AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

# A-level
# FURTHER MATHEMATICS

Paper 2

Thursday 4 June 2020 Afternoon Time allowed: 2 hours

**Materials**
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
|---|---|
| Question | Mark |
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| **TOTAL** | |

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Answer **all** questions in the spaces provided.

**1** Three of the four expressions below are equivalent to each other.

Which of the four expressions is **not** equivalent to any of the others?

Circle your answer.

**[1 mark]**

$\mathbf{a} \times (\mathbf{a} + \mathbf{b})$ $(\mathbf{a} + \mathbf{b}) \times \mathbf{b}$ $(\mathbf{a} - \mathbf{b}) \times \mathbf{b}$ $\mathbf{a} \times (\mathbf{a} - \mathbf{b})$

**2** Given that $\arg(a + b\mathrm{i}) = \varphi$, where $a$ and $b$ are positive real numbers and $0 < \varphi < \frac{\pi}{2}$, three of the following four statements are correct.

Which statement is **not** correct?

Tick (✓) **one** box.

**[1 mark]**

$\arg(-a - b\mathrm{i}) = \pi - \varphi$ ☐

$\arg(a - b\mathrm{i}) = -\varphi$ ☐

$\arg(b + a\mathrm{i}) = \frac{\pi}{2} - \varphi$ ☐

$\arg(b - a\mathrm{i}) = \varphi - \frac{\pi}{2}$ ☐

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**3** Find the gradient of the tangent to the curve

$$y = \sin^{-1} x$$

at the point where $x = \frac{1}{5}$

Circle your answer.

**[1 mark]**

$\frac{5\sqrt{6}}{12}$ $\frac{2\sqrt{6}}{5}$ $\frac{4\sqrt{3}}{25}$ $\frac{25}{24}$

**Turn over for the next question**

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**4** The matrices **A** and **B** are defined as follows:

$$\mathbf{A} = \begin{bmatrix} x+1 & 2 \\ x+2 & -3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} x-4 & x-2 \\ 0 & -2 \end{bmatrix}$$

Show that there is a value of $x$ for which $\mathbf{AB} = k\mathbf{I}$, where $\mathbf{I}$ is the $2 \times 2$ identity matrix and $k$ is an integer to be found.

**[3 marks]**

**5** Solve the inequality

$$\frac{2x+3}{x-1} \leq x+5$$

**[5 marks]**

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**6** Find the sum of all the integers from 1 to 999 inclusive that are not square or cube numbers.

**[5 marks]**

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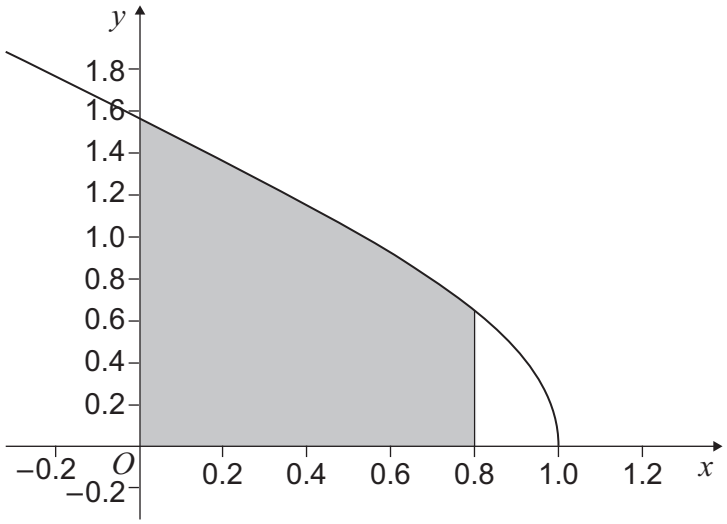
**7** The diagram shows part of the graph of $y = \cos^{-1} x$




The finite region enclosed by the graph of $y = \cos^{-1} x$, the $y$-axis, the $x$-axis and the line $x = 0.8$ is rotated by $2\pi$ radians about the $x$-axis.

Use Simpson's rule with five ordinates to estimate the volume of the solid formed. Give your answer to four decimal places.

**[5 marks]**

**8 (a)** Factorise $\begin{vmatrix} 2a+b+x & x+b & x^2+b^2 \\ 0 & a & -a^2 \\ a+b & b & b^2 \end{vmatrix}$ as fully as possible.

**[6 marks]**

**8 (b)** The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} 13+x & x+3 & x^2+9 \\ 0 & 5 & -25 \\ 8 & 3 & 9 \end{bmatrix}$$

Under the transformation represented by **M**, a solid of volume $0.625\,\text{m}^3$ becomes a solid of volume $300\,\text{m}^3$

Use your answer to part **(a)** to find the possible values of $x$.

