

# **GCE**

**Further Mathematics B (MEI)** 

Y435/01: Extra pure

Advanced GCE

Mark Scheme for Autumn 2021

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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### Annotations and abbreviations

Annotation in scoris	Meaning
√and <b>≭</b>	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0,B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in	Meaning
mark scheme	
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction <b>In this question you must show detailed reasoning</b> appears in the question.

(	Question	n	Answer	Marks	AO	Guid	dance
1	(a)		DR $z = f(2, y) = 8 + 4y - 2y^2$	M1	1.1	Deriving correct equation of graph of section.	
			= $10 - 2(y-1)^2 \Rightarrow \max \text{ at } (1, 10) \text{ or } (2, 1, 10)$	A1	1.1	Finding TP by completing the square, use of "-b/2a", differentiation or mid-point between roots.	Working must be shown.
				B1	1.1	∩-shaped parabola which <u>crosses</u> horizontal axis twice.	Condone incorrect variable names on axes (eg $x$ - $y$ for $y$ - $z$ ).
			Crossing z-axis at $8$ , y-axis at $1 \pm \sqrt{5}$ and showing $(1,10)$ as a max	A1	1.1	Coordinates of intercepts and max must be shown on graph or apparent in working. Allow decimal values (awrt –1.2 and 3.2) for the <i>y</i> -intercepts.	z intercept must be shown as positive and max in 1st quadrant. However, scale is unimportant except that the negative y-intercept must be closer to O than the positive one.
				[4]			

(b)	2-	B1	1.1		
(b)	$\frac{\partial z}{\partial x} = 3x^2 + 2xy$	Di	1.1		
	$\frac{\partial z}{\partial y} = x^2 - 4y$	B1	1.1		
	· ·	M1	1.1	Setting a partial derivative to 0	Or
	$\frac{\partial z}{\partial x} = 0$			and deriving condition(s) on $x$ and/or $y$ .	$\frac{\partial z}{\partial y} = 0$
	$\Rightarrow 3x^2 + 2xy = 0$			and/or y.	•
	$\Rightarrow$ either $x = 0$ or $x = -\frac{2}{3}y$ or $y = -\frac{3}{2}x$				$\Rightarrow y = \frac{1}{4}x^2 \text{ or } x^2 = 4y \text{ or } x = \pm 2\sqrt{y}$
	$\frac{\partial z}{\partial y} = 0, \ x = -\frac{2}{3}y \Rightarrow 4y = \frac{4}{9}y^2 \Rightarrow y = 9$	M1	1.1	Substituting condition into other partial derivative equation to	$\frac{\partial z}{\partial x} = 0, \ y = \frac{1}{4}x^2$
	$\frac{\partial y}{\partial x}$ 3 9			derive a non-zero value for $x$ or $y$ .	i .
				$\frac{\partial z}{\partial y} = 0, \ y = -\frac{3}{2}x$	$3x^2 + \frac{1}{2}x^3 = 0 \Rightarrow x = -6$
					or $\frac{\partial z}{\partial z}$ or $\frac{12}{\sqrt{z}}$
					$\frac{\partial z}{\partial x} = 0, \ x = \pm 2\sqrt{y}$
					$12y \pm 4y^{\frac{3}{2}} = 0 \Rightarrow y = 9$
	x = -6	A1	1.1		<i>y</i> = 9
	z = -54 so $(-6, 9, -54)$	<b>A1</b>	1.1	From correct working only.	
	or $x = 0 \Rightarrow y = 0$ so $(0, 0, 0)$	A1	1.1	Derived from both $\frac{\partial z}{\partial r} = 0$ and	If an extra SP is presented then A1 can be awarded for either SP correct
				C.A	and then <b>A0</b> .
				$\frac{\partial z}{\partial y} = 0$ (could be by observation).	
		[7]			

