

A-level MATHEMATICS 7357/2

Paper 2

Mark scheme

June 2020

Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aga.org.uk

Copyright information

AQA retains the copyright on all its publications. However, registered schools/colleges for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre. Copyright © 2020 AQA and its licensors. All rights reserved

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
Α	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
Е	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description					
	AO1.1a	Select routine procedures					
AO1	AO1.1b	Correctly carry out routine procedures					
	AO1.2	Accurately recall facts, terminology and definitions					
AO2.1 Construct rigorous mathematical arguments (in		Construct rigorous mathematical arguments (including proofs)					
	AO2.2a	Make deductions					
AO2	AO2.2b	Make inferences					
AUZ	AO2.3	Assess the validity of mathematical arguments					
	AO2.4	Explain their reasoning					
	AO2.5	Use mathematical language and notation correctly					
	AO3.1a	Translate problems in mathematical contexts into mathematical processes					
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes					
	AO3.2a	Interpret solutions to problems in their original context					
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems					
AO3	AO3.3	Translate situations in context into mathematical models					
	AO3.4	Use mathematical models					
	AO3.5a	Evaluate the outcomes of modelling in context					
	AO3.5b	Recognise the limitations of models					
	AO3.5c	Where appropriate, explain how to refine models					

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	2.2a	B1	$f(x) = -e^{x-1}$
	Total		1	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks correct box	2.2a	B1	$\sec x = 0$
	Total		1	

Q	Marking instructions	AO	Marks	Typical solution
3	Uses the product of $(2x)^5$ and $\left(\pm\frac{3}{x}\right)^3$ terms condone sign error and/or omission of ${}^{\rm n}{\rm C_r}$ term Or Obtains any two correct terms (unsimplified) term	3.1a	M1	${}^{8}C_{3}(2x)^{5} \times \left(-\frac{3}{x}\right)^{3} = 56 \times 32 \times -27x^{2}$ ∴ coefficient is -48384
	Multiplies their $(2x)^5$ and $\left(-\frac{3}{x}\right)^3$ by ${}^8\text{C}_3$ or ${}^8\text{C}_5$ or 56 OE For this mark condone $(2x)^3$ and $\left(-\frac{3}{x}\right)^5$	1.1a	M1	
	Obtains correct coefficient of x^2 -48384 condone inclusion x^2	1.1b	A1	
	Total		3	

Q	Marking instructions	AO	Marks	Typical solution
4	Uses or states small angle approximation for $\tan 5x \approx 5x$	1.1b	B1	$\frac{x \tan 5x}{\cos 4x - 1} \approx \frac{x \times 5x}{1 - \frac{\left(4x\right)^2}{2} - 1}$
	Uses or states small angle approximation for $\cos 4x \approx 1 - \frac{\left(4x\right)^2}{2}$ Condone omission of bracket	1.1b	B1	$\approx \frac{5x^2}{-8x^2}$ $\approx -\frac{5}{8}$
	Substitutes their expressions Of the form $\tan 5x \approx mx$ and $\cos 4x \approx 1 - \frac{nx^2}{2}$ into $\frac{x \tan 5x}{\cos 4x - 1}$ Condone correct extra terms	1.1b	M1	0
	Deduces $A = -\frac{5}{8}$ from a reasoned argument CSO	2.2a	R1	
	Total		4	

Q	Marking instructions	AO	Marks	Typical solution
5	Uses a suitable substitution $u = 4x + 1$ or $u = \sqrt{4x + 1}$ OE	3.1a	M1	$u = 4x + 1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4$
	Differentiates their substitution correctly	1.1b	A1	$x = -\frac{1}{4} \Rightarrow u = 0$
	Completes substitution to obtain correct integrand for their suitable substitution. Can be unsimplified.	1.1a	M1	$x = 6 \Rightarrow u = 25$ $x = \frac{u - 1}{4}$
	Correctly integrates their simplified integrand provided it is of the form $A\left(u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)$ or $B\left(u^4-u^2\right)$	1.1a	A1	$\int_{-\frac{1}{4}}^{6} x \sqrt{4x+1} dx = \int_{0}^{25} \frac{u-1}{4} \sqrt{u} \frac{1}{4} du$
			N44	$=\frac{1}{16}\int_{1}^{25}u^{\frac{3}{2}}-u^{\frac{1}{2}}du$
	Substitutes correct limits for their substitution or 6 and -1/4 for <i>x</i>	1.1a	M1	$=\frac{1}{16} \left[\frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3} \right]_{0}^{25}$
				$=\frac{1}{8}\left(\frac{25^{\frac{5}{2}}}{5}-\frac{25^{\frac{3}{2}}}{3}\right)$
				$=\frac{875}{12}$
	Completes rigorous argument to show the required result.	2.1	A1	
	Total		6	

