Oxford Cambridge and RSA

GCE

Further Mathematics A

Y543/01: Mechanics

Advanced GCE

Mark Scheme for Autumn 2021

Oxford Cambridge and RSA Examinations

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Question			Answer	Marks	AO	Guidance	
1			Initial Elastic PE = $= \dfrac{24 \times 0.9^2}{2 \times 0.6}$	B1	1.1	Use of $\dfrac{\lambda x^2}{2l}$ with attempt at finding extension (ie not just $x = 1.5$)	16.2 J
			Final Elastic PE = $= \dfrac{24 \times 0.4^2}{2 \times 0.6}$	B1	1.1	Use of $\dfrac{\lambda x^2}{2l}$ with attempt at finding extension (ie not just $x = 1$)	3.2 J
			Increase in PE = $0.4g \times 2.5$	M1	1.1	Attempt at use of "mgh" to find the increase of gravitational PE from initial position to ceiling	9.8 J
			"16.2" = "3.2" + $\frac{1}{2} \times 0.4v^2$ + "9.8"	M1	1.1	Attempt at conservation of energy with consideration of KE and their PE	8.624 J
			$v^2 = 16 \Rightarrow$ speed is 4 m s^{-1}	A1 [5]	1.1	Not ±. Units required.	

Question			Answer	Marks	AO	Guidance	
2	(a)		$\mathbf{I} = m\mathbf{v} - m\mathbf{u} = 2(-3\mathbf{i} + \mathbf{j} - (5\mathbf{i} + 16\mathbf{j}))$	M1	1.1	Correct use of formula (award if $m\mathbf{u} - m\mathbf{v}$)	or using the cosine rule on vectors **u**,**v**,**I** to reach $\|I\| = 34$
			$= 2(-8\mathbf{i} - 15\mathbf{j})$	A1	1.1	Allow $16\mathbf{i} + 30\mathbf{j}$	
			$I = 2\sqrt{(-8)^2 + (-15)^2}$	M1	1.1	or $\sqrt{(-16)^2 + (-30)^2}$ oe	
			$= 2\sqrt{289} = 34$	A1	1.1		
			$\cos\theta = \frac{\mathbf{I}.\mathbf{i}}{\|\mathbf{I}\|\|\mathbf{i}\|} = \frac{-16\times 1}{34\times 1}$	M1	1.1	Attempting to use the dot product of **I** and **i** to find the required angle	or use of ordinary trigonometry eg $\tan\theta = \frac{-30}{-16}$
			$\theta = \cos^{-1}\frac{-8}{17} = 118.1°$ or 2.06 rad	A1	1.1		
				[6]			
2	(b)		Init KE $= \frac{1}{2}\times 2\times\left(5^2 + 16^2\right)$	M1	1.1	281 J	
			Final KE $= \frac{1}{2}\times 2\times\left((-3)^2 + 1^2\right)$	M1	1.1	10 J	
			Loss = 281 – 10 = 271 J	A1 [3]	1.1		

Question			Answer	Marks	AO	Guidance	
3	(a)		$[F] = \mathrm{MLT^{-2}}$	B1	1.1		
			$\left[mv\frac{\mathrm{d}v}{\mathrm{d}x}\right] = \frac{[m][v][v]}{[x]} = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{L}} = \mathrm{MLT^{-2}}$	B1 [2]	2.1	Correctly finding the dimensions of both sides is sufficient for B1B1; an explicit conclusion is not necessary.	
3	(b)		Only quantities with the same dimensions can be added (or subtracted) [so $[a^2] = [x^2]$ which means that $[a] = [x]$]	B1 [1]	2.4		
3	(c)		$[k]\mathrm{M}^{-\frac{1}{2}}(\mathrm{L}^2)^{\frac{1}{2}} = \mathrm{LT^{-1}}$	M1	2.2a	Use of formula for v to derive dimensional equation for $[k]$	
			$[k] = \mathrm{M^{\frac{1}{2}}T^{-1}}$	A1	1.1		
			Alternative solution	M1			
			$v = km^{-\frac{1}{2}}\sqrt{a^2 - x^2} \Rightarrow k = \frac{vm^{\frac{1}{2}}}{\sqrt{a^2 - x^2}}$ so the units of k are $\mathrm{kg^{\frac{1}{2}}s^{-1}}$			Use of formula for v to derive units of k.	
			$[k] = \mathrm{M^{\frac{1}{2}}T^{-1}}$	A1			
				[2]			
3	(d)		$\frac{\mathrm{d}v}{\mathrm{d}x} = km^{-\frac{1}{2}}(-2x)\frac{1}{2}(a^2 - x^2)^{-\frac{1}{2}}$	M1	1.1	Use of chain rule to differentiate v wrt x	$\frac{\mathrm{d}v}{\mathrm{d}x} = -km^{-\frac{1}{2}}x(a^2 - x^2)^{-\frac{1}{2}}$
			$\therefore F = mv\frac{\mathrm{d}v}{\mathrm{d}x}$ $= m \times km^{-\frac{1}{2}}(a^2 - x^2)^{\frac{1}{2}}km^{-\frac{1}{2}}(-2x)\frac{1}{2}(a^2 - x^2)^{-\frac{1}{2}}$	M1	1.1	Use of formula for F with m, v and their $\frac{\mathrm{d}v}{\mathrm{d}x}$ substituted in.	
			$\therefore F = -k^2x$	A1 [3]	1.1		