2 (a) From Lagrange's Theorem the order of any subgroup of $G$ must be a factor of $S$ and $S$ is not a factor of $S$ i					T	<b>1</b>	·
a factor of 8  bull plants be a factor of the order of the group and 6 is not a factor of 8 by Lagrange's Theorem there can be no subgroup of $G$ of order 6" but $\mathbf{B0}$ for eg "By Lagrange's Theorem there can be no subgroup of $G$ of order 6".  bull plants be a factor of 8  bull plants be	2	(a)	From Lagrange's Theorem the order of any	<b>B</b> 1	2.4	Or "order of any subgroup of $G$	If referenced, Lagrange's Theorem
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$g \leftrightarrow 3, g^2 \leftrightarrow 6, g^3 \leftrightarrow 1, g^5 \leftrightarrow 7, g^6 \leftrightarrow 2, g^7 \leftrightarrow 5 \text{ and} $ Other three. Ignore repeats. $g^4 \leftrightarrow 4 \text{ does need not be seen again}$ $g \leftrightarrow 5, g^2 \leftrightarrow 2, g^3 \leftrightarrow 7, g^5 \leftrightarrow 1, g^6 \leftrightarrow 6, g^7 \leftrightarrow 3 \text{ and} $ $g \leftrightarrow 7, g^2 \leftrightarrow 6, g^3 \leftrightarrow 5, g^5 \leftrightarrow 3, g^6 \leftrightarrow 2, g^7 \leftrightarrow 1$						any one isomorphism	
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$g \leftrightarrow 7, g^2 \leftrightarrow 6, g^3 \leftrightarrow 5, g^5 \leftrightarrow 3, g^6 \leftrightarrow 2, g^7 \leftrightarrow 1$			$g \leftrightarrow 5, g^2 \leftrightarrow 2, g^3 \leftrightarrow 7, g^5 \leftrightarrow 1, g^6 \leftrightarrow 6, g^7 \leftrightarrow 3$ and				
r							
				[4]			

3	(a)	$\det(\mathbf{A} - \lambda \mathbf{I}) = \begin{vmatrix} 3 - \lambda & 3 & 0 \\ 0 & 2 - \lambda & 2 \\ 1 & 3 & 4 - \lambda \end{vmatrix}$	M1	1.1a	Formation of appropriate determinant soi.	
		$= (3 - \lambda)[(2 - \lambda)(4 - \lambda) - 2 \times 3] - 3(0 - 2 \times 1) \text{ oe}$	M1	1.1	Attempt to expand determinant. Allow one slip.	May see eg expansion by 1st col: $(3-\lambda)[(2-\lambda)(4-\lambda)-6]+1(6-0)$ Or other formulation eg: $((3-\lambda)(2-\lambda)(4-\lambda)+6+0)$ $-(0+6(3-\lambda)+0)$
		$=-\lambda^3+9\lambda^2-20\lambda+12=0$	A1 [3]	1.1	Must be an equation. ISW.	
3	(b)	1, 2 and 6 substituted into (a) equation to verify	B1 [1]	1.1	eg checking trace is insufficient.	
3	(c)	3a + 3b = a  or  2a  or  6a and $2b + 2c = b \text{ or } 2b \text{ or } 6b$ and $a + 3b + 4c = c \text{ or } 2c \text{ or } 6c$ $\lambda = 1: 2a = -3b, b = -2c$ or $\lambda = 2: c = 0, a = -3b$ or $\lambda = 6: a = b, c = 2b$	M1	1.1	Correctly forming 3 equations in 3 unknowns for one of their eigenvalues. May see explicit choice of eg $c = 1$ to form 3 equations in 2 unknowns. Attempt to solve equations for at least one of their eigenvalues leading to two unknowns in terms of $3^{rd}$ .	Or formation of appropriate $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 - \lambda & -4 \\ 0 & -1 & 3 - \lambda \end{vmatrix}.$ Attempt to expand determinant (might be in terms of $\lambda$ ) eg $\begin{pmatrix} 8 - 7\lambda + \lambda^2 \\ 6 - 2\lambda \\ 2 \end{pmatrix}.$ Can be inferred by
		$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \text{ or } \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	A1	1.1	or any non-zero multiple.	2 correct coefficients.
		$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	A1	1.1	or any non-zero multiple.	
			[4]			