Q	Marking instructions	AO	Marks	Typical solution
Q 6(a)	Begins a valid method to find the coordinates Uses gradient of L to find gradient of perpendicular radius Or Forms equation of circle with unknown radius and solves simultaneously with equation of L Or differentiates equation of circle implicitly Uses $(7, 9)$ to find the equation of the radius Or Uses $(7, 9)$ correctly in their equation of circle Or Uses $\frac{-12}{5}$ after their implicit differentiation Obtains $12y - 5x = 73 \text{OE}$	1.1a	Marks M1	Typical solution $5y+12x = 298$ $y = \frac{-12}{5}x + \frac{298}{5}$ $y-9 = \frac{5}{12}(x-7)$ $-12y-5x = 73$ $x = 19$ $y = 14$ $(19,14)$
	Or Correctly eliminates a variable to obtain a quadratic in x or y for example obtain a quadratic in x or y $(x-7)^2 + \left(\frac{298-12x}{5} - 9\right)^2 = k$ $\Rightarrow (x-7)^2 + \left(\frac{253-12x}{5}\right)^2 = k$ $\left(\frac{298-5y}{12} - 7\right)^2 + (y-9)^2 = k$ $\Rightarrow \left(\frac{214-5y}{12}\right)^2 + (y-9)^2 = k$			
	Equates discriminant to zero PI By correct answer or Solves their simultaneous equations of tangent and radius PI by correct answer	3.1a	M1	
	Obtains correct values for x and y (19,14)	1.1b	A1	
	Subtotal		5	

6(b)	Obtains $(x-7)^2 + (y-9)^2 = r^2$ PI if r^2 is replaced with correct value using their point from (a)	1.1a	M1	$(x-7)^{2} + (y-9)^{2} = r^{2}$ $(19-7)^{2} + (14-9)^{2} = r^{2}$ $12^{2} + 5^{2} = 169$
	Uses their point to find radius or radius squared. Must have obtained a point in part (a)	1.1a	M1	$(x-7)^2 + (y-9)^2 = 169$
	Obtains correct equation of circle CSO ACF	1.1b	A1	
	Subtotal		3	
	Question Total		8	

Q	Marking instructions	AO	Mark	Typical solution
7(a)(i)	Identifies the error lies in step 1 without contradiction.	2.3	E1	Mistake is $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$
	Subtotal		1	
7(a)(ii)	Recalls correct addition $Accept \frac{b}{ab} + \frac{a}{ab}$	1.1b	M1	$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$ a + b is rational and ab is rational and therefore
	Completes rigorous argument to complete proof. Must state that ab is rational (and nonzero) and $a+b$ is rational and conclude that $\frac{1}{a} + \frac{1}{b}$ or $\frac{a+b}{ab}$ is rational	2.1	R1	and therefore $\frac{1}{a} + \frac{1}{b}$ is rational.
	Subtotal		2	
7(b)	States assumption to begin proof by contradiction may PI by $\frac{a}{b} - x = \frac{c}{d} or x - \frac{a}{b} = \frac{c}{d}$	3.1a	M1	Assume that the difference between a rational and an irrational number is rational. $\frac{a}{b} - x = \frac{c}{d}$
	Uses language and notation correctly to state initial assumptions: States their a,b,c and d are integers and x is irrational do not accept the irrational written as a fraction Condone missing $b,d \neq 0$	2.5	A1	b d Where a,b,c and d are integers, $b,d \neq 0$ and x is irrational $x = \frac{a}{b} - \frac{c}{d}$
	Demonstrates that x can be expressed as a rational number by obtaining $x = \frac{ad - cb}{bd}$ OE	1.1b	M1	$b d$ $= \frac{ad}{bd} - \frac{cb}{bd}$ $= \frac{ad - cb}{d}$
	Completes rigorous argument to prove the required result, clearly explaining where the contradiction lies with ALL assumptions correct at the start (including $b, d \neq 0$)	2.1	R1	Hence <i>x</i> is rational. This is a contradiction hence the difference of any rational number and any irrational number is irrational.
	Subtotal		4	
	Question Total		7	

