)(-13)(0)	M1	1.1	Three further vector product		
			$ = \begin{pmatrix} 14 \\ 4 \\ -6 \end{pmatrix} + \begin{pmatrix} -1 \\ 11 \\ 31 \end{pmatrix} + \begin{pmatrix} -13 \\ -15 \\ -25 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} $	A1 [3]	1.1	attempts (at least one FT correct) AG Correct final answer from fully correct <i>and visible</i> working		
2			$\frac{\partial z}{\partial x} = 3x^2 - 4x + 3y, \frac{\partial z}{\partial y} = 3y^2 - 10y + 3x$	M1* A1	1.1 1.1	Attempt at both first partial derivatives; correct		
			$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0 \implies \text{(e.g.)}$	M1 dep*	3.1a	Setting both 1 st partial derivatives to zero		
			$y = \frac{4}{3}x - x^{2} \Rightarrow$ $3\left(\frac{16}{9}x^{2} - \frac{8}{3}x^{3} + x^{4}\right) - 10\left(\frac{4}{3}x - x^{2}\right) + 3x = 0$ $\Rightarrow \frac{1}{3}x(9x^{3} - 24x^{2} + 46x - 31) = 0$	M1	1.1	Creating polynomial of degree four in one variable by substitution.	OR $x = \frac{10}{3}y - y^2 \implies$ $\frac{1}{3}y(9y^3 - 60y^2 +$ + 112y - 31) = 0	

(Question		Answer		AO	Guidance	
			$\Rightarrow x(x-1)(9x^2-15x+31)=0$	M1	1.1	Solving (possibly BC)	SC1 to replace the final three marks if all three coordinates of <i>A</i> correct without working
			giving $x = 1$, $y = \frac{1}{3}$ and $z = -\frac{14}{27}$	A1 [6]	1.1	All three coordinates of A correct	

Ç	Questic	on	Answer	Marks	AO	Guidance	
3			n 1 2 3 4 5 6 7	M1	1.1	$u_3 = -1 + 0 + n - 1$ where $n = 1,2$ or 3 with visible substitution	(Possibly BC)
3	a		$U_n = 0 - 1 - 1 - 1 = 0 = 2 = 6$	A1	1.1	All of U_3 to U_7 correct (fully correct implies M1)	NB U_1 and U_2 are given
				[2]		,	
	b	i	F _n 1 1 2 3 5 8 13	M1	1.1a	Any method for comparing the two sequences	Listing elements of U_n and F_n is not sufficient
			$U_n + n = F_n$	A1	2.2b		
				[2]			
		ii	$F_{n+2} = U_{n+2} + n + 2$ and $F_{n+1} = U_{n+1} + n + 1$	B1	2.2a	Seen or used somewhere	FT their result 3(b).
			$F_{n+2} = F_{n+1} + F_n$	M1	2.1	Their F_{n+2} , F_{n+1} and F_n substd. into the usual Fibonacci r.r.	
			$\Rightarrow U_{n+2} = U_{n+1} + U_n + n - 1$	A1	2.3	Given result correctly shown.	
			Alternative method $u_1 + 1 = F_1$ true and $u_k + k = F_k$	B1		Baseline case and assumption that statement is true for $n = k$	

Question	Answer		AO	Guidance	
	$\begin{aligned} U_{k+1} + k + 1 &= F_{k+1} \\ F_{k+2} &= F_{k+1} + F_k = \\ u_{k+1} + k + 1 + u_k + k = \\ u_{k+2} - (k-1) + k + 1 + k \\ &= U_{k+2} + k + 2 \end{aligned}$	M1		Attempt at $n = k + 1$ Inductive reasoning all correctly shown and conclusion	
		[3]			