					1		
3	(d)		(3 -3 1)	M1	3.1a	Forming matrix of their	
			-2			eigenvectors, E.	
			$\begin{bmatrix} 3 & 3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$				
				4 1 E/E	2.1	DC E' 1' ' C1 '	N. 1 . 1 . 1 C
			$\begin{bmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}^{-1} = \frac{1}{10} \begin{bmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{bmatrix} $ oe	A1FT	3.1a	BC. Finding inverse of their	May be in decimal form:
			$\begin{vmatrix} -2 & 1 & 1 \end{vmatrix} = \frac{1}{-5} \begin{vmatrix} -5 & -5 & 5 \end{vmatrix}$ oe			matrix of eigenvectors.	(-0.2  -0.6  0.4)
			$\begin{bmatrix} 1 & 1 & 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 10 & 1 & 3 & 3 \end{bmatrix}$				-0.5 -0.5 0.5
							$ \begin{pmatrix} -0.2 & -0.6 & 0.4 \\ -0.5 & -0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 \end{pmatrix} $
			$ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} $	<b>B</b> 1	3.1a	Matrix of eigenvalues must be	
			$\begin{vmatrix} 0 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 2^n & 0 \end{vmatrix}$			consistent with matrix of	
						eigenvectors. Allow 1 <sup>n</sup> .	
			$ \begin{vmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} \frac{1}{10} \begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix} $	M1	3.1a	Forming $\mathbf{E} \mathbf{\Lambda}^n \mathbf{E}^{-1}$ . Can be awarded	
			$\begin{vmatrix} 1 & -2 & 1 & 1 & 0 & 2^n & 0 & \frac{1}{2} & -5 & -5 & 5 \end{vmatrix}$			if $\Lambda^n$ incorrect or uncalculated but	
			$\begin{bmatrix} 1 & 1 & 1 & 0 & 2 & 0 & 10 & 1 & 2 & 3 & 1 \end{bmatrix}$			eigenvectors must be in same	
			$\begin{pmatrix} 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 6^n \end{pmatrix} \begin{pmatrix} 1 & 3 & 3 \end{pmatrix}$			order as eigenvalues.	
			$(1 \ 0 \ 0)(-2 \ -6 \ 4)$	M1	1.1	Proper attempt to multiply either	(3 -3 1)(1 0 0)
			$\begin{bmatrix} 0 & 2^n & 0 \end{bmatrix} \begin{bmatrix} -2 & -0 & 4 \\ 5 & 5 & 5 \end{bmatrix}$			the first two or the last two (of 3)	$\begin{bmatrix} 1 & 3 & 3 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2^n & 0 \end{bmatrix}$
						in the correct order (with or	
			$ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} \begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix} = $			without $\frac{1}{10}$ ).	$(1 \ 0 \ 2)(0 \ 0 \ 6^n)$
						10 /	or $(2  2  2  2^n  6^n)$
			$\begin{bmatrix} -2 & -6 & 4 \\ -5 \times 2^n & -5 \times 2^n & 5 \times 2^n \end{bmatrix}$				3 -3×2 0
			$\begin{bmatrix} -3 \times 2^n & -3 \times 2^n & 3 \times 2^n \end{bmatrix}$				$  -2  2^n  6^n  $
			$\left(\begin{array}{ccc} 6^n & 3 \times 6^n & 3 \times 6^n \end{array}\right)$				$\begin{bmatrix} 1 & 0 & 2 \times 6^n \end{bmatrix}$
				A 1			$\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} = $ or $\begin{pmatrix} 3 & -3 \times 2^n & 6^n \\ -2 & 2^n & 6^n \\ 1 & 0 & 2 \times 6^n \end{pmatrix}$
			$ \begin{vmatrix} \frac{1}{10} \begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} -2 & -6 & 4 \\ -5 \times 2^n & -5 \times 2^n & 5 \times 2^n \\ 6^n & 3 \times 6^n & 3 \times 6^n \end{vmatrix} = $	<b>A1</b>	1.1	OI	