	Q	Marking instructions	AO	Mark	Typical solution
Writes the Cartesian equation in the required form Subtotal Subtotal Subtotal Subtotal Subtotal Subtotal Or Differentiates both $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$ with at least one correct Or Differentiates their $y^2 = 4x$ to obtain $y \frac{dy}{dx} = A$ Or rearranges and differentiates $y = 2\sqrt{x}$ and obtains $\frac{dy}{dx} = Ax^{\frac{1}{2}}$ OE Obtains correct $\frac{dy}{dx}$ at $t = a$ Explains that the gradient of a line is the tangen of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan \theta$ and concludes $\tan \theta = \frac{1}{a}$ Subtotal Subtotal Subtotal Subtotal 1.1a M1 $\tan \phi = \frac{2a - 0}{a^2 - 1}$ $= \frac{2a}{a^2 - 1}$	•		,	aii	- Jp.ou. colution
	8(a)			M1	$v = v^2$
Subtotal28(b)(i)Differentiates both $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$ with at least one correct Or3.1aM1 $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$ Differentiates their $y^2 = 4x$ to obtain $y \frac{dy}{dx} = A$ The gradient of a line is equal to the tangent of the angle between the line and the horizontal hence $\tan \theta = \frac{1}{a}$ OEObtains correct $\frac{dy}{dx}$ at $t = a$ 1.1bA1Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan \theta$ and concludes $\tan \theta = \frac{1}{a}$ 2.4E1Subtotal38(b)(ii)Uses formula for gradient of straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator Obtains correct $\tan \phi$ 1.1bA1		•	1.1b	A1	$\frac{5}{2} = t, x = \frac{5}{4}$
8(b)(i)Differentiates both $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$ with at least one correct Or3.1aM1 $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$ $\frac{dy}{dx} = \frac{2}{2a} = \frac{1}{a}$ The gradient of a line is equal to the tangent of the angle between the line and the horizontal hence tan $\theta = \frac{1}{a}$ OEObtains correct $\frac{dy}{dx}$ at $t = a$ 1.1bA1Explains that the gradient of aline is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan \theta$ and concludes $\tan \theta = \frac{1}{a}$ 2.4E18(b)(ii)Uses formula for gradient of straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator Obtains correct $\tan \phi$ 1.1bA1		in the required form			$y^2 = 4x$
$\frac{dy}{dt} = 2 \text{ with at least one correct} \\ \text{Or} \\ \text{Differentiates their } y^2 = 4x \text{ to} \\ \text{obtain } y \frac{dy}{dx} = A \\ \text{Or rearranges and differentiates} \\ y = 2\sqrt{x} \text{ and obtains} \\ \frac{dy}{dx} = Ax^{\frac{1}{2}} \\ \text{OE} \\ \hline \text{Obtains correct } \frac{dy}{dx} \text{ at } t = a \\ \hline Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to \tan \theta and \cot \theta $		Subtotal		2	
Differentiates their $y^2 = 4x$ to obtain $y\frac{\mathrm{d}y}{\mathrm{d}x} = A$ Or rearranges and differentiates $y = 2\sqrt{x}$ and obtains $\frac{\mathrm{d}y}{\mathrm{d}x} = Ax^{-\frac{1}{2}}$ OE Obtains correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $t = a$ Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan\theta$ and $\tan\theta$ concludes $\tan\theta = \frac{1}{a}$ Subtotal 8(b)(ii) Uses formula for gradient of straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator Obtains correct $\tan\theta$ 1.1b A1 The gradient of a line is equal to the tangle between the line and the horizontal land $\tan\theta = \frac{1}{a}$ E1 $\tan\theta = \frac{1}{a}$ The gradient of a line is equal to the tangle between the line and the horizontal hence $\tan\theta = \frac{1}{a}$ E1 $\tan\theta = \frac{1}{a}$ $\tan\theta = \frac{1}{a}$ $\tan\theta = \frac{1}{a}$ $\tan\theta = \frac{2a - 0}{a^2 - 1}$ $\tan\theta = \frac{2a - 0}{a^2 - 1}$	8(b)(i)	$\frac{dy}{dt} = 2$ with at least one correct	3.1a	M1	
$y = 2\sqrt{x} \text{ and obtains}$ $\frac{dy}{dx} = Ax^{-\frac{1}{2}}$ OE Obtains correct $\frac{dy}{dx}$ at $t = a$ Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan \theta$ and concludes $\tan \theta = \frac{1}{a}$ Subtotal 8(b)(ii) Uses formula for gradient of straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator Obtains correct $\tan \phi$ 1.1b A1		Differentiates their $y^2 = 4x$ to obtain $y \frac{dy}{dx} = A$			the tangent of the angle between the line and the horizontal hence
Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan \theta$ and concludes $\tan \theta = \frac{1}{a}$ Subtotal Subtotal 3 8(b)(ii) Uses formula for gradient of straight line with points A and B Must have a^2-1 or $1-a^2$ in denominator Obtains correct $\tan \phi$ 1.1b A1		$y = 2\sqrt{x}$ and obtains $\frac{\mathrm{d}y}{\mathrm{d}x} = Ax^{-\frac{1}{2}}$			а
Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan \theta$ and concludes $\tan \theta = \frac{1}{a}$ Subtotal 3 8(b)(ii) Uses formula for gradient of straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator Obtains correct $\tan \phi$ 1.1b A1		Obtains correct $\frac{dy}{dt}$ at $t = a$	1.1b	A1	
8(b)(ii) Uses formula for gradient of straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator Obtains correct $\tan \phi$ 1.1a M1 $\tan \phi = \frac{2a - 0}{a^2 - 1}$ $= \frac{2a}{a^2 - 1}$		Explains that the gradient of a line is the tangent of the angle between the line and the horizontal or shows on right-angled triangle on diagram and links to $\tan \theta$ and $\cot \theta = \frac{1}{a}$	2.4		
straight line with points A and B Must have $a^2 - 1$ or $1 - a^2$ in denominator Obtains correct $\tan \phi$ 1.1b $\tan \phi = \frac{a^2 - 1}{a^2 - 1}$ $= \frac{2a}{a^2 - 1}$	-	Subtotal		3	
Obtains correct $\tan \varphi$	8(b)(ii)	straight line with points A and B Must have a^2-1 or $1-a^2$ in		M1	
		Obtains correct $\tan \phi$	1.1b	A1	u^{-1}
Subtotal 2				2	