Q)uesti	on	Answer	Marks	AO	Guidance	2
4	a		3N - M = 30a + 3b - a - 3b = 29a	M1 A1	3.1a 1.1	Linear combination of <i>N</i> and <i>M</i> attempted Multiple of 29 stated or shown	or $2N + 9M = 29(a + b)$ or $N + 19M = 29(a + b)$ 2b
			Then $29 \mid N \Rightarrow 29 \mid M = 3N - 29a$	M1	2.1	1 direction of an ⇔ proof attempted	Alt. "or" forms above lead to exactly similar
				A1	1.1	Shown convincingly	working; suggest the hcf
			and $29 \mid M \Rightarrow 29 \mid 3N = M + 29a$	A1	2.2a	Including justification since hcf(3, 29) = 1	business only required once in the $2N + 9M$ case
			Alternative method Let $N = 29n$. Then $M = a + 3b = a + 3(N - 10a)$ = 3N - 29a = 29(3n - a)	M1 M1 A1		First direction of ⇔ proof	
			and if $M = 29m$ then $N = 10(M - 3b) + b$ $\Rightarrow N = 10M - 29b = 29(10m - b)$	M1 A1		Second direction of ⇔ proof	
				[5]			
	b		$899\ 364\ 47\ \ 2\ \to\ 899\ 364\ 5\ \ 3$	M1	1.1	1 st step of iterative process correct (i.e. $10a + b$ replaced by $a + 3b$)	
			→ 899 365 4 → 899 37 7 $ → 899 5 8 → 901 9 → 92 8$	M1	1.1	Method used repeatedly at least twice correctly	
			$\rightarrow 11 \mid 6 \rightarrow 29$	A1 [3]	2.2a	Ending at 29 or at $116 = 4 \times 29$	

Q	uesti	on	Answer	Marks	AO	Guidance	
5	a		It gives lines of peaks (etc.) extending in both <i>x</i> - and <i>y</i> -directions	B1	3.4		
			Restrict the domain to $-\pi/2 \le y \le 3\pi/2$	B1	3.3	Allow other intervals of length 2π Swapping x , y throughout is OK	
			All the peaks are the same height	B 1	3.5a		
			Introduce a (damping) factor in the <i>x</i> -direction, such as e^{-ax}	B1	3.5c	OR a separate damping factor to each	Allow 'ratio' or just 'factor' in conjunction with 'decreasing'
				[4]			
	b		Max. $(\sin x + \sin y) = 2$ so contour $z = 2$ contains only (an infinite	B1	3.1a		
			number of) (isolated) points (the very peak	B 1	3.2a		
			of each island mountain)				
				[2]			

Q	uesti	on	Answer	Marks	AO	Guidance	
6	a		$u_2 = u_1 + 30, u_3 = u_2 + 300,$ $u_4 = u_3 + 3000, \dots$	M1	1.1a	Any sensible method (may be implied by a correct, or nearly correct, answer)	OR $u_2 = 10u_1 - 33$, $u_3 = 10u_2 - 33$,
			$u_{n+1} = u_n + 3 \times 10^n \ (n \ge 1) \text{ with } u_1 = 7$	A1	2.2b	Must include first term. Condone missing "range" of <i>n</i> .	OR $u_{n+1} = 10u_n - 33$ with $u_1 = 7$
				[2]			
	b		CS is $u_1 = C$	B1	1.1		
			For PS try $u_n = a \times 10^n$	M1	1.1	Including substn. to determine a	
			$a = \frac{1}{3}$	A1	1.1		
			$u_n = \frac{1}{3} \times 10^n + C$ and $u_1 = 7$ used	M1	1.1	Their $GS = CS + PS$ and attempt to find a numerical value for C	
			$u_n = \frac{1}{3} \times 10^n + \frac{11}{3}$	A1	1.1	AG	
				[5]			
	c		$\mod 37, \{u_n\} = \{7, 0, 4, 7, 0 \dots\}$	B1	2.2a	Noting that every third term is (or appears to be) a multiple of 37	
			Baseline case: u_2 is a multiple of 37	B1	1.1	Noted or used	
			Assume u_k is a multiple of 37 for some k	M1	2.1		
			Then $u_{k+1} = 3 \times 10^n + u_k$ $\Rightarrow u_{k+2} = 3 \times 10^{k+1} + 3 \times 10^k + u_k \text{ and}$ $u_{k+3} = 3 \times 10^{k+2} + 3 \times 10^{k+1} + 3 \times 10^k + u_k$	M1*	3.1a	Attempt to employ the recurrence relation across three steps	
			$\Rightarrow u_{k+3} = 111 \times 3 \times 10^k + u_k$	A1	1.1		

