



Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

A-level FURTHER MATHEMATICS

Paper 3 Discrete

Thursday 11 June 2020

Afternoon

Time allowed: 2 hours

Materials

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Mechanics **or** Statistics). You will have 2 hours to complete **both** papers.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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7	
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TOTAL	



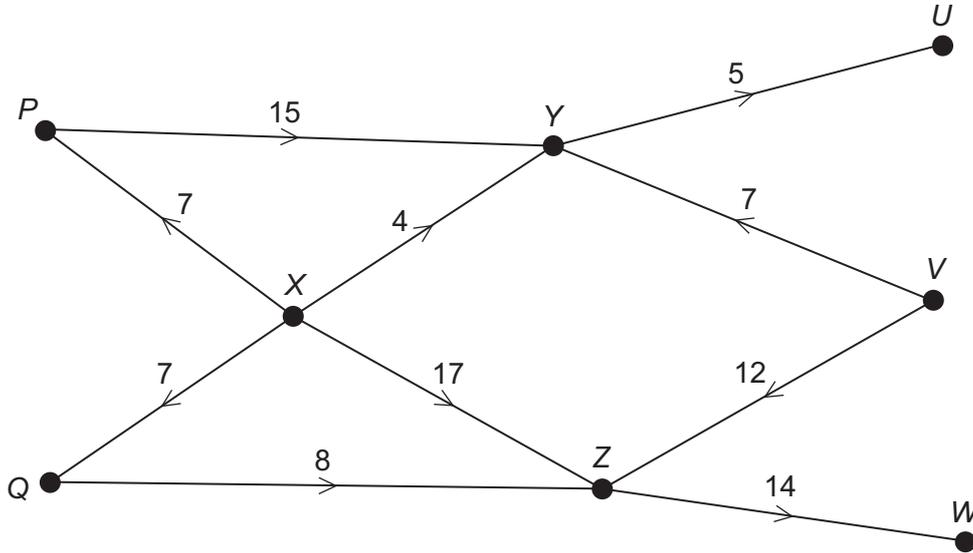
J U N 2 0 7 3 6 7 3 D 0 1

PB/Jun20/E6

7367/3D

Answer **all** questions in the spaces provided.

1 The diagram below shows a network of pipes with their capacities.



A supersource and a supersink will be added to the network.

To which nodes should the supersource and supersink be connected?

Tick (✓) **one** box.

[1 mark]

Supersource	Supersink
P, Q	U, V, W
U, V, W	P, Q
V, X	U, W
U, W	V, X



2 Which of the following statements is true about the operation of matrix multiplication on the set of all 2×2 real matrices?

Tick (✓) **one** box.

[1 mark]

Matrix multiplication is associative and commutative.

Matrix multiplication is associative but not commutative.

Matrix multiplication is commutative but not associative.

Matrix multiplication is not commutative and not associative.

Turn over for the next question

Turn over ►



3 (a) (ii) Suggest a reason why the total cost to the council for installing the internal telephone network is likely to be different from your answer to part **(a)(i)**.

[1 mark]

3 (b) Before the company starts installing the cable, it is told that the Education office cannot be connected directly to the Transport office due to issues with the building.

Explain the possible impact of this on your answer to part **(a)(i)**.

[2 marks]

Turn over for the next question

Turn over ►



- 4** Joe, a courier, is required to deliver parcels to six different locations, *A*, *B*, *C*, *D*, *E* and *F*.

Joe needs to start and finish his journey at the depot.

The distances, in miles, between the depot and the six different locations are shown in the table below.

	Depot	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
Depot	–	18	17	15	16	19	30
<i>A</i>	18	–	29	20	25	35	21
<i>B</i>	17	29	–	26	30	16	14
<i>C</i>	15	20	26	–	28	31	27
<i>D</i>	16	25	30	28	–	34	24
<i>E</i>	19	35	16	31	34	–	28
<i>F</i>	30	21	14	27	24	28	–

The minimum total distance that Joe can travel in order to make all six deliveries, starting and finishing at the depot, is L miles.

- 4 (a)** Using the nearest neighbour algorithm starting from the depot, find an upper bound for L .

[2 marks]



4 (b) By deleting the depot, find a lower bound for L .

[3 marks]

4 (c) Joe starts from the depot, delivers parcels to all six different locations and arrives back at the depot, covering 134 miles in the process.

Joe claims that this is the minimum total distance that is possible for the journey.

Comment on Joe's claim.

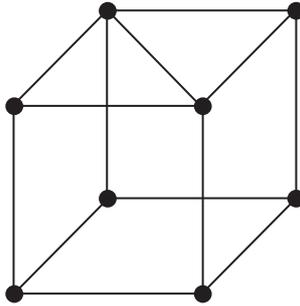
[1 mark]

Turn over for the next question

Turn over ►



5 The planar graph P is shown below.



5 (a) Determine the number of faces of P .

[2 marks]

5 (b) Akwasi claims that P is semi-Eulerian as it is connected and it has exactly two vertices with even degree.

Comment on the validity of Akwasi's claim.

[2 marks]



Turn over for the next question

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0 9

6 (b) Complete the Cayley table for elements of G .