8(b)(iii)	States double angle formula for $\tan 2\theta$	1.2	B1	$\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta}$
	Substitutes $\tan \theta = \frac{1}{a}$ into their $\tan 2\theta = \frac{2 \tan \theta}{1 \pm \tan^2 \theta}$	1.1a	M1	$=\frac{2\times\frac{1}{a}}{1-\left(\frac{1}{a}\right)^2}$
	Simplifies and completes argument to show required result	2.1	R1	$= \frac{2a}{a^2 - 1}$ $= \tan \phi$
	Subtotal		3	
	Question Total		10	

	Moulting instructions	A 0	M	Typical calution
Q	Marking instructions	AO	Marks	Typical solution
9(a)	Identifies and clearly defines variable for radius of cylinder. Can be shown on diagram or can be implied by use in $V = \pi r^2 h$	2.5	B1	Radius of cylinder = r $h^2 + r^2 = R^2$ $V = \pi r^2 h$
	Uses Pythagoras to connect h, r and R	3.1a	M1	
	Eliminates the radius variable to form an expression for the volume of the cylinder in terms of h , completing argument to show given result. Condone undefined r	2.1	R1	$V = \pi (R^2 - h^2) h$ $= \pi R^2 h - \pi h^3$
	Subtotal		3	
9(b)	Differentiates the expression for volume w.r.t. <i>h</i> with at least one term correct.	3.1a	M1	$\frac{\mathrm{d}V}{\mathrm{d}h} = \pi R^2 - 3\pi h^2$
	Obtains correct $\frac{\mathrm{d}V}{\mathrm{d}h}$	1.1b	A1	For maximum volume $\frac{dV}{dh} = 0$
	Explains that their derivative w.r.t <i>h</i> equals zero for a maximum or stationary point	2.4	E1	$\Rightarrow R^2 - 3h^2 = 0$ $-h^2 = \frac{R^2}{3} \Rightarrow h = \frac{R}{\sqrt{3}}$
	Equates volume derivative w.r.t. <i>h</i> to zero and correctly obtains a value for <i>h</i> in terms of <i>R</i>	1.1a	M1	$n - \frac{1}{3} \rightarrow n - \frac{1}{\sqrt{3}}$
	Substitutes their <i>h</i> , in terms of <i>R</i> , from derivative w.r.t. <i>h</i> into volume formula.	1.1a	M1	Hence volume $R = R^2 R - (R)$
	Obtains the correct max volume in the form $kR^3 - pR^3$ or better	3.2a	A1	$V = \pi R^2 \frac{R}{\sqrt{3}} - \pi \left(\frac{R}{\sqrt{3}}\right)$
	Justifies correct volume in the form $kR^3 - pR^3$ or better form is the maximum eg:	2.1	R1	$=\frac{2\sqrt{3}\pi R^3}{9}$
	 V = 0 when h=0 or R and V>0 in between. Sketches shape of graph passing through the origin with (min on negative side) and max on positive side Obtains d²V/dh² = -6πh < 0 			$\frac{\mathrm{d}^2 V}{\mathrm{d}h^2} = -6\pi h$ When $h = \frac{R}{\sqrt{3}}$ $\frac{\mathrm{d}^2 V}{\mathrm{d}h^2} < 0$ Therefore maximum
	NB R1 can be awarded even if E1 is not awarded.			

Subtotal	7	
Question Total	10	

Q	Marking Instructions	AO	Marks	Typical Solution
10	Ticks correct box	1.2	B1	The resultant force acting on the vehicle is zero
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	1.1b	B1	$\binom{-2}{2}$ N
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Circles correct answer	1.1b	B1	-6
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	Finds a moment of a force about any point. Must have the form force x distance. Can be awarded for example for 2w, 7w, 5R, 2R Condone use of 9.8, 9.81, 10 instead of g	3.4	M1	$2W = 1.5 \times 4g$ $W = 3g$
