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# Level 2 Certificate

# FURTHER MATHEMATICS

# 8365/1

Paper 1 Non-Calculator

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Mark scheme

June 2021

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Version: 1.0 Final



2 1 6 G 8 3 6 5 / 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

<b>M</b>	Method marks are awarded for a correct method which could lead to a correct answer.
<b>M dep</b>	A method mark dependent on a previous method mark being awarded.
<b>A</b>	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
<b>B</b>	Marks awarded independent of method.
<b>B dep</b>	A mark that can only be awarded if a previous independent mark has been awarded.
<b>ft</b>	Follow through marks. Marks awarded following a mistake in an earlier step.
<b>SC</b>	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
<b>oe</b>	Or equivalent. Accept answers that are equivalent.  eg accept 0.5 as well as $\frac{1}{2}$
<b>[a, b]</b>	Accept values between $a$ and $b$ inclusive.
<b>3.14...</b>	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

***Diagrams***

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

***Responses which appear to come from incorrect methods***

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

***Questions which ask candidates to show working***

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

***Questions which do not ask candidates to show working***

As a general principle, a correct response is awarded full marks.

***Misread or miscopy***

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

***Further work***

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

***Choice***

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

***Work not replaced***

Erased or crossed out work that is still legible should be marked.

***Work replaced***

Erased or crossed out work that has been replaced is not awarded marks.

***Premature approximation***

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

***Continental notation***

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Question	Answer	Mark	Comments
1	$(5 - -3)^2 + (1 - 7)^2$ or $8^2 + (-6)^2$	M1	oe eg $8^2 + 6^2$ or $\sqrt{100}$
	10	A1	
	<b>Additional Guidance</b>		

Question	Answer	Mark	Comments
2	$2x^5 - 7x^4$	M1	
	$10x^4$ or $(-) 28x^3$	M1	oe eg $5 \times 2x^{5-1}$
	$\left(\frac{dy}{dx} = \right) 10x^4 - 28x^3$ with no additional terms	A1	do not award for $y =$ or $\frac{d^2y}{dx^2} =$ on the answer line SC2 $2x^4 - 7x^3 + 8x^4 - 21x^3$ SC1 $2x^4 - 7x^3 + x(8x^3 - 21x^2)$
	<b>Additional Guidance</b>		
	Allow $y = \dots$ for M marks but must be recovered for A1 $\left(\frac{dy}{dx} = \right) 10x^4 - 28x^3 + c$		M2A0

Question	Answer	Mark	Comments
3	B	B1	
	<b>Additional Guidance</b>		

Q	Answer	Mark	Comments
4(a)	$6 < x < 8$ or $x < 8$ and $x > 6$	B1	oe eg $8 > x > 6$ word 'and' must be included if writing two inequalities
	<b>Additional Guidance</b>		
	[6, 8]		B1
	Condone eg $x = 6 < x < 8$		B1
	Incorrect inequality symbol eg $6 \leq x < 8$ or $6 < x > 8$		B0
	$6 < f(x) < 8$		B0
	B1 response with a list of integers on answer line		B0
Only a list of integers		B0	

Q	Answer	Mark	Comments	
4(b)	$-4 \leq g(x) < 5$ or $g(x) < 5$ and $g(x) \geq -4$	B2	oe eg $5 > g(x) \geq -4$ word 'and' must be included if writing two inequalities for B2 or B1 or SC1 B1 $-4 < g(x) < 5$ or $-4 < g(x) \leq 5$ or $-4 \leq g(x) \leq 5$ or $g(x) < 5$ and $g(x) > -4$ or $g(x) \leq 5$ and $g(x) > -4$ or $g(x) \leq 5$ and $g(x) \geq -4$ or $k < g(x) < 5$ where k is less than 5 or $k \leq g(x) < 5$ where k is less than 5 or $-4 \leq g(x) < m$ where m is greater than $-4$ SC1 $-4 \leq x < 5$ or $x < 5$ and $x \geq -4$ or only $-4$ and $5$ seen (condone 9 given as a range in this case)	
	<b>Additional Guidance</b>			
	Condone $g(x)$ replaced by eg $y$ or $g$ or $gx$ or $f$ or $fx$ or $G$ or $Gx$ or $x^2 - 4$ eg1 $-4 \leq f(x) < 5$ eg2 $-4 \leq y \leq 5$		B2 B1	
	$[-4, 5)$		B2	
	$(-4, 5)$ or $(-4, 5]$ or $[-4, 5]$		B1	
	Condone eg $g(x) = -4 \leq g(x) < 5$		B2	
	Condone eg $g(x) = -4 < g(x) < 5$		B1	
	B2 response with a list of integers on answer line		B1	
	B1 response with a list of integers on answer line		B0	
	Only a list of integers		B0	