Question		Answer	Marks	AO	Guidance	
4	(a)	KE of $P = \frac{1}{2}mv^2$	B1	1.2		SSU – change C to R if a better reflection of candidate solutions
		$\updownarrow$ $C\sin\theta = mg$	M1	3.3	Balancing forces in the vertical. C must be resolved	In this solution, C is the normal contact force between P and the cone and θ is the semi-vertical angle of the cone
		$\leftrightarrow$ $C\cos\theta = ma$	M1	3.3	NII in the horizontal using a resolved component of C	
		$\frac{\cos\theta}{\sin\theta} = \frac{a}{g} = \frac{v^2}{rg}$	M1	3.4	Eliminating C (and m) between the two equations and using a correct form for a	May see $v^2 = gh$ here and used later
		PE of P (exceeds that of Q by) $mgh = mg\frac{r}{\tan\theta} = mg\frac{r\cos\theta}{\sin\theta} = mg\frac{v^2}{g} = mv^2$ soi	M1	3.4	Using the relationship to find the (excess) PE of P in terms of m and v (and possibly g) only	h is the vertical height of P above Q
		So total ME of P exceeds that of Q by $= mv^2 + \frac{1}{2}mv^2 = \frac{3}{2}mv^2$ J	A1 [6]	2.2a	AG. Or total ME of $Q = 0$ but some justification of excess for PE at least must be seen in the solution	Use R instead of C?
4	(b)	One of: - We have assumed that the radius of the circle which P moves in is the same as the radius of the cone at that level - Q is at V [neither of which is quite true if P and Q do not have a negligible radius]	B1 [1]	3.5b	Also accept e.g. - CofM of P lies on the edge of the cone - CofM of Q lies at V	V is the vertex of the cone
4	(c)	Resistance to the motion of P should be included in the model.	B1 [1]	3.5c	eg air resistance. Allow friction.	

Question			Answer	Marks	AO	Guidance	
5	(a)		$F \propto \frac{1}{(t+1)^2}$ $\therefore F = \frac{k}{(t+1)^2} = ma = 3\frac{\mathrm{d}v}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{k}{3(t+1)^2}$	B1 [1]	3.1b	AG	
5	(b)		$\therefore v = \frac{k}{3}\int \frac{1}{(1+t)^2}\,\mathrm{d}x = \frac{-k}{3(1+t)} + u$	M1	3.1b	Separating variables correctly and integrating to $\frac{C}{1+t}$; award if "$+ u$" missing	May use + c instead of u
			$t = 0,\ v = 0 \Rightarrow k = 3u$	M1	3.1b	Substituting initial values to determine a relationship between k and u.	NB The units of k are N s^2 or kg m but these are not required.
			$t = 1,\ v = 2 \Rightarrow\ 2 = \frac{-k}{3(1+1)} + u$	M1	3.1b	Substituting $t = 1$ to determine a second relationship between k and u oe.	
			$\Rightarrow u = 4,\ k = 12 \Rightarrow v = 4 - \frac{4}{1+t}$ oe	A1 [4]	1.1	eg $v = \frac{4t}{1+t}$	
5	(c)		$\frac{\mathrm{d}x}{\mathrm{d}t} = 4 - \frac{4}{1+t} \Rightarrow x = 4t - 4\ln(1+t) + c$	M1	1.1	For integrating their 'v' to reach an expression involving $k\ln(1+t)$ oe Can be awarded even if no "$+ c$"	
			$t = 0,\ x = 1 \Rightarrow c = 1$ so $x = 4t - 4\ln(1+t) + 1$	A1 [2]	1.1		
5	(d)		95% of $v_T = 0.95 \times 4 = 3.8$	B1	2.2a		
			$v = 3.8 \Rightarrow 3.8 = 4 - \frac{4}{1+t}$	M1	3.1b	Setting their v to their 3.8 in the appropriate equation	
			$\Rightarrow 0.2 = \frac{4}{1+t} \Rightarrow 1 + t = 20 \Rightarrow t = 19$	A1	1.1		