	Obtains w= 3 <i>g</i> CAO	1.1b	A1	
	Subtotal		2	
13(b)	Explains that the weight of the rod acts through the midpoint or the centre of mass is at the midpoint (of the rod) OE	3.5a	E1	The rod is uniform so its weight acts at the centre
	Subtotal		1	
	Question Total		3	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Differentiates to find expression for v with at least one component correct	3.4	M1	$\mathbf{v} = (3t^2 - 10t)\mathbf{i} + (8 - 2t)\mathbf{j}$
	Substitutes <i>t</i> = 2 into their v which has at least one component correct.	1.1a	M1	So when $t = 2$
	Uses correct process to find the magnitude of their expression for v provided both of their components are non-zero	1.1a	M1	$\mathbf{v} = -8\mathbf{i} + 4\mathbf{j}$ $Speed = \mathbf{v} = \sqrt{8^2 + 4^2}$
	Obtains correct answer 4√5 ISW ACF CSO	1.1b	A1	$=4\sqrt{5}$ m s ⁻¹
	Subtotal		4	
14(b)	Differentiates <i>their</i> vector v to find expression for a with at least one component correct.	3.4	M1	$\mathbf{a} = (6t - 10)\mathbf{i} - 2\mathbf{j}$
	Do not award for a = r			$ \mathbf{a} = \sqrt{(6t-10)^2 + 2^2}$
	Uses a valid explanation as to why the magnitude must be positive for all values of t This could be earned by: forming a quadratic from acceleration components, allow sign error or Stating that the \mathbf{j} component is always -2 Deduces that $ \mathbf{a} \geq 2$ OE or that Bella is correct or $ \mathbf{a} > 0$. Deduction must be from completely correct reasoning.	2.4 2.2a	E1	Since a ≥ 2 for any value of <i>t</i> the magnitude of the acceleration is never zero Therefore Bella's claim is correct.
	Subtotal		3	
	Question Total		7	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	States strip width <i>h</i> = 20 PI by correct y values and not contradicted.	1.1b	B1	h = 20
	States five y values PI $y_0 = 131$, $y_1 = 140$, $y_2 = 120$, $y_3 = 80$, $y_4 = 0$ If they use h=25 condone $y_0 = 6$, $y_1 = 135$, $y_2 = 134$, $y_3 = 94$, $y_4 = 0$ If they use five strips between 20 <t<100 <math="" condone="">y_0 = 131, $y_1 = 140$, $y_2 = 132$, $y_3 = 108$, $y_4 = 67$, $y_5 = 0$</t<100>	1.1a	M1	$y_0 = 131, y_1 = 140,$ $y_2 = 120, y_3 = 80, y_4 = 0$ $Area = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4)$ Distance = 8110 m
	Accept ±2 on all y values Applies correct trapezium rule formula to their values (this mark could be achieved with B0M0 so far)	1.1a	M1	
	Applies trapezium rule with four strips and obtains correct value for distance. (accept values between 7970m and less than 8250m)	1.1a	A1	
	Subtotal		4	
15(b)	Explains means of gaining a more accurate estimate For example, "integrate the quadratic between 20 and 100" Must include limits for integration. Or Use the quadratic to calculate y values in the trapezium rule or	2.4	E1	Integrates the quadratic between the limits 20 and 100.
	other appropriate numerical method.			
	Subtotal		1	
	Question Total		5	