Question	Answer	Mark	Comments
4(c)	<b>Alternative method 1</b>		
	$x = 2h(x) - 3$ or $x = 2y - 3$	M1	oe
	$2x - 3$	A1	
	<b>Alternative method 2</b>		
	$x = \frac{3 + h^{-1}(x)}{2}$ or $x = \frac{3 + y}{2}$	M1	oe
	$2x - 3$	A1	
	<b>Additional Guidance</b>		
	Answer left as $y = 2x - 3$		M1A0

Question	Answer	Mark	Comments
5(a)	$2n + 47 = 5(n + 1)$ or $2n + 47 = 5n + 5$	M1	oe equation with fraction eliminated
	14	A1	
	<b>Additional Guidance</b>		
	$n = 14$ from trial and error		M1A1

Question	Answer	Mark	Comments
5(b)	2	B1	
	<b>Additional Guidance</b>		
	Do not allow $n \rightarrow 2$		

Question	Answer	Mark	Comments
6	40 and 140 with no other values	B2	B1 40 or 140
	<b>Additional Guidance</b>		
	sin 40 and sin 140 with no other incorrect answers		B1
	40 and 140 but with other values (incorrect or outside range)		B1

Question	Answer	Mark	Comments
7	$2x^2 - x - 3$ or $2x^2 - 3x + 2x - 3$	M1	
	$4 > -x - 3$	M1dep	oe eg $7 > -x$
	$x > -7$ or $-7 < x$	A1	
	<b>Additional Guidance</b>		
	= used instead of > throughout and not recovered on answer line		M2A0

Question	Answer	Mark	Comments
<b>8</b>	<b>Alternative method 1</b>		
	$\sqrt{225} + \sqrt{144}$ or $15 + 12$	M1	
	27	A1	
	<b>Alternative method 2</b>		
	$5\sqrt{3} + 4\sqrt{3}$ or $9\sqrt{3}$ or $9 \times 3$ or $15 + 12$	M1	
	27	A1	
	<b>Additional Guidance</b>		

Question	Answer	Mark	Comments
9	Expanding second and third bracket $3x^2 + 6x - 4x - 8$ with at least three terms correct or $3x^2 + 2x + k$ where $k$ is a non-zero constant	M1	oe expansion attempt of one pair of brackets eg1 first and second brackets $6x^2 - 8x - 15x + 20$ with at least three terms correct or $6x^2 - 23x + k$ where $k$ is a non-zero constant eg2 first and third brackets $2x^2 + 4x - 5x - 10$ with at least three terms correct or $(2x^2 - x + k)$ where $k$ is a non-zero constant
	$6x^3 + 12x^2 - 8x^2 - 16x$ or $6x^3 + 4x^2 - 16x$ or $-15x^2 - 30x + 20x + 40$ or $-15x^2 - 10x + 40$	M1dep	attempt at a full expansion with correct multiplication of their 3 or 4 terms by one of the terms in the remaining bracket oe eg1 $6x^3 - 8x^2 - 15x^2 + 20x$ or $6x^3 - 23x^2 + 20x$ or $12x^2 - 16x - 30x + 40$ or $12x^2 - 46x + 40$ eg2 $6x^3 + 12x^2 - 15x^2 - 30x$ or $6x^3 - 3x^2 - 30x$ or $-8x^2 - 16x + 20x + 40$ or $-8x^2 + 4x + 40$
	$6x^3 - 11x^2 - 26x + 40$	A1	fully correct simplified expansion allow terms written in a different order
	<b>Additional Guidance</b>		
	For the M marks terms may be seen in a grid		
First M1 do not allow any omissions or extra terms eg1 $3x^2 + 6x - 8$ eg2 $3x^2 + 6x - 4x - 8 + x^2$			M0 M0

