



Oxford Cambridge and RSA

Thursday 21 October 2021 – Afternoon

AS Level Further Mathematics A

Y535/01 Additional Pure Mathematics

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 The points A , B and C have position vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ respectively, relative to the origin O .
- (a) (i) Calculate $\mathbf{a} \times \mathbf{b}$, giving your answer as a multiple of $\mathbf{c}$. [1]
- (ii) Explain, geometrically, why $\mathbf{a} \times \mathbf{b}$ must be a multiple of $\mathbf{c}$. [1]
- (b) Use a vector product method to calculate the area of triangle ABC . [3]
- 2 The surface S is given by $z = x^2 + 4xy$ for $-6 \leq x \leq 6$ and $-2 \leq y \leq 2$.
- (a) (i) Write down the equation of any **one** section of S which is parallel to the x - z plane. [1]
- (ii) Sketch the section of (a)(i) on the axes provided in the Printed Answer Booklet. [2]
- (b) Write down the equation of any **one** contour of S which does **not** include the origin. [1]
- 3 For positive integers n , the sequence of Fibonacci numbers, $\{F_n\}$, starts with the terms $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, ... and is given by the recurrence relation $F_n = F_{n-1} + F_{n-2}$ ($n \geq 3$).
- (a) Show that $F_{3k+3} = 2F_{3k+1} + F_{3k}$, where k is a positive integer. [2]
- (b) Prove by induction that F_{3n} is even for all positive integers n . [4]
- 4 (a) Let $a = 1071$ and $b = 67$.
- (i) Find the unique integers q and r such that $a = bq + r$, where $q > 0$ and $0 \leq r < b$. [1]
- (ii) Hence express the answer to (a)(i) in the form of a linear congruence modulo b . [1]
- (b) Use the fact that $358 \times 715 - 239 \times 1071 = 1$ to prove that 715 and 1071 are co-prime. [4]

- 5 A trading company deals in two goods. The formula used to estimate z , the total weekly cost to the company of trading the two goods, in tens of thousands of pounds, is

$$z = 0.9x + \frac{0.096y}{x} - x^2y^2,$$

where x and y are the masses, in thousands of tonnes, of the two goods.

You are given that $x > 0$ and $y > 0$.

- (a) In the first week of trading, it was found that the values of x and y corresponded to the stationary value of z .

Determine the total cost to the company for this week. [7]

- (b) For the second week, the company intends to make a small change in either x or y in order to reduce the total weekly cost.

Determine whether the company should change x or y . (You are not expected to say by how much the company should reduce its costs.) [4]

- 6 The set S consists of the following four complex numbers.

$$\sqrt{3} + i \quad -\sqrt{3} - i \quad 1 - i\sqrt{3} \quad -1 + i\sqrt{3}$$

For $z_1, z_2 \in S$, the binary operation $\circ$ is defined by $z_1 \circ z_2 = \frac{1}{4}(1 + i\sqrt{3})z_1z_2$.

- (a) (i) Complete the Cayley table for $(S, \circ)$ given in the Printed Answer Booklet. [3]

(ii) Verify that $(S, \circ)$ is a group. [4]

(iii) State the order of each element of $(S, \circ)$. [1]

- (b) Write down the only proper subgroup of $(S, \circ)$. [1]

- (c) (i) Explain why $(S, \circ)$ is a cyclic group. [1]

(ii) List all possible generators of $(S, \circ)$. [1]

Turn over for questions 7 and 8

7 (a) Let $f(n) = 2^{4n+3} + 3^{3n+1}$.

Use arithmetic modulo 11 to prove that $f(n) \equiv 0 \pmod{11}$ for all integers $n \geq 0$. [4]

(b) Use the standard test for divisibility by 11 to prove the following statements.

(i) $10^{33} + 1$ is divisible by 11 [2]

(ii) $10^{33} + 1$ is divisible by 121 [4]

8 A sequence $\{u_n\}$ is defined by the recurrence system

$$u_1 = 1 \text{ and } u_{n+1} = a - \frac{a^2}{2u_n} \text{ for } n \geq 1, \text{ where } a \text{ is a positive constant.}$$

Determine with justification the behaviour of the sequence for all possible values of a . [7]

END OF QUESTION PAPER

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