AQA

AS
FURTHER MATHEMATICS
7366/2D

Paper 2 Discrete

Mark scheme

June 2020

Version: 1.0 Final Mark Scheme

206A7366/2d/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

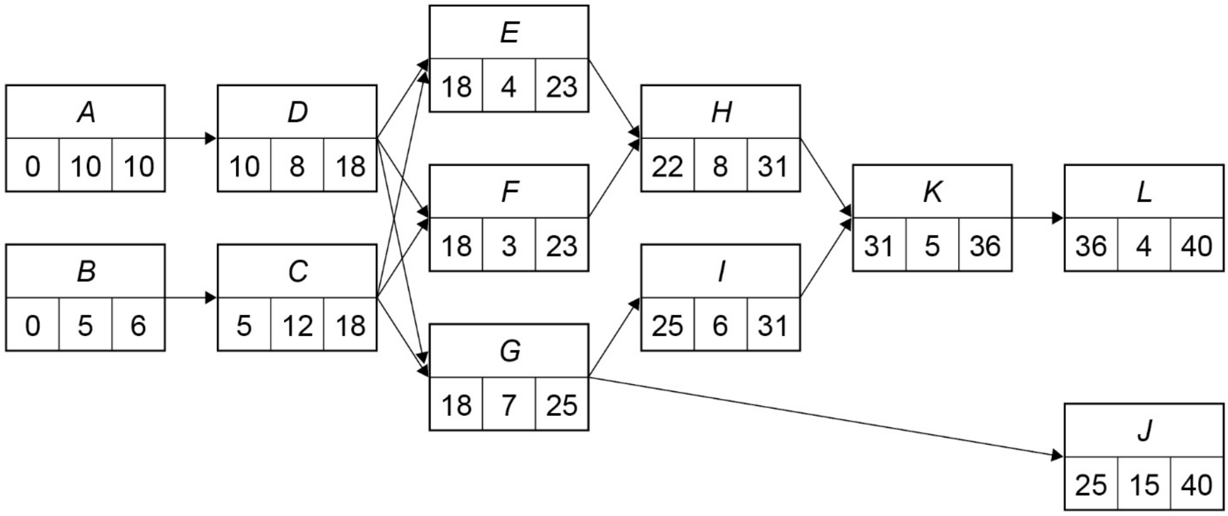
Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer.	2.5	B1	$M \leq 60$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Ticks correct box.	1.2	B1	*G* is semi-Eulerian and Hamiltonian
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Identifies correctly the row minima or column maxima. OR Identifies at least one dominated strategy.	1.1a	M1	row minima: (–4, –12, 4) column maxima: (10, 4, 10) max(row minima) = 4 = min(column maxima) Therefore a stable solution exists.
	Finds correctly max(row minima) and min(column maxima). OR Finds a 2 × 2 pay-off matrix by removing dominated strategies.	1.1b	A1	
	Completes a reasoned argument to show that the game has a stable solution.	2.1	R1	
3(b)(i)	States the value of the game for Summer.	1.1b	B1	Value of the game for Summer = 4
3(b)(ii)	States the play-safe strategy for each player.	1.1b	B1	Play safe strategy for Summer: S_3 Play safe strategy for Haf: H_2
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Uses Euler's formula for connected planar graphs (PI).	3.1a	M1	As the graph is planar and connected then $v - e + f = 2$
	Forms a correct unsimplified equation in x using Euler's formula.	1.1a	M1	$3x + 6 - (x^2 + 8x) + 2x^2 + 2 = 2$ $x^2 - 5x + 6 = 0$
	Solves quadratic equation and gives two solutions (CAO).	1.1b	A1	$x = 3$ or $x = 2$ However, an Eulerian graph cannot have a vertex of degree 3
	Deduces that $x = 2$ by completing a reasoned argument that only even values of x are valid because P is an Eulerian graph. (FT their odd and even values of x)	2.2a	R1F	Therefore $x = 2$
	Total		**4**	