Q	Answer	Mark	Comments
10	<b>Alternative method 1</b>		
	(Second differences $\Rightarrow$ ) $-2$ or $-n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
	$0 - -1 \quad 1 - -4 \quad 0 - -9 \quad (-3 - -16)$ or $1 \quad 5 \quad 9 \quad (13)$ or $-1 - 0 \quad -4 - 1 \quad -9 - 0 \quad (-16 - -3)$ or $-1 \quad -5 \quad -9 \quad (-13)$	M1dep	subtracts $-n^2$ from the given terms or subtracts the given terms from $-n^2$
	$-n^2 + 4n - 3$	A1	oe eg $4n - 3 - n^2$
	<b>Alternative method 2</b>		
	Any three of $a + b + c = 0$ $4a + 2b + c = 1$ $9a + 3b + c = 0$ $16a + 4b + c = -3$	M1	using $n$ th term $= an^2 + bn + c$
	$3a + b = 1$ and $5a + b = -1$ or $a = -1$ and $b = 4$	M1dep	oe obtains two equations in the same two variables
	$-n^2 + 4n - 3$	A1	oe eg $4n - 3 - n^2$

**Mark scheme and Additional Guidance continue on the next page**

<b>10 cont</b>	<b>Alternative method 3</b>		
	(Second differences =) $-2$ or $-n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
	$3a + b = 1$ and substitutes $a = -1$	M1dep	oe eg $-3 + b = 1$ or $b = 4$
	$-n^2 + 4n - 3$	A1	oe eg $4n - 3 - n^2$
	<b>Additional Guidance</b>		
	Condone use of $U_n$		M2A1
	Condone working in different variable(s) eg $-n^2 + 4x - 3$		M2A1
	Answer $-n^2 \dots$ scores at least M1		
	Condone $-n^2 + 4n - 3 = 0$ or $n = -n^2 + 4n - 3$		M2A1

Question	Answer	Mark	Comments
11	$\begin{bmatrix} 2a & 2b + 0.4 \\ 0 & 1.2 \end{bmatrix}$ or $2a = k$ or $k = 1.2$ or $2b + 0.4 = 0$	M1	oe any 3 terms correct in correct position could be implied from second M mark
	$2a = k$ and $2b + 0.4 = 0$	M1dep	oe eg $2a = 1.2$ and $2b + 0.4 = 0$
	$a = 0.6$ or $b = -0.2$	M1	oe
	$a = 0.6$ and $b = -0.2$	A1	oe
	<b>Additional Guidance</b>		

Question	Answer	Mark	Comments
12(a)	$(x - 4)^2 + (y + 2)^2 = 20$	B2	B1 $(x - 4)^2 + (y + 2)^2$ or 20
	<b>Additional Guidance</b>		
	$(x + 4)^2 + (y - 2)^2 = 20$		B1
	$(x - 4)^2 + (y + 2)^2 = 4^2 + (-2)^2$		B1
	$(x - 4)^2 + (y + 2)^2 = \sqrt{20}$		B1
	$(x - 4)^2 - (y + 2)^2 = 20$		B1
	$(x - 4)^2 + (y - -2)^2 = 20$		B2
	$(x - 4)^2 + (y - -2)^2 = (\sqrt{20})^2$		B2
	ignore further working		

Question	Answer	Mark	Comments
12(b)	(Gradient AC =) $\frac{0 - -2}{8 - 4}$ or $\frac{2}{4}$	M1	oe
	(Gradient of tangent =) negative reciprocal of their $\frac{2}{4}$ or -2	M1	oe ft their gradient AC only gradient -2 seen is M2
	$y = -2x + 16$	A1	oe
	<b>Additional Guidance</b>		
	It is possible to find an incorrect gradient of AC and then get the second M mark for finding the negative reciprocal of this		

Question	Answer	Mark	Comments
13(a)	$k^2 = \frac{49}{16}$ or $k = \sqrt{\frac{49}{16}}$	M1	oe
	$\frac{7}{4}$	A1	oe
	<b>Additional Guidance</b>		
	$-\frac{7}{4}$ or $\pm\frac{7}{4}$ will not gain the A mark unless recovered		M1A0

Question	Answer	Mark	Comments
13(b)	$\frac{4}{7}$	B1ft	oe allow decimal rounded to 2dp or better ft their (a) but not if $k = 0$ or $1$
	<b>Additional Guidance</b>		
	$\frac{1}{4}$ $\frac{1}{7}$		B0

