AQA

A-level

FURTHER MATHEMATICS

7367/3M

Paper 3 Mechanics

Mark scheme

June 2020

Version: 1.1 Final Mark Scheme

2 0 6 A 7 3 6 7 / 3 M / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer.	1.1b	B1	2.4 metres
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer.	1.1b	B1	1.2 J
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Forms a dimensional analysis equation using the formula given.	1.1a	M1	$[v] = [r][\omega]$ $LT^{-1} = L[\omega]$ $[\omega] = T^{-1}$
	Completes a clear argument to show that the dimensions of angular speed are T^{-1}.	2.1	R1	
	Total		**2**	

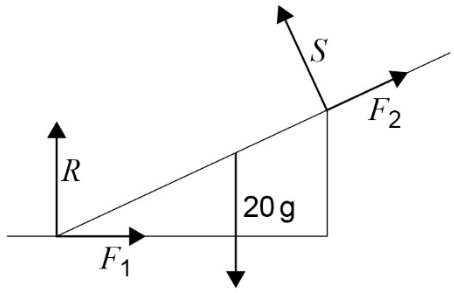
Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Models the resistance force using speed and a constant of proportionality.	3.3	M1	Resistance $= kv$ $600 = 20k$ $k = 30$ $P = 30 \times 48 \times 48$ $P = 69120$ W
	Obtains the correct maximum power 69120 W from a complete argument by applying the formula $P=Fv$	2.1	R1	
4(b)	Forms a three-term equation of motion for the car. Condone sign error.	1.1a	M1	$F = 1000a + 30 \times 25$ $69120 = (1000a + 30 \times 25) \times 25$ $a = 2.01$ m s^{-2}
	Uses $P=Fv$ to form an equation to find the maximum driving force.	3.4	M1	
	Obtains the correct maximum acceleration. AWRT 2 m s^{-2} FT their constant of proportionality. Condone missing units.	1.1b	A1	
4(c)	Obtains the correct maximum acceleration. AWRT 23 m s^{-2} FT their constant of proportionality. Condone missing units.	1.1b	B1	$F = 1000a + 30 \times 3$ $69120 = (1000a + 30 \times 3) \times 3$ $a = 23.0$ m s^{-2}
4(d)	Identifies that the acceleration in (c) is very large. OE e.g. car will reach maximum speed in about 2 seconds.	3.5a	E1	Acceleration very high when speed = 3 m s^{-1} Model only seems valid for higher speeds.
	Suggests that the model is not valid at lower speeds.	3.5b	E1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Models the impulse of the force by using an integral of the form $\int F\,\mathrm{d}t$	3.3	M1	$I=\int_0^{0.1} kt^2(0.1-t)^2 dt$
	Integrates $t^2(0.1-t)^2$	1.1a	M1	
	Obtains $\int_0^{0.1} kt^2(0.1-t)^2 dt = \frac{1}{3000000}$	1.1b	A1	$I=k\int_0^{0.1}\left(\frac{t^2}{100}-\frac{t^3}{5}+t^4\right)dt$
	Completes a rigorous argument linking (magnitude of) impulse to $\frac{k}{3000000}$. Units not required	2.1	R1	$I=\frac{k}{3000000}$
5(b)	Explains that the ball must rebound with the same velocity component perpendicular to the wall for the maximum magnitude of the impulse or uses $e \le 1$ or $e=1$.	3.3	E1	For maximum impulse the ball rebounds with same magnitude of velocity perpendicular to the wall.
	Explains that the impulse must be greater than the impulse needed to reduce the perpendicular component of the velocity to zero or uses $0 < e$ or $e=0$.	3.3	E1	As the ball rebounds the minimum impulse must be greater than the impulse required to bring the magnitude of velocity perpendicular to the wall to zero.
	Calculates the maximum and minimum magnitudes for the impulse or velocity.	1.1b	B1	$I_{max}=2(0.3\times 4\sin 30°)=1.2$ $I_{min}>0.3\times 4\sin 30°=0.6$
	Uses their impulse to deduce inequality for k.	2.2a	M1	$0.6<\frac{k}{3000000}\le 1.2$
	Reaches the required conclusion from a complete argument.	2.1	R1	$1800000<k\le 3600000$