			$\left  \frac{1}{10} \right ^{-2} = \frac{1}{10} \left  \frac{1}{10} \right ^{-5 \times 2^n} = \frac{1}{10} \left  \frac{1}{10} \right ^$			$\begin{bmatrix} \frac{1}{10} \begin{pmatrix} 3 & -3 \times 2^n & 6^n \\ -2 & 2^n & 6^n \\ 1 & 0 & 2 \times 6^n \end{pmatrix} \begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix} =$	
						$\left  \frac{1}{10} \right  -2  2^n  6^n  \left  -5  -5  5 \right  =$	
			$ \frac{1}{10} \begin{pmatrix} -6+15\times 2^n + 6^n & -18+15\times 2^n + 3\times 6^n & 12-15\times 2^n + 3\times 6^n \\ 4-5\times 2^n + 6^n & 12-5\times 2^n + 3\times 6^n & -8+5\times 2^n + 3\times 6^n \\ -2+2\times 6^n & -6+6^{n+1} & 4+6^{n+1} \end{pmatrix} $			$\begin{bmatrix} 10 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 2 \times 6^n \\ \end{bmatrix}$ $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$	
			$\frac{1}{10}$ $4-5\times 2^n+6^n$ $12-5\times 2^n+3\times 6^n$ $-8+5\times 2^n+3\times 6^n$			etc.	
			$\begin{vmatrix} 10 \\ -2 + 2 \times 6^n \\ -6 + 6^{n+1} \\ 4 + 6^{n+1} \end{vmatrix}$				
						Condone $6 \times 6^n$ unsimplified.	
				[6]			
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4	(a)	CF: $u_{n+2} - 3u_{n+1} - 10u_n = 0$ and $u_n = \alpha r^n$	M1	1.1a	Deriving the auxiliary equation	
-	(4)	$\Rightarrow r^2 - 3r - 10 = 0$	1,11	11111	(allow one sign error).	
		$\Rightarrow r = 5 \text{ or } r = -2$	A1FT	1.1	FT correct roots of their AE to	Condone missing brackets around
		CF is $\alpha 5^n + \beta (-2)^n$	11111	111	form CF (do not ISW).	-2 unless misused.
		Trial function: $u_n = an + b$	B1	1.1a	Correct form.	Other forms eg $an^2 + bn + c$ are
		a(n+2)+b-3[a(n+1)+b]-10(an+b)	M1	1.1	Substituting their form correctly	allowable provided $a = 0$ derived.
		$\begin{vmatrix} a(n+2)+b-3[a(n+1)+b]-10(an+b) \\ = 24n-10 \end{vmatrix}$	IVII	1.1	into recurrence relation.	
		$\Rightarrow (a-3a-10a)=24$	M1	1.1	Deriving two equations in $a$ and $b$	
		and $2a + b - 3a - 3b - 10b = -10$	1,11	1.1	using a correct method (eg	
					comparing coefficients)	
		a = -2 and $b = 1$ so GS is	<b>A1</b>	1.1	Full form of GS, including $u_n =$ ,	cao
		$u_n = 1 - 2n + \alpha 5^n + \beta (-2)^n$			must be seen.	
			[6]			
4	(b)	Either: $n = 0 = 1 + \alpha + \beta = 6$	M1	1.1	Substituting $n = 0$ or $n = 1$ in their	This mark can be awarded if one of
		or: $n = 1 = 1 - 2 + 5\alpha - 2\beta = 10$			GS to derive an equation in $\alpha \& \beta$ .	their equations is wrong.
		$\alpha + \beta = 5$ and $5\alpha - 2\beta = 11$	M1	1.1	Deriving 2 equations from	Attempt to solve can be implied by
		$=> 2\alpha + 2\beta = 10 => 7\alpha = 21$			substituting $n = 0 \& 1$ , at least one	correct answer or valid algebra but
					correct for their GS, and	incorrect answer with no working
			, 4 P/F		attempting to solve.	M0
		$\alpha = 3$ and $\beta = 2$ so	A1FT	1.1	FT from their GS. Allow non-	
		$u_n = 1 - 2n + 3 \times 5^n + 2 \times (-2)^n$			embedded values if GS seen in (a).	
			[2]		Do not ISW.	
4	(c)	From recurrence relation:	[3] B1	2.5	Both expressions properly seen (ie	
		$u_2 = 3u_1 + 10u_0 + 24 \times 0 - 10$		2.5	it must be clear that candidates are	
		$= 3 \times 10 + 10 \times 6 - 10 = 80$			correctly using two different	
		From particular solution:			methods to find $u_2$ ).	
		$u_2 = 1 - 2 \times 2 + 3 \times 5^2 + 2 \times (-2)^2$			ĺ	
		= 1 - 4 + 75 + 8 = 80				
			[1]			

