



Oxford Cambridge and RSA

Thursday 15 October 2020 – Afternoon

AS Level Further Mathematics B (MEI)

Y412/01 Statistics a

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1** The random variable X represents the number of cars arriving at a car wash per 10-minute period. From observations over a number of days, an estimate was made of the probability distribution of X . Table 1 shows this estimated probability distribution.

r	0	1	2	3	4	>4
$P(X = r)$	0.30	0.38	0.19	0.08	0.05	0

Table 1

- (a)** In this question you must show detailed reasoning.

Use Table 1 to calculate estimates of each of the following.

- $E(X)$
- $\text{Var}(X)$ **[5]**

- (b)** Explain how your answers to part **(a)** indicate that a Poisson distribution may be a suitable model for X . **[1]**

You should now assume that X can be modelled by a Poisson distribution with mean equal to the value which you calculated in part **(a)**.

- (c)** Find each of the following.

- $P(X = 2)$
- $P(X > 3)$ **[3]**

- (d)** Given that the probability that there is at least 1 car arriving in a period of k minutes is at least 0.99, find the least possible value of k . **[3]**

- 2 A researcher is investigating the concentration of bacteria and fungi in the air in buildings. The researcher selects a random sample of 12 buildings and measures the concentrations of bacteria, x , and fungi, y , in the air in each building. Both concentrations are measured in the same standard units. Fig. 2 illustrates the data collected. The researcher wishes to test for a relationship between x and y .

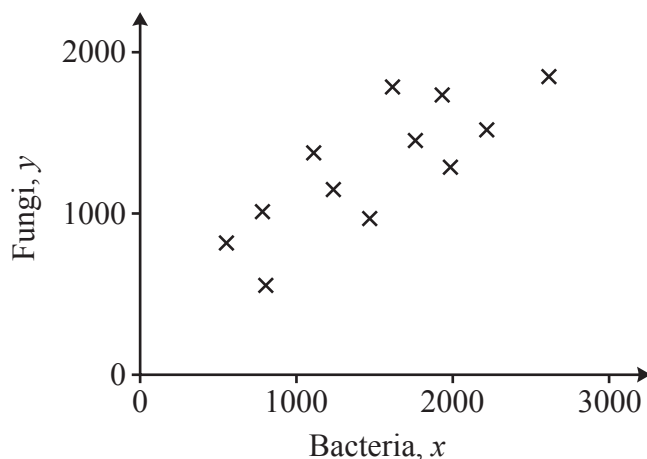


Fig. 2

- (a) Explain why a test based on the product moment correlation coefficient is likely to be appropriate for these data. [2]

Summary statistics for the data are as follows.

$$n = 12 \quad \Sigma x = 18\,030 \quad \Sigma y = 15\,550 \quad \Sigma x^2 = 31\,458\,700 \quad \Sigma y^2 = 21\,980\,500 \quad \Sigma xy = 25\,626\,800$$

- (b) **In this question you must show detailed reasoning.**

Calculate the product moment correlation coefficient between x and y . [4]

- (c) Carry out a test at the 5% significance level based on the product moment correlation coefficient to investigate whether there is any correlation between concentrations of bacteria and fungi. [5]
- (d) Explain why, in order for proper inference to be undertaken, the sample should be chosen randomly. [1]

- 3** A child is trying to draw court cards from an ordinary pack of 52 cards (court cards are Kings, Queens and Jacks; there are 12 in a pack). She draws cards, one at a time, with replacement, from the pack.

Find the probabilities of the following events.

- (a) She draws a court card for the first time on the sixth try. [2]
- (b) She draws a court card at least once in the first six tries. [2]
- (c) She draws a court card for the second time on the sixth try. [2]
- (d) She draws at least two court cards in the first six tries. [2]

- 4** A fair 8-sided dice has faces labelled 1, 2, ..., 8. The random variable X represents the score when the dice is rolled once.

- (a) State the distribution of X . [2]
- (b) Find $P(X < 4)$. [1]
- (c) Find each of the following.
 - $E(X)$
 - $\text{Var}(X)$ [2]
- (d) The random variable Y is defined by $Y = 10X + 5$. Find each of the following.
 - $E(Y)$
 - $\text{Var}(Y)$ [3]

- 5 A doctor is investigating the relationship between the levels in the blood of a particular hormone and of calcium in healthy adults. The levels of the hormone and of calcium, each measured in suitable units, are denoted by x and y respectively.

The doctor selects a random sample of 14 adults and measures the hormone and calcium levels in each of them. The spreadsheet in Fig. 5 shows the values obtained, together with a scatter diagram which illustrates the data. The equation of the regression line of y on x is shown on the scatter diagram, together with the value of the square of the product moment correlation coefficient.