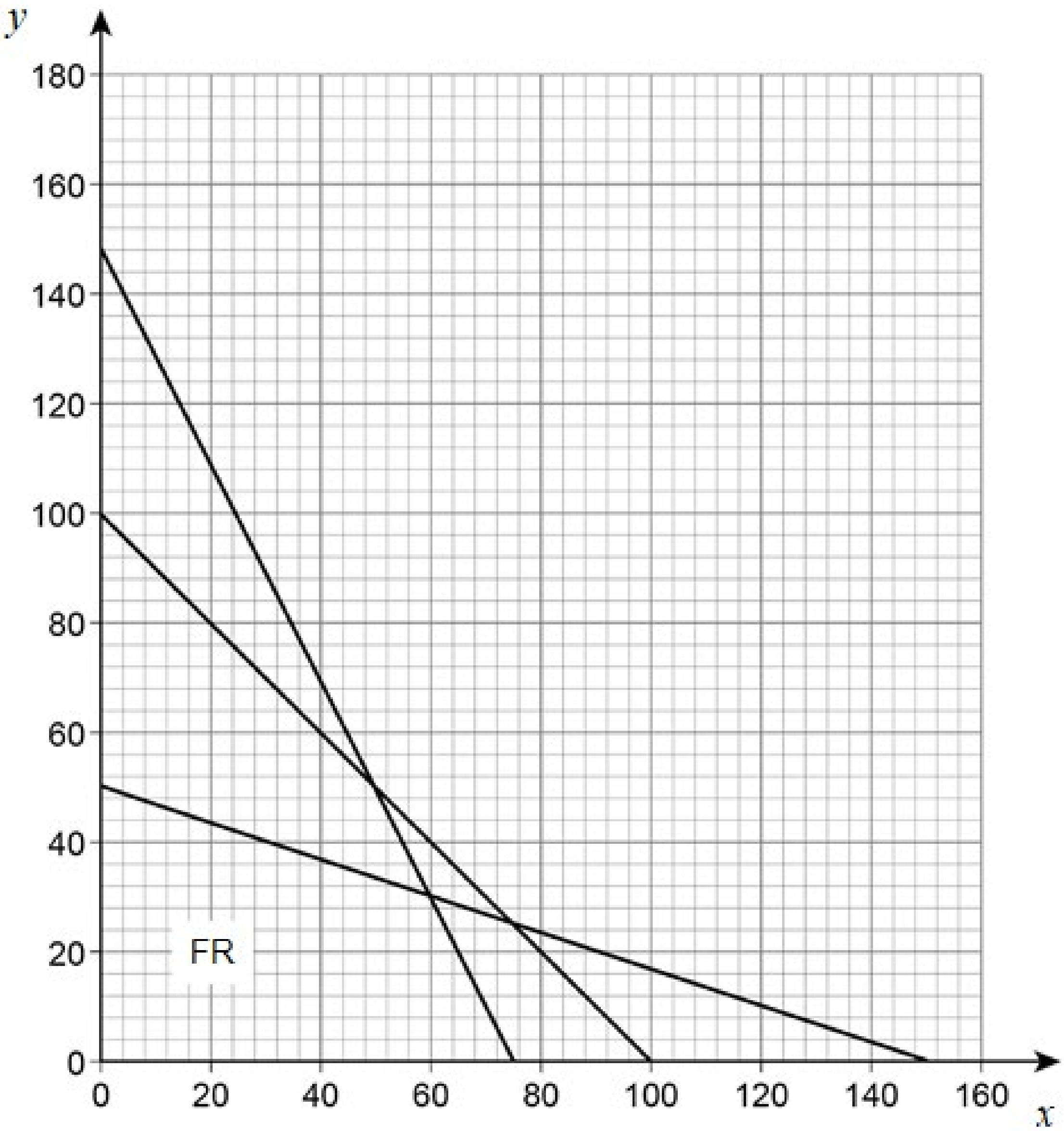
Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Constructs an activity network with activities *A* to *G* drawn with correct arcs.	1.1a	M1	(See diagram below)
	Completes activity network with all activities and correct arcs (condone extra ‘End’ activity with duration 0).	1.1b	A1	
	Finds correctly the earliest start time for each activity on the correct network.	1.1b	B1	
	Finds correctly the latest finish time for each activity on their network. (FT their earliest start times)	1.1b	B1F	