4	(d)	$v_n = \frac{1-2n}{p^n} + 3\left(\frac{5}{p}\right)^n + 2\left(\frac{-2}{p}\right)^n$	M1	3.1a	Writing $v_n$ in a form which enables the limit to be deduced.	
		If $ p  < 5$ then $v_n \to \infty$ while if $ p  > 5$ then $v_n \to 0$ as $n \to \infty$	B1	2.1	Convincing argument. FT for GS of the form: $c - dn + \alpha s^n + \beta t^n$ (where $ s  >  t $ ).	At most one of $c$ and $d$ is 0. s and $t$ are not equal and both not 0. Both $\alpha$ and $\beta$ are not 0. Either $ s  > 1$ or $ t  > 1$ (or both).
		$   \begin{array}{c}     p = 5 \\     q = 3   \end{array} $	A1 A1 [4]	2.2a 2.2a	FT. $p = s$ (must be a number). FT. $q = \alpha$ (must be a number).	<b>A0</b> If $s = -t$ . <b>A0</b> If $s = -t$ . If <b>M0</b> then <b>SC2</b> for $p = 5$ , $q = 3$ .

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5	$\partial g$ $\partial g$ $\partial g$ $\partial g$ $\partial g$	M1	3.1a	$g(x, y, z) = x^2 + y^2 + 2z^2$ and	May be rewritten as
	$\frac{\partial g}{\partial x} = 2x \text{ or } \frac{\partial g}{\partial y} = 2y \text{ or } \frac{\partial g}{\partial z} = 4z$			surface is $g = 126$ . Finding one	$z = f(x, y) = \sqrt{63 - \frac{1}{2}y^2 - \frac{1}{2}x^2}$
				correct partial derivative.	but condone ±.
	(2x)	<b>A1</b>	3.1a	Finding the normal vector.	
	$\nabla g = \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix}$ is the normal to the tangent plane at				$\left  -{2z} \right $
	(4z)				$\nabla g = \left  -\frac{y}{2} \right $ oe
	each point.				$\nabla g = \begin{pmatrix} -\frac{x}{2z} \\ -\frac{y}{2z} \\ -1 \end{pmatrix} \text{ oe }$
					-1
	(0.) (0)	N/I 1	2.1-	D-44:	
	$\begin{pmatrix} 2x \\ 0 \end{pmatrix}$	M1	3.1a	Dotting normal with normal to <i>x-y</i> plane.	
	$\nabla g.\mathbf{n} = \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 4z$			plane.	
	(2x) $(0)$	M1	2.2a	Expressing dot product in other form using correct value of angle.	
	$ = \begin{pmatrix} 2x \\ 2y \\ 4\pi \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cos \frac{\pi}{3} $			form using correct value of angle.	
		3.54			
	$= \sqrt{(2x)^2 + (2y)^2 + (4z)^2} \times 1 \times \frac{1}{2}$	M1	1.1	Using $\cos \frac{\pi}{3} = \frac{1}{2}$ , forming	
	<u> </u>			magnitude of both normals and	
	$=2\sqrt{x^2+y^2+4z^2}\times\frac{1}{2}=\sqrt{126-2z^2+4z^2}$			reducing to form $\sqrt{a+bz^2}$ oe	or $\sqrt{a+bx^2+by^2}$ (could see eg
	$=\sqrt{126+2z^2}$			(could be done after squaring).	$x^2 + y^2 = 108$ oe after equating to
	$=\sqrt{126+2z^2}$			(court of done arter squaring).	x + y = 108 oe after equating to $4z$ and eliminating $z$ ).
	$\sqrt{126 + 2z^2} = 4z \Rightarrow 126 + 2z^2 = 16z^2$	A1	3.2a	Not ± in final answer.	72 and chilinating 2).
	'				
	$\Rightarrow 14z^2 = 126 \Rightarrow z^2 = 9 \Rightarrow z = \pm 3$				
	$z \ge 0 \Rightarrow z = 3$ which is the equation of $\Pi$ .				
		[6]			

