AQA

AS
FURTHER MATHEMATICS
7366/2S

Paper 2 Statistics

Mark scheme

June 2020

Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer.	1.1b	B1	2
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer.	1.1b	B1	6
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Obtains E(X) = 4.5 OE	1.1b	B1	E(X) = $\frac{8+1}{2} = 4.5$
3(b)	Obtains Var (X) = 5.25 OE	1.1b	B1	Var (X) = $\frac{8^2-1}{12} = 5.25$
3(c)	Obtains P($X \geq 6$) = 0.375 OE	1.1b	B1	P($X \geq 6$) = $3 \times \frac{1}{8} = 0.375$
3(d)	Evaluates the results and suggests or implies the dice is biased or that a uniform distribution is not suitable because the frequencies are not approximately equal.	3.5a	E1	The dice appears to be biased. The random variable X would be modelled with a discrete random variable where the probabilities are estimated using relative frequencies.
	Explains that the probabilities would be estimated using relative frequencies.	3.5c	E1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Obtains correct z-value AWRT 2.17 PI by a correct upper or lower limit of the confidence interval.	1.1a	B1	$z = 2.17$
	Uses formula for upper or lower limit of a confidence interval using their z-value. Condone use of $\sqrt{5100}$ Condone use of t-value.	1.1a	M1	$\bar{x} \pm z\sqrt{\frac{s^2}{n}}$ $= 28500 \pm 2.17 \times \frac{5100}{\sqrt{200}}$ $= (27\,717, 29\,283)$
	Obtains the correct confidence interval. CAO	1.1b	A1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Obtains P($X \leq 6$) = 0.9 OE	1.1b	B1	P($X \leq 6$) = 0.2 + 0.6 + 0.1 = 0.9
5(b)	Applies formula for E(X) or obtains the values of $3X + 2$ PI by correct Var (X)	1.1a	M1	E(X) = 2 × 0.2 + 4 × 0.6 + 6 × 0.1 + 9 × 0.1 = 4.3 Var (X) = 2^2 × 0.2 + 4^2 × 0.6 + 6^2 × 0.1 + 9^2 × 0.1 – 4.3^2 = 3.61 Var (Y) = Var ($3X + 2$) = 3^2 Var (X) = 9 × 3.61 = 32.49
	Uses the formula for Var (X) PI or uses formula for E(3X + 2)	1.1a	M1	
	Obtains Var (X) = 3.61 OE or E($3X + 2$) = 14.9 OE	1.1b	A1	
	Applies Var ($3X + 2$) = 3^2 Var (X) or E($(3X + 2)^2$) – (E($3X + 2$))^2	1.1a	M1	
	Correctly shows that Var (Y) = 32.49	2.1	R1	
5(c)	Obtains Var ($T + Y$) = 37.49 OE	1.1b	B1	Var ($T + Y$) = Var (T) + Var (Y) = 5 + 32.49 = 37.49
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Uses an integral of $f(x)$ with one limit of 2	1.1a	M1	$P(X<2)=\int_1^2 \frac{4}{45}(x^3-10x^2+29x-20)\,dx$
	Obtains $P(X<2)=\frac{47}{135}$ or AWRT 0.348	1.1b	A1	$=\frac{4}{45}\left[\frac{x^4}{4}-\frac{10x^3}{3}+\frac{29x^2}{2}-20x\right]_1^2$ $=\frac{47}{135}$
6(b)	Integrates $\int_1^m \frac{4}{45}(x^3-10x^2+29x-20)\,dx$ and compares with 0.5	1.1a	M1	$\int_1^m \frac{4}{45}(x^3-10x^2+29x-20)\,dx=0.5$
	Obtains correct quartic equation in terms of the median. PI by sight of 2.28	1.1b	A1	$\frac{4}{45}\left[\frac{x^4}{4}-\frac{10x^3}{3}+\frac{29x^2}{2}-20x\right]_1^m=0.5$
	Obtains equation such that one side is equal to zero. PI by sight of 2.28	1.1b	A1	$\frac{1}{45}m^4-\frac{8}{27}m^3+\frac{58}{45}m^2-\frac{16}{9}m+\frac{103}{135}=0.5$ $\frac{1}{45}m^4-\frac{8}{27}m^3+\frac{58}{45}m^2-\frac{16}{9}m+\frac{71}{270}=0$
	Concludes correctly that the median of X is 2.3 to two significant figures because either the median is equal to AWRT 2.28 or that there is a sign change OE when a value of m such that $2.25 \le m < 2.3$ and a value of m such that $2.3 < m \le 2.35$ are substituted into the equation.	2.1	R1	m = 2.282817 Therefore, the median of X is 2.3 to two significant figures.