Question			Answer	Marks	AO	Guidance	
			so $x = 4 \times 19 - 4\ln(1+19) + 1$	M1	1.1	Substituting their t into the equation for x	
			$x = 77 - 4\ln 20$ so distance moved is $76 - 4\ln 20$ m or awrt 64 m	A1	1.1		
				[5]			

Question			Answer	Marks	AO	Guidance	
6	(a)		$20 = 4u \Rightarrow u = 5$	B1	1.1		
			Initial energy = $\frac{1}{2}\times 4\times 5^2$	B1	1.1	$= 50$	Assuming zero PE level at initial level of P
			Energy at $\theta = \frac{1}{2}\times 4\times v^2 + 4g\times 0.8(1-\cos\theta)$	M1	1.1	Attempt to derive total ME at general or specific angle	
			$2v^2 + 15.68 = 50 \Rightarrow v^2 = 17.16$	A1	1.1	Equating energies to derive a value for v^2	$v = 4.142...$
			Radial: $a_r = \frac{v^2}{0.8} = \frac{17.16}{0.8}$	M1	3.1b	Correct form for centripetal acceleration and use of v^2	$a_r = 21.45$
			Tangential: $ma_t = -mg\sin\frac{\pi}{3}$	M1	3.1b	NII for tangential direction with weight resolved (– not necessary)	$a_t = -\frac{\sqrt{3}g}{2} = -8.4870...$
			$a = \sqrt{\left(-\frac{\sqrt{3}g}{2}\right)^2 + \left(\frac{429}{20}\right)^2} = 23.067...$ so the magnitude of the acceleration is 23.1 m s^{-2} (3 sf)	A1	1.1		
				[7]			
6	(b)		Radial: $T - 4g\cos\theta = \frac{4v^2}{0.8}$	M1	2.1	NII for radial direction. T could be set to 0. Correct form of a_r.	
			$v^2 = 5^2 - 2g\times 0.8(1-\cos\theta)$	M1	2.1	v^2 in terms of $\cos\theta$ from conservation of energy	$v^2 = 9.32 + 15.68\cos\theta$
			$-7.84\cos\theta = 9.32 + 15.68\cos\theta$ $\therefore \cos\theta = -\frac{9.32}{23.52}$ $\therefore \theta = 113.3°$ or 1.98 rads	A1	3.2a		
				[3]			