Question	Answer	Marks	AO	Guidance
	Since $111 = 3 \times 37$, it follows that if $37 \mid u_k$ then $37 \mid u_{k+3}$ also, and proof follows by induction	M1dep * A1	2.1	Inductive reasoning attempted from their u_{k+3} All correctly explained (for final A , must note that this implies that $\{u_n\}$ contains infinitely many multiples of 37)
	Alternative method 1 mod 37, $\{u_n\} = \{7, 0, 4, 7, 0\}$ Baseline case: u_2 is a multiple of 37	B1 B1		Noting that every third term is (or appears to be) a multiple of 37 Noted or used
	Assume u_{3k-1} is a multiple of 37 for some k Then $u_{3k+2} = \frac{1}{3}(10^{3k+2} + 11) = u_{3k-1} + 111 \times 3 \times 10^{3k-1}$	M1 M1*		Attempt to write <i>their</i> u_{3k+2} in terms of u_{3k-1}
	Since $111 = 3 \times 37$, it follows that if $37 \mid u_k$ then $37 \mid u_{k+3}$ also, and proof follows by induction	M1dep *		Inductive reasoning attempted from their u_{k+3} All correctly explained (for final A , must note that this implies that $\{u_n\}$ contains infinitely many multiples of 37)

Question	Answer Marks AO Guidance				
	Alternative method 2				
	The sequence $\{10^n + 11\}$ runs	D1		Working with the sequence (or with	
	{21, 111, 1011, 10011, 100011}	B1		the original one)	
	Noting that $111 = 3 \times 37$ and $100011 =$			Must make the conclusion that $\{u_n\}$	
	2703×37 and noting that every third term	B1		contains infinitely many multiples of	
	is (appears to be) a multiple of 37, so			37 at some stage	
	Testing $10^{3n+2} + 11$	M1		Directly or inductively	Or $10^{3n-1} + 11$
	$= (10^3)^n \times 10^2 + 11$	M1		Use of the laws of indices	
	$= 100 \times 1000^n + 11$	A1			
	Since 999 is a multiple of 37,	M1		Use of modular arithmetic	
	$1000 \equiv 1 \pmod{37}$	IVII		Osc of modular aritimetic	
	$10^{3n+2} + 11 \equiv 100 \times 1^n + 11 = 111$				
	$= 3 \times 37 \equiv 0 \pmod{37}$	A1		All correctly concluded	
		[7]			

Q	uestio	n	Answer	Marks	AO	Guidance		
7	a		$i = A ^{C} B \qquad a = A ^{B} C \qquad b = C ^{A} B$ $c = B ^{C} A \qquad p = C ^{B} A \qquad q = B ^{A} C$	B1 B1 B1	1.1 1.1 1.1	1 st B for correct <i>i</i> and ≥ 1 of <i>a</i> , <i>b</i> , <i>c</i> 2nd B for all of <i>i</i> , <i>a</i> , <i>b</i> , <i>c</i> correct 3 rd B for <i>p</i> , <i>q</i> correct		
				[3]				
	b		i a b c p q i i a b c p q a a i p q b c b b q i p c a c c p q i a b p p c a b q i	B1 B1	1.1	First row and first column are given Correct composition of reflections (3×3 square) Correct composition of rotations	NB The main diagonal & composition of rotations are obvious; one composition of reflections gives the others by the LSP;	
				B1	1.1	(2×2 square) All the rest correct (white cells)	similarly for each of the white rectangles.	
	С		$\{i, a\}, \{i, b\}, \{i, c\} \text{ and } \{i, p, q\}$	[3] B1 B1 B1	1.1 1.1	Any one subgroup of order 2 The subgroup of order 3 All four correct with no extras	Condone inclusion of $\{i\}$ and/or D_3	
				[3]				
	d	i	NOT cyclic, as D_3 has no element of order 6 or IF they first explain that D_3 is nonabelian (with valid reason) then allow the claim "non-abelian \Rightarrow non-cyclic"	B1	2.4	Or since elements have orders: $a, b, c \dots 2$ and $p, q \dots 3$	Just 'no generator' not sufficient	
			•	[1]				
		ii	NOT abelian, as (e.g.) $ab = p$ but $ba = q$	B1	2.4	Pull the mark forward from d i if they have done this there	Accept no symmetry in the leading diagonal in the Cayley table	
				[1]				

Question		Answer	Marks	AO	Guidance	
e		H is cyclic (while D_3 isn't) H is abelian (while D_3 isn't)	B1 B1	2.5 2.2a	Any two differences clearly stated or described on group structure or order of elements or	
			[2]		subgroup order	

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