6(c)	Uses a correct integral of the form $\int x\,f(x)\,dx$ with any limits.	1.1a	M1	$E(X)=\int_1^4 \frac{4}{45}x(x^3-10x^2+29x-20)\,dx$
	Obtains $E(X)=2.32$ OE	1.1b	A1	$=\frac{4}{45}\left[\frac{x^5}{5}-\frac{10x^4}{4}+\frac{29x^3}{3}-\frac{20x^2}{2}\right]_1^4$ $=2.32$
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	States both hypotheses using correct language.	2.5	B1	H_0: There is no association between meal ordered and age of customer. H_1: There is an association between meal ordered and age of customer.
7(b)	Obtains the correct critical value (43.773) or *p*-value (0.0467).	1.1b	B1	
	Evaluates the χ^2-test statistic by correctly comparing their critical value with the test statistic (or their *p*-value with 0.05).	3.5a	R1	χ^2 cv for 30 df = 43.773 44.1 > 43.773
	Infers H_0 rejected. FT their comparison.	2.2b	E1	Reject H_0 Sylvia was not correct to accept the null hypothesis.
	Concludes that Sylvia was not correct to accept the null hypothesis. FT their comparison	2.1	E1	
7(c)	States their conclusion to the hypothesis test in context. (Conclusion must not be definite) FT their null and alternative hypotheses in 7(a) and their conclusion in 7(b).	3.2a	E1	Significant evidence to suggest that there is an association between meal ordered and gender of customer.
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	States both hypotheses using correct language.	2.5	B1	H_0: $\lambda = 25$ H_1: $\lambda \neq 25$ X ~ Po(25) P($X \leq 16$) $= 0.038 > 0.025$ Accept H_0 There is no significant evidence to suggest that the total birth rate in the two hospitals has changed.
	Selects and uses a Poisson model with $\lambda = 25$ to find P(number of births ≤ 16) or P(number of births < 16) or the lower or upper part of the critical region ($X \leq 15$ or $X \geq 36$)	3.3	M1	
	Obtains 0.038 (AWRT) or both parts of the critical region ($X \leq 15$ and $X \geq 36$)	3.4	A1	
	Evaluates the Poisson model by comparing 'their' *p*-value with 0.025 or the sample value with 'their' critical region	3.5a	R1	
	Infers H_0 not rejected. FT comparison of 'their' *p*-value with 0.025 or 0.05	2.2b	E1F	
	Concludes in context. (The conclusion must not be definite.) FT their incorrect rejection of H_0 if stated or 'their' comparison of *p*-value and 0.025/0.05 if not	3.2a	E1	

Q	Marking Instructions	AO	Marks	Typical Solution
8(b)	Selects a method to determine the probability of a Type I error by considering the probability of the upper or lower part of the critical region e.g. by finding P($X \leq 15$) or P($X \leq 16$) FT one tailed test used in 8(a). FT their Po(20) or Po(5) if used in 8(a).	3.1a	M1	P($X \leq 15$) = 0.02229 P($X \leq 16$) = 0.03775 P($X \geq 35$) = 0.03384 P($X \geq 36$) = 0.02246 P(Type I error) = 0.02229 + 0.02246 = 0.045
	Develops their method by considering the other part of the critical region e.g. by finding P($X \geq 35$) or P($X \geq 36$) Do not FT one tailed test used in 8(a). FT their Po(20) or Po(5) if used in 8(a).	3.1a	M1	
	Obtains P(Type I error) = AWRT 0.045 FT one tailed test used in 8(a) . FT their Po(20) or Po(5) if used in 8(a).	1.1b	A1	
	Total		**9**	

	Paper total		**40**	