6	(a)	$\frac{1}{(q+1)} + \frac{1}{(q+1)(q+2)} + \frac{1}{(q+1)(q+2)(q+3)} + \dots$ $< \frac{1}{(q+1)} + \frac{1}{(q+1)^2} + \frac{1}{(q+1)^3} + \dots$	M1	2.1	Correct statement that given series is less than an infinite GP (could be eg $\frac{1}{q} + \frac{1}{q^2} + \dots$ or $\frac{1}{3} + \frac{1}{3^2} + \dots$ ).	
		$=\frac{\frac{1}{q+1}}{1-\frac{1}{q+1}}$	A1	2.1	FT on their $\frac{a}{1-r}$ .	
		$=\frac{1}{q+1-1}=\frac{1}{q}$	A1 [3]	2.1	AG. Intermediate step must be seen.	
6	(b)	$q \ge 1 \Rightarrow \frac{1}{a} \le 1$	M1	2.2a		
		But $S < \frac{1}{q} \Rightarrow S < 1$ ; clearly $S > 0$ so $0 < S < 1$ so	A1	2.2a	AG. $S > 0$ must be stated but need not be justified.	Since $0 < \frac{1}{q} \le 1$ and $S < \frac{1}{q}$ then
		$S otin \square$ .	[2]			$0 < S < 1$ and $\therefore S \notin \square$ .
6	(c)	$e = \sum_{r=0}^{\infty} \frac{1}{r!} = \frac{p}{q} \Rightarrow eq! = \sum_{r=0}^{\infty} \frac{q!}{r!} = p(q-1)!$	M1	3.1a	Multiplying both sides by $q!$ No need to mention $q \ge 1$ in this part.	
		$\therefore p(q-1)! = \sum_{r=0}^{\infty} \frac{q!}{r!} = \sum_{r=0}^{q} \frac{q!}{r!} + \sum_{r=q+1}^{\infty} \frac{q!}{r!}$	M1	2.1	Rewriting to a form in which it is clear that every term on both sides, except <i>S</i> , is an integer.	
		$=q!+q!+\frac{q!}{2!}++\frac{q!}{q!}+S$				
		$= 2q! + q(q-1) \times 3 + q(q-1) \times 4 + + 1 + S$ $p(q-1)! \text{ and } q! + q! + \frac{q!}{2!} + + 1 \text{ are all integers}$ but S is not which is a contradiction.	A1	3.2a	AG	

Alternative Method: $S = \sum_{r=q+1}^{\infty} \frac{q!}{r!}$	M1	Expressing $S$ as an infinite sum in terms of factorials.
$S = q! \sum_{r=0}^{\infty} \frac{1}{r!} - \sum_{r=0}^{q} \frac{q!}{r!} = q! e - q! - \sum_{r=1}^{q} \frac{q!}{r!}$ $= p(q-1)! - q! - \sum_{r=0}^{q} q(q-1)(q-r+1)$	M1	Rewriting to a form in which it is clear that every term on both sides, except <i>S</i> , is an integer.
since $1 \le r \le q$ p(q-1)!, q! and $q(q-1)(q-r+1)$ are all integers but $S$ is not which is a contradiction.	A1	AG

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