Question			Answer	Marks	AO	Guidance	
7	(a)		$u_{Ax}=3,\ u_{Bx}=-2$	B1	3.3	Resolving horizontal components of u_A and u_B. Accept $u_A = 5\cos\alpha$ and $u_B = -4\cos\frac{\pi}{3}$ but must have opposite signs or directions indicated on diagram.	Signs may be reversed throughout
			$m_A\times3+m_B\times-2=m_Av_{Ax}+m_B\times0$	M1	3.4	Conservation of momentum	May be seen in **(b)**
			$v_{Ax}=3-\dfrac{2m_B}{m_A}$	A1	1.1		
			$e=\dfrac{0-v_{Ax}}{3--2}$ or $v_{Ax}=-5e$	M1	3.4	Restitution	$e=\dfrac{0-\left(3-\dfrac{2m_B}{m_A}\right)}{3--2}=\dfrac{2m_B}{5m_A}-\dfrac{3}{5}$
			$e\ge0\Rightarrow\dfrac{2m_B}{5m_A}-\dfrac{3}{5}\ge0\Rightarrow\dfrac{m_B}{m_A}\ge\dfrac{3}{2}$	A1	2.1	AG	
			$e\le1\Rightarrow\dfrac{2m_B}{5m_A}-\dfrac{3}{5}\le1\Rightarrow\dfrac{m_B}{m_A}\le4$	A1	2.1	AG	
				[6]			
7	(b)		Total initial KE = $\frac{1}{2}\times2\times5^2+\frac{1}{2}\times6\times4^2=73$	B1	1.1		
			$v_{Ay}=u_{Ay},\ v_{By}=u_{By}=2\sqrt{3}$	M1	3.4	Perpendicular components found and unchanged	
			$v_{Ax}=-3$	M1	3.4	Using their formula for v_{Ax} from **(a)**.	NB If method mark for conservation of momentum not seen in **(a)** then award M1 in **(a)** if either $m_A\times3+m_B\times-2=m_Av_{Ax}$ or $2\times3+6\times-2=2v_{Ax}$ seen here If method mark for restitution not seen in (a) then award M1 in (a) if seen here.
‘			KE Loss = $73-\left(\frac{1}{2}\times2\times(3^2+4^2)+\frac{1}{2}\times6\times\left(2\sqrt{3}\right)^2\right)=12$ J	A1	1.1		
				[4]			

Question			Answer	Marks	AO	Guidance	
8	(a)		$\bar{x} = \frac{12a \times M + x \times m}{M+m} = \frac{12Ma + mx}{M+m}$	B1 **[1]**	1.1	AG. www	
8	(b)		$\bar{y} = \frac{3a \times M + y \times m}{M+m} = \frac{3Ma + my}{M+m}$	B1 **[1]**	1.1		
8	(c)		If P is at O, $\bar{x} = \frac{12Ma}{M+m}$ and $\bar{y} = \frac{3Ma}{M+m}$	B1ft	3.3	FT their expression for $\bar{y}$	Alternative: B1 for correct expressions for $\bar{x}, \bar{y}$ M1: forming 2 inequalities with $2a$ and $6a$ (must be right way around) M1: simplifying or manipulating both inequalities so that they can be combined or compared A1: fully correct and conclusion www
			$\bar{y} < 2a \Rightarrow 3M < 2M + 2m \Rightarrow m > \frac{1}{2}M$	M1	3.4		
			$\bar{x} < 6a \Rightarrow 12M < 6M + 6m \Rightarrow m > M$	M1	3.4		
			Conclusion: $m > \frac{1}{2}M$	A1 **[4]**	2.4	AG.	
8	(d)		$\bar{x} = \frac{12Ma + m \times 12ak}{M+m}$ used	B1	3.3		
			$\frac{12Ma + m \times 12ak}{M+m} = 6a$	M1	3.4	Their $\bar{x}$ equated to $6a$	Ignore working with $\bar{y}$
			$k = \frac{m-M}{2m}$ oe	A1 **[3]**	1.1	$k = \frac{1}{2}\left(1 - \frac{M}{m}\right)$	Ignore working with $\bar{y}$ unless this affects final answer
8	(e)		$m = \frac{3}{2}M \Rightarrow k_{OC} = \frac{1}{6}$	B1	3.3	$k_{OC} = \frac{3}{18} = 0.1\dot{6}$	
			$\bar{y} = \frac{3Ma + \frac{3}{2}M \times 6ak}{M + \frac{3}{2}M}$	M1	3.4	Substituting $y = 6ak$ and $m = \frac{3}{2}M$ into their $\bar{y}$	

Question			Answer	Marks	AO	Guidance	
			$\bar{y} = 2a \Rightarrow \frac{6a+18ak}{5} = 2a \Rightarrow k_{OA} = \frac{2}{9}$	A1	3.4	$k_{OA} = \frac{4}{18} = 0.\dot{2}$	
			(k changes from 1 to 0 and $k_{OA} > k_{OC}$ so) lamina topples over edge OA.	A1 **[4]**	2.2a	www	

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