[3 marks]

\blacktriangle	e	r	r^2	q	qr	qr^2
e	e	r	r^2	q	qr	qr^2
r	r	r^2	e			
r^2	r^2	e	r			
q	q	qr	qr^2	e		
qr	qr	qr^2	q	r^2		
qr^2	qr^2	q	qr	r	r^2	e

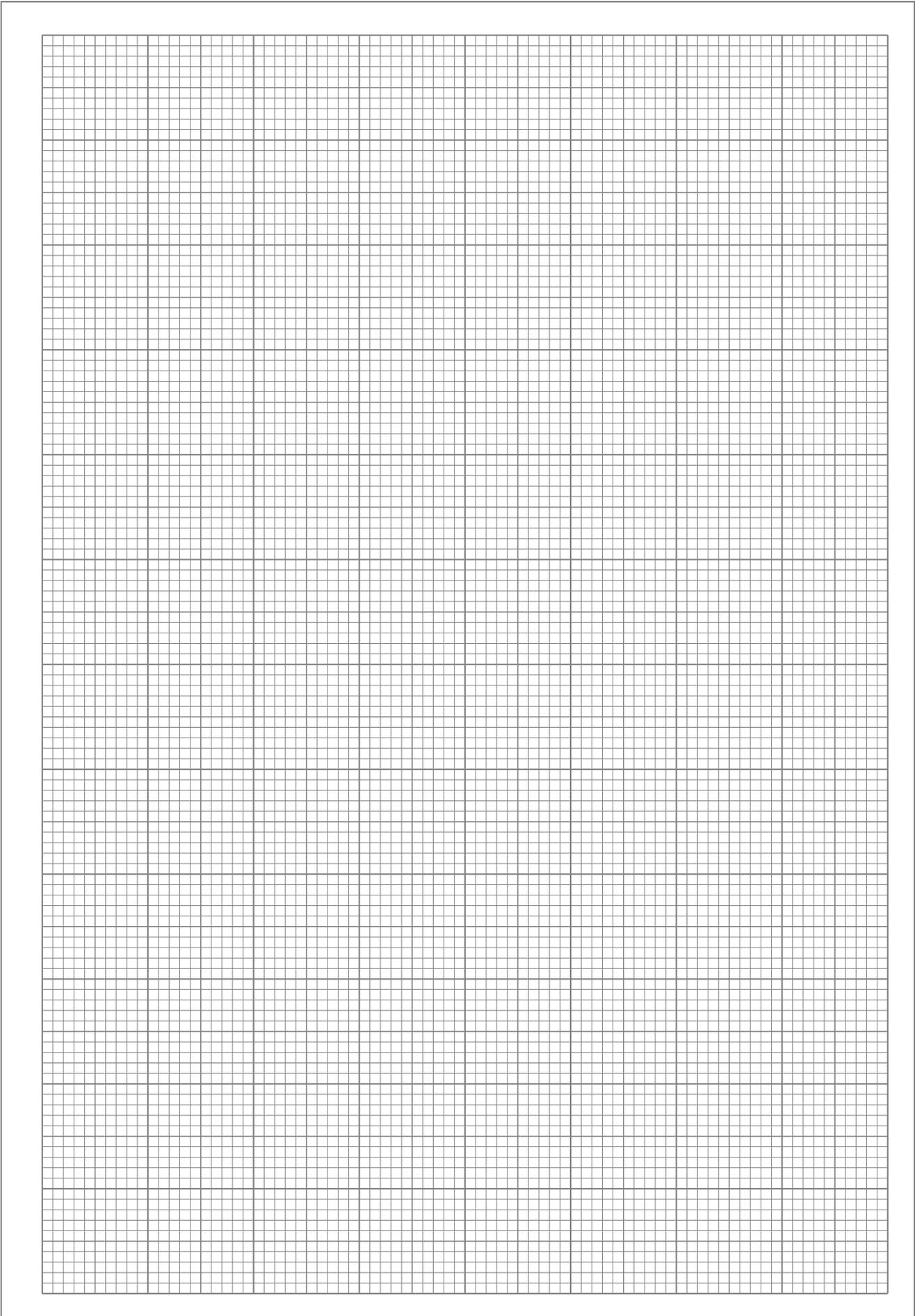
6 (c) State the name of a group which is isomorphic to G .

[1 mark]

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7 (b) Give a reason why the engineering company may not be able to make the number of each kit that you found in part **(a)**.

[1 mark]

7 (c) During one particular month the engineering company removes the need to make at least 100 of each type of kit.

Explain whether or not this has an effect on your answer to part **(a)**.

[2 marks]

Turn over for the next question

Turn over ►



8 Daryl and Clare play a zero-sum game.

The game is represented by the following pay-off matrix for Daryl.

		Clare		
		X	Y	Z
Daryl	A	-1	3	4
	B	3	2	1

8 (a) (i) Show that the game does not have a stable solution.

[3 marks]

8 (a) (ii) The value of the game for Daryl is V .

Using your answer to part **(a)(i)**, state the interval in which V lies.

[1 mark]



8 (b) Daryl plays strategy **A** with probability $\sin^2 \theta$ and strategy **B** with probability $\cos^2 \theta$, where $0^\circ \leq \theta \leq 90^\circ$

8 (b) (i) Show that the expected gain for Daryl when Clare plays strategy **X** is

$$3 - 4 \sin^2 \theta$$

[2 marks]

8 (b) (ii) Find an expression in terms of $\sin^2 \theta$ for the expected gain for Daryl when Clare plays strategy **Y**.

[1 mark]

8 (b) (iii) Find an expression in terms of $\sin^2 \theta$ for the expected gain for Daryl when Clare plays strategy **Z**.

[1 mark]

Question 8 continues on the next page

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