Q	Marking Instructions	AO	Marks	Typical Solution
16	Uses $s = ut + \frac{1}{2}at^2$ for particle A With u=0 t=5 and a=g (g can be 9.8 9.81, or 10)	3.1b	M1	$s = ut + \frac{1}{2}at^{2}$ $s = h, a = g, u = 0 \text{ and } t = 5$
	Finds h in terms of g Condone use of g, 9.8, 9.81, or 10 Can achieve this mark for $h = 12.5g, 122.5, 122.625, 125$	1.1b	A1	$h = \frac{25}{2}g$ $kh = \frac{1}{2}g(5-t)^2$
	Uses $kh = ut + \frac{1}{2}at^2$ for particle B With u=0 and a=g (g can be 9.8 9.81, or 10)	1.1a	M1	$\frac{25}{2}gk = \frac{1}{2}g(5-t)^{2}$ $5\sqrt{k} = 5 - t, \text{ since } 0 < t < 5$
	Deduces "time for B" + t =5	3.3	B1	$t = 5(1 - \sqrt{k})$
	Eliminates h and completes reasoned argument to obtain $t=5\left(1-\sqrt{k}\right)$ with consistent use of g or value for g and no slips AG	2.1	R1	
	Total		5	

Q	Marking Instructions	AO	Marks	Typical Solution
17	Models vertical motion using a suitable constant acceleration equation, to find the time of flight. Condone consistent sine/ cosine error	3.3	M1	Vertically: $0 = t \times u \sin \theta - 0.5 \times g \times t^{2}$ $2u \sin \theta$
	Obtains correct equation or inequality for t Must use g not numerical value	1.1b	A1	$t = \frac{2u\sin\theta}{g}$ Horizontally:
	Models horizontal displacement Condone consistent sine/ cosine error	3.3	M1	$x = u t \cos \theta$ $x = u \frac{2u \sin \theta}{g} \cos \theta,$
	Obtains correct equation or inequality to model horizontal displacement	1.1b	A1	using $\sin 2\theta = 2\sin \theta \cos \theta$ $x = \frac{u^2 \sin 2\theta}{a}$
	Eliminates t , where t is time of flight, from their horizontal and vertical models to obtain an expression in terms of u, g and θ only.	3.4	M1	Since $x \ge d$
	Completes reasoned argument to obtain stated result. Must justify stated inequality must have used correct resolution of u throughout	2.1	R1	$\frac{u^2 \sin 2\theta}{g} \ge d$ $\sin 2\theta \ge \frac{dg}{u^2}$
	Total		6	