**[3 marks]**

**9** The matrix $\mathbf{C} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, where $a$ and $b$ are positive real numbers,

and $\mathbf{C}^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Use $\mathbf{C}$ to show that $\cos\frac{\pi}{12}$ can be written in the form $\frac{\sqrt{\sqrt{m}+n}}{2}$, where $m$ and $n$ are integers.

**[7 marks]**

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**Turn over for the next question**

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**10** The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 0 \qquad u_{n+1} = \frac{5}{6 - u_n}$$

Prove by induction that, for all integers $n \geq 1$,

$$u_n = \frac{5^n - 5}{5^n - 1}$$

**[6 marks]**

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**11 (a)** Starting from the series given in the formulae booklet, show that the general term of the Maclaurin series for

$$\frac{\sin x}{x} - \cos x$$

is

$$(-1)^{r+1}\frac{2r}{(2r+1)!}x^{2r}$$

**[4 marks]**

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**11 (b)** Show that

$$\lim_{x \to 0} \left[ \frac{\frac{\sin x}{x} - \cos x}{1 - \cos x} \right] = \frac{2}{3}$$

**[4 marks]**

**12 (a)** Given that $I = \int_a^b e^{2t} \sin t \, dt$, show that

$$I = \left[ q e^{2t} \sin t + r e^{2t} \cos t \right]_a^b$$

where $q$ and $r$ are rational numbers to be found.

**[6 marks]**

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**12 (b)** A small object is initially at rest. The subsequent motion of the object is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} + v = 5\mathrm{e}^{t}\sin t$$

where $v$ is the velocity at time $t$.

Find the speed of the object when $t = 2\pi$, giving your answer in exact form.

**[6 marks]**

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**13** Charlotte is trying to solve this mathematical problem:

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 10e^{-2x}$$

Charlotte's solution starts as follows:

Particular integral: $y = \lambda e^{-2x}$

so

$$\frac{dy}{dx} = -2\lambda e^{-2x}$$

and

$$\frac{d^2y}{dx^2} = 4\lambda e^{-2x}$$

**13 (a)** Show that Charlotte's method will fail to find a particular integral for the differential equation.

**[2 marks]**

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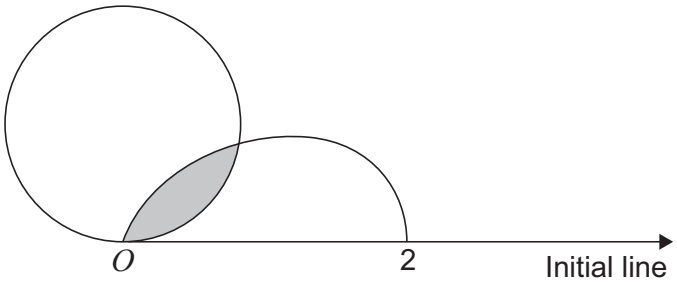
**13 (b)** Explain how Charlotte should have started her solution differently and find the general solution of the differential equation.

**[8 marks]**

**14** The diagram shows the polar curve $C_1$ with equation $r = 2\sin\theta$

The diagram also shows part of the polar curve $C_2$ with equation $r = 1 + \cos 2\theta$




**14 (a)** On the diagram above, complete the sketch of $C_2$

**[2 marks]**

**14 (b)** Show that the area of the region shaded in the diagram is equal to

$$k\pi + m\alpha - \sin 2\alpha + q\sin 4\alpha$$

where $\alpha = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$, and $k$, $m$ and $q$ are rational numbers.

**[9 marks]**

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**15** The points $A(7, 2, 8)$, $B(7, -4, 0)$ and $C(3, 3.2, 9.6)$ all lie in the plane $\Pi$.

**15 (a)** Find a Cartesian equation of the plane $\Pi$.

**[3 marks]**

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**15 (b)** The line $L_1$ has equation $\mathbf{r} = \begin{bmatrix} 5 \\ -0.4 \\ 4.8 \end{bmatrix} + \mu \begin{bmatrix} 15 \\ 3 \\ 4 \end{bmatrix}$

**15 (b) (i)** Show that $L_1$ lies in the plane $\Pi$.

**[2 marks]**

**15 (b) (ii)** Show that every point on $L_1$ is equidistant from $B$ and $C$.

**[4 marks]**

**15 (c)** The line $L_2$ lies in the plane $\Pi$, and every point on $L_2$ is equidistant from $A$ and $B$.

Find an equation of the line $L_2$

**[4 marks]**

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**15 (d)** The points $A$, $B$ and $C$ all lie on a circle $G$.
The point $D$ is the centre of circle $G$.

Find the coordinates of $D$.

**[3 marks]**

**END OF QUESTIONS**

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