Q	Answer	Mark	Comments
14	<b>Alternative method 1</b> Eliminates $b$ from first two equations before eliminating a second variable		
	Correct attempt to eliminate $b$ from LHS of first two equations	M1	eg $2(4a - b + 3c) + 3a + 2b - c$ or $11a + 5c$ adding or subtracting the two equations can be implied from two terms correct
	Correct attempt to eliminate $a$ or $c$ from LHS of third equation and their equation in $a$ and $c$	M1dep	eg $11a + 5c + 2a - 5c$ or $2(11a + 5c) - 11(2a - 5c)$
	Correct equation in $a$ or $c$	M1dep	eg $13a = 52$ or $65c = 195$ implied by $a = 4$ or $c = 3$ with M2
	Two correct values with M3	A1	eg $a = 4$ and $c = 3$ with M3
	$a = 4$ and $b = -2$ and $c = 3$ with M3	A1	
	<b>Alternative method 2</b> Eliminates $a$ or $c$ before eliminating a second variable		
	Two correct attempts to eliminate the same variable ( $a$ or $c$ ) from LHS	M1	eg (eliminating $a$ ) $4a - b + 3c - 2(2a - 5c)$ and $2(3a + 2b - c) - 3(2a - 5c)$ or $-b + 13c$ and $4b + 13c$
	Correct attempt to eliminate a second variable from LHS of their two equations	M1dep	eg $-b + 13c - (4b + 13c)$
	Correct equation in one variable	M1dep	eg $-5b = 10$ implied by $b = -2$ with M2
	Two correct values with M3	A1	eg $b = -2$ and $a = 4$ with M3 or $b = -2$ and $c = 3$ with M3
	$a = 4$ and $b = -2$ and $c = 3$ with M3	A1	

Additional Guidance is on the next page

		<b>Additional Guidance</b>	
<b>14 cont</b>		For the first two marks ignore the RHS of the equations	
		First two method marks may be seen in one attempt eg Alt1 $2(4a - b + 3c) + 3a + 2b - c + 2a - 5c$	M1M1
		Elimination may be seen from other approaches eg1 Alt 1 (equates expressions for $2b$ from first two equations) $2(4a + 3c - 27) = 5 - 3a + c$ eg2 Alt 2 (rearranges third equation to $a = 2.5c - 3.5$ and substitutes into first two equations) $4(2.5c - 3.5) - b + 3c$ and $3(2.5c - 3.5) + 2b - c$	M1  M1
		Correct values with no working	M0A0

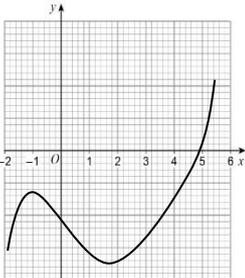
Question	Answer	Mark	Comments
15	$\tan x = (\pm)\frac{1}{\sqrt{3}}$ or $\tan x = (\pm)\frac{\sqrt{3}}{3}$	M1	
	30 with no incorrect solutions within the given range	A1	ignore correct solutions outside the given range.
	<b>Additional Guidance</b>		

Q	Answer	Mark	Comments	
16(a)	$200\left(-\frac{1}{2}\right)^3 + 100\left(-\frac{1}{2}\right)^2 - 18\left(-\frac{1}{2}\right) - 9$	M1	oe eg $200\left(-\frac{1}{8}\right) + 100\left(\frac{1}{4}\right) - 18\left(-\frac{1}{2}\right) - 9$	
	$-25 + 25 + 9 - 9 = 0$ with M1 seen	A1	must evaluate each term and equate to zero	
	<b>Additional Guidance</b>			
	Condone $\left(\frac{1}{2}\right)^2$ for $\left(-\frac{1}{2}\right)^2$			
	$200\left(-\frac{1}{2}\right)^3 + 100\left(-\frac{1}{2}\right)^2 - 18\left(-\frac{1}{2}\right) - 9 = 0$		M1A0	

Question	Answer	Mark	Comments
16(b)	$(100x^2 - 9)$	M1	
	$(10x - 3)(10x + 3)$ or $(x = )\sqrt{\frac{9}{100}}$	M1dep	oe eg $(x = )\sqrt{0.09}$
	-0.5 and -0.3 and 0.3	A1	oe eg fractions
	<b>Additional Guidance</b>		
	-0.5 and -0.3 or -0.5 and 0.3 with the other solution missing implies $(100x^2 - 9)$ -0.3 and 0.3 on answer line implies $(10x - 3)(10x + 3)$		M1M0A0 M2A0