5(c)	Obtains correct component of velocity parallel to the wall.	3.4	B1	Parallel component $= 4\cos 30°$
	Applies their impulse to motion perpendicular to the wall.	3.3	M1	$I = 0.8$
	Obtains correct rebound velocity component perpendicular to the wall.	1.1b	A1	$0.3v - (-0.3 \times 4\sin 30°) = 0.8$ $v = \frac{2}{3}$
	Obtains their correct speed using their perpendicular and parallel components. Condone missing units.	1.1b	A1F	Speed $= \sqrt{\left(\frac{2}{3}\right)^2 + (4\cos 30°)^2}$ $= 3.53$ m s^{-1}
	Total		**13**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Forms an equation to obtain the angular speed.	1.1a	M1	$4\omega = 2\pi$ $\omega = \frac{\pi}{2}$ Angular speed = $\frac{\pi}{2}$ rad s^{-1}
	Obtains the correct angular speed. Condone missing units.	1.1b	A1	
6(b)	States a position vector of the form $\mathbf{r} = 2\sin/\cos(\omega t)\mathbf{i} + (2\sin/\cos(\omega t)+\mathbf{c})\mathbf{j}$ with their value of ω. Allow $c = 0$	3.3	M1	$\mathbf{r} = 2\sin\left(\frac{\pi t}{2}\right)\mathbf{i} + \left(2 - 2\cos\left(\frac{\pi t}{2}\right)\right)\mathbf{j}$ metres
	Obtains the correct position vector. FT their value of ω or just ω if no value given. Condone missing units.	1.1b	A1F	
6(c)	Differentiates their position vector to find the velocity.	1.1a	M1	$\mathbf{v} = \pi\cos\left(\frac{\pi t}{2}\right)\mathbf{i} + \pi\sin\left(\frac{\pi t}{2}\right)\mathbf{j}$ $\mathbf{a} = -\frac{\pi^2}{2}\sin\left(\frac{\pi t}{2}\right)\mathbf{i} + \frac{\pi^2}{2}\cos\left(\frac{\pi t}{2}\right)\mathbf{j}$ ms^{-2}
	Differentiates their velocity to find the acceleration.	1.1a	M1	
	Obtains their correct acceleration from fully correct working. Condone missing units.	2.1	A1F	
6(d)	States the correct magnitude of their acceleration. Condone missing units.	1.1b	B1F	$\frac{\pi^2}{2}$ ms^{-2}
6(e)	Deduces correctly that the acceleration is directed towards the origin at 2 seconds. Condone missing units.	2.2a	B1	2 seconds
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Obtains a correct expression for the work done by the tension.	1.1b	B1	Work done by tension $= 50\cos 40°x$
	Finds the normal reaction force using the vertical component of the tension.	3.3	M1	Resolving vertically $R = 8 \times 9.8 - 50\sin 40°$
	Obtains the correct expression for the work done against friction.	1.1b	A1	Work done against friction $= 0.4(78.4 - 50\sin 40°)x$
	Sets up a model for the motion using the work-energy principle, including initial and final KE and work done by/against at least one force.	3.4	M1	Work-energy principle $50\cos 40°x - 0.4(78.4 - 50\sin 40°)x$ $= \frac{1}{2} \times 8 \times 5^2 - \frac{1}{2} \times 8 \times 2^2$
	Obtains a fully correct equation using the work-energy principle.	1.1b	A1	$84 = 19.80x$
	Obtains the correct value for x. AWRT 4.2	1.1b	A1	$x = 4.2$
7(b)	States a valid refinement.	3.5c	M1	Include air resistance, which would increase the value of x because more work would need to be done to reach the required speed.
	Correctly deduces how this would change the value of x with a valid explanation referring to work/energy.	2.2a	E1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
8	States that the ladder is assumed to be uniform.	3.3	B1	The ladder is uniform.
	Resolves horizontally or parallel to the ladder to obtain an equation.	3.3	M1	
	Resolves vertically or perpendicular to the ladder to obtain an equation.	3.3	M1	
	Obtains two correct force equations.	1.1b	A1	Resolving horizontally $F_2 \cos\alpha + F_1 = S\sin\alpha$
	Takes moments about the base of the ladder, or any other point, using their distance from the base of the ladder to the top of the wall.	3.3	M1	Resolving vertically $R + S\cos\alpha + F_2 \sin\alpha = 20\text{g}$
	Obtains a correct moments equation.	1.1b	A1	Taking moments about the base of the ladder. $20\text{g} \times 2\cos\alpha = S \times \dfrac{1.5}{\sin\alpha}$
	Uses $F = \mu R$ and obtains equations in α and one force by eliminating three forces.	3.4	M1	$R = 20\text{g} - S\cos\alpha - \dfrac{1}{2}S\sin\alpha$
	Obtains the required expression from a complete argument. **AG**	2.1	R1	$\dfrac{S}{2}\cos\alpha + 10\text{g} - \dfrac{S}{2}\cos\alpha - \dfrac{S}{4}\sin\alpha = S\sin\alpha$ $S = \dfrac{8\text{g}}{\sin\alpha}$ $40\text{g}\cos\alpha = \dfrac{12\text{g}}{\sin^2\alpha}$ $\cos\alpha\sin^2\alpha = \dfrac{3}{10}$
	Total		**8**	