Q	Marking Instructions	AO	Marks	Typical Solution
18(a)	Uses F = ma to form a four-term	3.3	M1	$T - W\sin\theta - Friction = ma$
	equation with at least three correct terms			$T - mg\sin\theta - \mu mg\cos\theta = ma$
	condone consistent swapping of			
	sine/cosine			$T - 0.2g(0.28) - 0.2\mu g(0.96)$
	Obtains perpendicular component	3.1b	B1	$=0.2 \times \frac{543}{625}g$
	of weight Obtains parallel component of	3.1b	B1	7.10
	weight			$2g - T = 2 \times \frac{543}{625}g$
	Recalls $F = \mu R$	1.2	B1	023
	Forms fully correct equation for	1.1b	A1	
	particle A			$2g - 0.2g(0.28) - 0.2\mu g(0.96)$
	$T - mg\sin\theta - \mu mg\cos\theta = ma$ Obtains M4. B4. B4. B4. A4.			$=2.2\times\frac{543}{625}g$
	Obtains M1, B1, B1, B1, A1 Forms fully correct equation for	3.3	B1	625
	particle B	0.0		$0.192\mu = 2 - 0.056 - 1.911(36)$
	2g - T = 2a or with acceleration			
	substituted	0.1	8.44	$\mu = 0.17$
	Eliminates <i>T</i> from their equations	3.4	M1	
	of motion with $a \neq 0$ Shows at least one step leading to	1.1b	A1	
	given μ value AWRT 0.170	1.10	AI	
	If exact values used throughout			
	with g=9.81 then $\mu = 0.17$			
	exactly		0	
	Subtotal		8	
18(b)(i)	Uses F = ma with T=0 to form a	3.4	M1	$-0.2 \times 9.81 \times (0.28)$
	three-term equation with at least			$-0.2(0.17) \times 9.81$
	two correct terms			$\times (0.96) = 0.2a$
	condone consistent sine/cosine			
	error condone one sign error Obtains a= -4.347792	1.1b	A1	a = -4.347792
	AWRT -4.35			Use $v^2 = u^2 + 2as$
	Uses $v^2 = u^2 + 2as$ with u=0.5 and	3.4	M1	030 v = u + 2us
	v=0 and their α value			0 = 0.25 - 8.696s
	Do not accept the following values			
	for <i>a</i> 543			s = 0.0288 m
	$a \neq 0, \frac{543}{625}g, 8.51424$			5 0.0200 III
	$a \neq 8.522928, 8.688$			
	Obtains 0.0288 m	3.2a	A1	
	Must have units			
	CAO		4	
	Subtotal		4	
18(b)(ii)	Describes assumption in context	3.5b	E1	A does not reach the pulley before
(-/(-/	of part (b)(i) eg No air resistance,			coming to rest
	String does not obstruct block			-
	Subtotal		1	
	Question Total		13	

Q	Marking Instructions	AO	Marks	Typical Solution
19(a)	Forms differential equation PI by correctly separated variables or $\int -\frac{1}{n^2} dv = \int 0.1 dt$ OE	3.4	B1	$\frac{dv}{dt} = -0.1v^2$
	Separates variables to integrate Condone sign error	1.1a	M1	$\int -\frac{1}{v^2} \ dv = \int 0.1 \ dt$
	Integrates with one side correct	1.1b	A1	1
	Fully correct integration Condone missing constant of integration	1.1b	A1	$\frac{1}{v} = 0.1t + c$
	Uses initial conditions to correctly obtain an equation in v and t only.	3.4	M1	$\frac{1}{4} = 0.1 \times (0) + c$
	Completes rigorous argument to show given expression AG	2.1	R1	$\frac{1}{v} = 0.1t + 0.25$
				$v = \frac{1}{0.1t + 0.25}$
				$v = \frac{20}{5 + 2t}$
	Subtotal		6	
19(b)	Substitutes $t = 5.5$ in $v = \frac{20}{5+2t}$	3.4	M1	$v = \frac{20}{5+11} = 1.25$
	Obtains correct acceleration =-0.15625 m s ⁻²	1.1b	A1	$a = \frac{-5}{32} = -0.15625$
	AWRT - 0.156			Acceleration = $-0.15625 \text{ m s}^{-2}$
	Subtotal		2	
	Question Total		8	