Question	Answer	Mark	Comments
17	$x - 4$ or $4 - x$ seen in working	M1	from a subtraction of the quadratic and linear
	$y = x - 4$ drawn	A1	
	5.3 and 1.7 and $y = x - 4$ drawn	A1	Allow [5.2, 5.4] and [1.6, 1.8]
	<b>Additional Guidance</b>		
	Solutions with correct graph not seen eg from formula Solutions from quadratic graph drawn		M0A0A0 M0A0A0

Question	Answer	Mark	Comments	
18	$7^2 = x^2 + 3^2 - 2 \times 3 \times x \cos 60^\circ$	M1	oe	
	$x^2 - 3x - 40 (= 0)$	A1		
	$(x - 8)(x + 5) (= 0)$ or $\frac{- -3 \pm \sqrt{(-3)^2 - 4 \times 1 \times -40}}{2 \times 1}$	M1	oe follow through their three term quadratic	
	8	A1		
	<b>Additional Guidance</b>			
	If -5 is also given as an answer then do not award final A mark			

Q	Answer	Mark	Comments
19	Cubic curve from $x = -2$ to $x = 6$ and maximum point at $(-1, a)$ where $a$ is negative and minimum point at $(2, b)$ where $b$ is less than $a$ and increasing through $(5, 0)$	B4	B3 curve from $x = -2$ to $x = 6$ and maximum point at $(-1, c)$ where $c$ is any value and minimum point at $(2, d)$ where $d$ is less than $c$ and $d$ is negative and increasing through $(5, 0)$ or a B4 response apart from cubic curve not drawn from $x = -2$ to $x = 6$  B2 curve with maximum point at $(-1, e)$ where $e$ is any value and minimum point at $(2, f)$ where $f$ is less than $e$  B1 curve with maximum point at $(-1, g)$ where $g$ is negative or curve with minimum point at $(2, h)$ where $h$ is negative or curve increasing through $(5, 0)$ SC2 max and min correct and increasing through $(5, 0)$ but with straight lines rather than a curve.
	<b>Additional Guidance</b>		
		B4	

Question	Answer	Mark	Comments
20	$5 \times \dots$	M1	oe eg listing the 5 possible first digits
	$5 \times 5 \times 4 \times 3$	M1dep	
	300	A1	SC1 ( $6 \times 5 \times 4 \times 3 =$ ) 360
	<b>Additional Guidance</b>		

Q	Answer	Mark	Comments
21	<b>Alternative method 1</b> Works out $\frac{1}{2} \times (6 + 2\sqrt{7})$		
	$\frac{1}{2} \times (6 + 2\sqrt{7}) \times AD$	M1	oe eg $(3 + \sqrt{7}) \times AD$ or $(3 + \sqrt{7}) \times AC \sin C$ may be implied
	$\frac{13 + 3\sqrt{7}}{3 + \sqrt{7}} \times \frac{3 - \sqrt{7}}{3 - \sqrt{7}}$	M1	ft their $\frac{13 + 3\sqrt{7}}{3 + \sqrt{7}}$ their denominator must have 2 terms
	(numerator =) $39 - 13\sqrt{7} + 9\sqrt{7} - 21$ or $18 - 4\sqrt{7}$	M1dep	ft their numerator which must have 2 terms oe dep on 2nd M1
	(denominator =) 2	M1dep	ft their denominator dep on 2nd M1
	$9 - 2\sqrt{7}$	A1	
	<b>Alternative method 2</b> Works out $2 \times (13 + 3\sqrt{7})$		
	$\frac{1}{2} \times (6 + 2\sqrt{7}) \times AD$	M1	oe eg $(6 + 2\sqrt{7}) \times AD = (26 + 6\sqrt{7})$ may be implied
	$\frac{26 + 6\sqrt{7}}{6 + 2\sqrt{7}} \times \frac{6 - 2\sqrt{7}}{6 - 2\sqrt{7}}$	M1	ft their $\frac{26 + 6\sqrt{7}}{6 + 2\sqrt{7}}$ their denominator must have 2 term
	(numerator =) $156 - 52\sqrt{7} + 36\sqrt{7} - 84$ or $72 - 16\sqrt{7}$	M1dep	ft their numerator which must have 2 terms oe dep on 2nd M1
	(denominator =) 8	M1dep	ft their denominator dep on 2nd M1
	$9 - 2\sqrt{7}$	A1	