Q	Marking Instructions	AO	Marks	Typical Solution
5(b)	Evaluates the effect of increasing the duration of activity *C* by 3 weeks in their activity network. (e.g. considers the float of activity *C* or amends their activity network in (a)). (PI by ‘42’).	3.1b	M1	Activity *C* has a float of 1 so becomes critical as it is delayed by 3 weeks. Delaying activity *C* increases the minimum completion time by 2 weeks. The new minimum completion time is 42 weeks.
	Determines the new minimum completion time as 42 weeks. (Condone ‘42’) (FT their ‘40’ from part (a))	3.2a	A1F	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Uses a model to explore the minimum number of wider paths needed.	3.3	B1	If one of the statues was connected to all of the other statues, there would be at least 6 paths connected to this statue. Therefore, a minimum of 6 paths need to be wider.
	Explains that all statues still need to be connected when using a minimum number of wider paths and reasons that the minimum is 6.	2.4	E1	
6(b)	Uses a model, such as a clear attempt at Prim's algorithm in the table (PI).	3.4	M1	Using Prim's algorithm, starting at *A*: *AB*: 4 *BD*: 2 *BE*: 3 *EC*: 3 *EF*: 3 *CG*: 4 As the sum of the weights for the arcs found by Prim's algorithm is 19, then 19 trees need to be removed.
	Finds the correct arcs for the minimum spanning tree.	1.1b	A1	
	Finds correct total weight of minimum spanning tree and fully justifies their answer by use of a minimum spanning tree algorithm.	1.1b	A1	
6(c)	Uses the model to consider the weights of arcs connected to *A*, *D* and *F* in the minimum spanning tree.	3.4	M1	In the minimum spanning tree, the arc with the largest weight that could be removed is *AB* and either *AD* or *AF* constructed.
	Explains that arc *AB* should not be used as this arc has the largest weight of all the arcs connected to *A*, *D* and *F*.	2.4	A1	
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)(i)	Introduces two variables and defines at least one of them as 'number of'.	3.3	B1	x = number of standard boxes y = number of luxury boxes The line drawn on the graph is $y = -2x + 150$ The line forms the boundary of the region $12x + 6y \leq 900$, which is the constraint for the total number of rolls.
	Finds an equation for the line or the inequality for the number of rolls (ACF).	1.1b	B1	
	Explains clearly how the line on the graph represents the boundary of the constraint for the total number of rolls.	2.4	E1	
7(a)(ii)	Finds at least one non-trivial inequality for the model by considering the total numbers of teacakes or croissants (PI). (May be seen in (a)(i))	3.3	M1	Teacakes: $6x + 6y \leq 600$ Croissants: $3x + 9y \leq 450$ Maximum profit at (60, 30) Maximum profit $= 60 \times 2.5 + 30 \times 2$ = £210
	Produces correct straight lines for the two non-trivial constraints with consistent use of variables.	1.1b	A1	
	Identifies their feasible region (PI).	1.1b	A1F	
	Uses an objective line or uses a vertex of their feasible region (PI).	1.1a	M1	
	Obtains the correct coordinates of optimal vertex (60,30) (PI).	1.1b	A1	
	Calculates the correct daily profit of £210.	3.4	A1	

7(a)(ii) (cont)				
7(b)	States a plausible assumption that recognises a limitation of the model. (e.g. refers to demand, capacity, resources)	3.5b	B1	To maximise the profit, the bakery must sell all the bakery boxes.
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)(i)	Begins proof by exhaustion by identifying that all the elements remain unchanged combined with c	1.1a	M1	$a \square c = c \square a = a$ $b \square c = c \square b = b$ $c \square c = c$ $d \square c = c \square d = d$ Every element remains unchanged when combined with c under the binary operation $\square$ c is the identity element of S under the binary operation $\square$
	Completes a reasoned argument to show that c is an identity through proof by exhaustion using both left and right multiplication or using the commutativity of the operation.	2.1	R1	
8(a)(ii)	States correct inverse element.	1.1b	B1	d
8(b)	Finds identity element for multiplication modulo 4.	1.1b	B1	From Figure 3, the identity element is 1 From Figure 3, $1 \times_4 1 = 1$, $3 \times_4 3 = 1$ There are no pairs of distinct elements that produce the identity element. However, from part (a), the inverse of b is d, a different member of the set. Therefore, the Cayley table in Figure 2 cannot represent multiplication modulo 4.
	Uses the identity element to search for pairs of inverse elements from Figure 3. OR Forms a mapping between all four elements of Figure 2 and Figure 3.	1.1a	M1	
	Completes a reasoned argument to conclude that Mali's statement is incorrect.	2.1	R1	
	Total		**6**	

	Paper total		**40**	