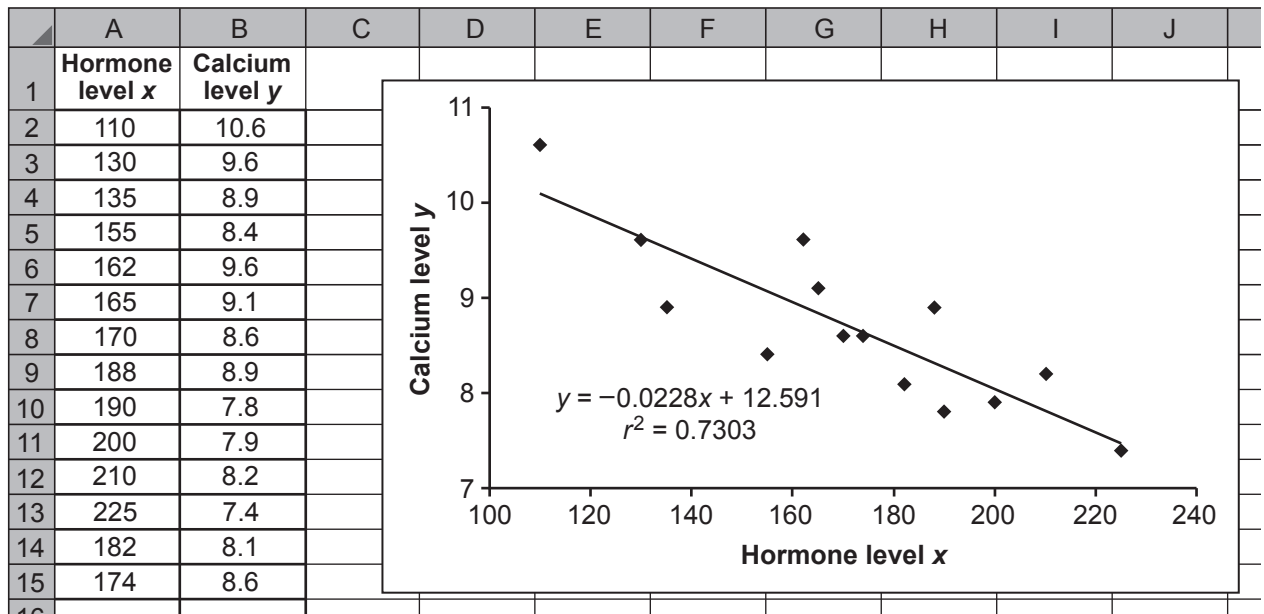


Fig. 5

- (a) Use the equation of the regression line to estimate the mean calcium level of people with the following hormone levels.
- 150
 - 250
- [2]
- (b) Explain which of your two estimates is likely to be more reliable.
- [1]
- (c) Comment on the goodness of fit of the regression line.
- [2]
- (d) Explain whether it would be appropriate to plot the scatter diagram the other way around with calcium level on the horizontal axis and hormone level on the vertical axis.
- [1]
- (e) Calculate the equation of a regression line which would be suitable for estimating the mean hormone level of people with a known calcium level.
- [2]

- 6 A researcher is investigating whether there is any relationship between whether a cyclist wears a helmet and the distance, x m, the cyclist is from the kerb (the edge of the road). Data are collected at a particular location for a random sample of 250 cyclists.

The researcher carries out a chi-squared test. Fig. 6 is a screenshot showing part of a spreadsheet used to analyse the data. Some values in the spreadsheet have been deliberately omitted.

	A	B	C	D	E	F	G
1			Observed frequency				
2			$x \leq 0.3$	$0.3 < x \leq 0.5$	$0.5 < x \leq 0.8$	$x > 0.8$	Totals
3	Wears helmet	Yes	26	27	23	46	122
4		No	45	31	21	31	128
5		Totals	71	58	44	77	250
6							
7			Expected frequency				
8			$x \leq 0.3$	$0.3 < x \leq 0.5$	$0.5 < x \leq 0.8$	$x > 0.8$	
9	Wears helmet	Yes	34.6480			37.5760	
10		No	36.3520			39.4240	
11							
12			Contribution to the test statistic				
13			$x \leq 0.3$	$0.3 < x \leq 0.5$	$0.5 < x \leq 0.8$	$x > 0.8$	
14	Wears helmet	Yes	2.1585	0.0601	0.1087	1.8885	
15		No	2.0573	0.0573		1.8000	
16							

Fig. 6

- (a) Showing your calculations, find the missing values in each of the following cells.

- E10
- E15

[3]

- (b) In this question you must show detailed reasoning.

Carry out a hypothesis test at the 10% significance level to investigate whether there is any association between helmet wearing and distance from the kerb.

[6]

- (c) Discuss briefly what the data suggest about helmet wearing for different distances from the kerb.

[3]

END OF QUESTION PAPER

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