Mark scheme and Additional Guidance continue on the next page

<b>21 cont</b>	<b>Alternative method 3</b> Using identities		
	$(6 + 2\sqrt{7}) \times AD = (26 + 6\sqrt{7})$	M1	oe
	$(6 + 2\sqrt{7}) \times (a + b\sqrt{7}) = (26 + 6\sqrt{7})$	M1	oe
	$6a + 14b = 26$ and $2a + 6b = 6$	M1dep	oe eg $3a + 7b = 13$ and $a + 3b = 3$
	$a = 9$ or $b = -2$	M1dep	
	$9 - 2\sqrt{7}$	A1	
	<b>Additional Guidance</b>		
	Alt1 $\frac{18 - 4\sqrt{7}}{2}$		M4
	Alt2 $\frac{72 - 16\sqrt{7}}{8}$		M4
	Omission of $\frac{1}{2}$ can score up to M0M1M1M1A0  eg $\frac{13 + 3\sqrt{7}}{6 + 2\sqrt{7}} \times \frac{6 - 2\sqrt{7}}{6 - 2\sqrt{7}}$  $\frac{78 - 26\sqrt{7} + 18\sqrt{7} - 42}{8}$  $4.5 - \sqrt{7}$  (If the $\frac{1}{2}$ is recovered then all 5 marks are possible)		MOM1  M1M1  A0
Missing brackets must be recovered			

Question	Answer	Mark	Comments
22	<b>Alternative method 1</b> Working with powers of 2		
	$2^{3x}$ or $2^{52}$	B1	from $8^x$ and $4^{26}$
	$2^{52}(2^4 - 1)$ or $2^{52} \times 15$ or $2 \times 8^x = 2^{52}$ or $2 \times 2^{3x} = 2^{52}$	M1	oe
	$2^{3x+1} = 2^{52}$ or $3x + 1 = 52$	M1dep	oe eg $2^{3x} = 2^{51}$
	17	A1	
	<b>Alternative method 2</b> Working with powers of 4		
	$4^{1.5x}$ or $4^{28}$	B1	from $8^x$ and $2^{56}$
	$4^{26}(4^2 - 1)$ or $4^{26} \times 15$ or $2 \times 4^{1.5x} = 4^{26}$	M1	oe
	$4^{1.5x} = \frac{4^{26}}{4^{0.5}}$ or $1.5x = 26 - 0.5$	M1dep	oe eg $4^{1.5x} = 4^{25.5}$
	17	A1	
	<b>Alternative method 3</b> Working with powers of 8		
	$\frac{56}{8^3}$ or $\frac{52}{8^3}$	B1	from $2^{56}$ and $4^{26}$
	$\frac{52}{8^3} (8^3 - 1)$ or $\frac{52}{8^3} \times 15$ or $2 \times 8^x = \frac{52}{8^3}$	M1	oe
	$8^x = \frac{52}{8^3} \div 8^{\frac{1}{3}}$ or $x = \frac{52}{3} - \frac{1}{3}$	M1dep	oe eg $8^x = 8^{17}$
	17	A1	
	<b>Additional Guidance</b>		

Question	Answer	Mark	Comments
23(a)	Angles in the same segment	B1	oe eg angles at the circumference are equal
	Alternate angles	B1	do not accept alternative or alternating
	<b>Additional Guidance</b>		
	Angles on the circumference from a chord		B1
	Angles in the same sector, opposite angles, parallel lines, angles from a chord, similar triangles, isosceles triangle, corresponding angles, triangles on a chord, intersecting chords, allied angles, alternate segment theorem		B0

Question	Answer	Mark	Comments
23(b)	$\angle HJF = 3y$ or $\angle JFG = 2x$ or $\angle HFL = 2x$	M1	may be on the diagram implied by one correct equation in $x$ and $y$
	$2x + 3y + 98 = 180$ and $4x + 7y = 180$	M1dep	two correct equations in $x$ and $y$
	A correct attempt to eliminate one of the variables from the two equations	M1dep	eg $(4x + 7y) - 2(2x + 3y)$
	$x = 17$ and $y = 16$	A1	
	<b>Additional Guidance</b>		