OCR
Oxford Cambridge and RSA

Thursday 22 October 2020 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Fig. 1.1 shows the graph of $y = \mathrm{f}(x)$ for $0 \leqslant x \leqslant 1$.

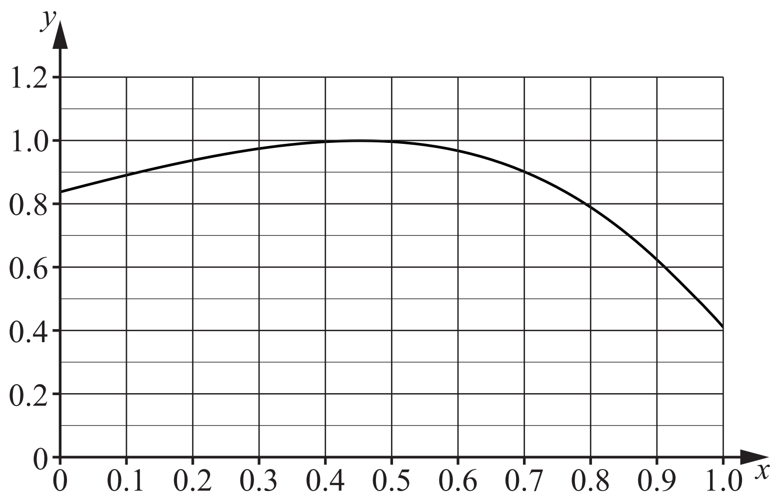

Fig. 1.1

Fig. 1.2 shows three values of x and the associated values of $\mathrm{f}(x)$.

x	0.6	0.8	1.0
$\mathrm{f}(x)$	0.968584	0.793203	0.410781

Fig. 1.2

(a) Use the forward difference method to calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 0.8$, giving your answer correct to 4 decimal places. **[2]**

(b) Use the central difference method to calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 0.8$, giving your answer correct to 4 decimal places. **[2]**

(c) Draw a suitable straight line on the diagram in the Printed Answer Booklet to show how the answer to part **(b)** is obtained using the central difference method. **[1]**

The central difference method is used to obtain further estimates of $\frac{dy}{dx}$ at $x = 0.8$.

These estimates, together with some further analysis, are shown in Fig. 1.3.

h	$f'(x)$	difference	ratio
0.1	–1.3657484		
0.05	–1.3579227	0.0078257	
0.025	–1.3559268	0.0019959	0.255045
0.0125	–1.3554254	0.0005014	0.251234
0.00625	–1.3552999	0.0001255	0.250307
0.003125	–1.3552685	3.139E-05	0.250077

Fig. 1.3

(d) Write 3.139E-05 in standard mathematical notation. **[1]**

(e) **Without** doing any further calculation, state the value of $\frac{dy}{dx}$ at $x = 0.8$ as accurately as you can, justifying the precision quoted. **[1]**

(f) Explain what the entries in column 4 of Fig. 1.3 demonstrate about the order of convergence of the central difference method. **[2]**

2 Fig. 2.1 shows the graph of $y = f(x)$ where $f(x) = e^{x^2}$ for $0 \leqslant x \leqslant 1.3$.

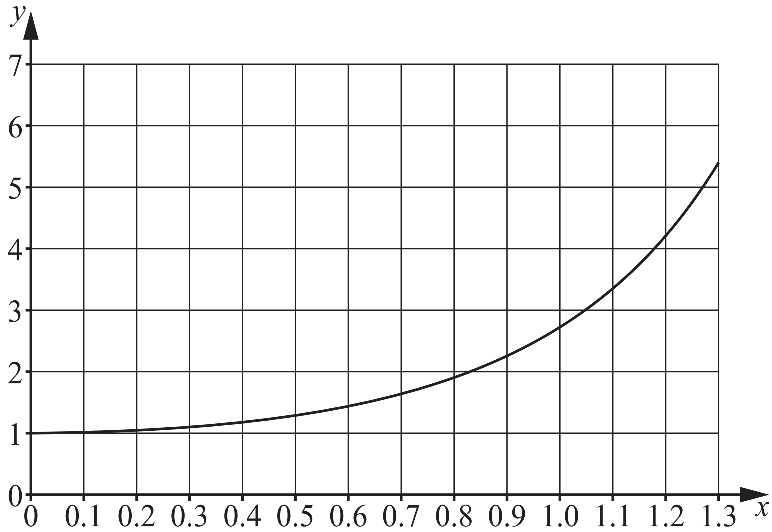

Fig. 2.1

Fig. 2.2 shows three values of x and the associated values of $f(x)$. The values of x are exact and the values of $f(x)$ are correct to 6 decimal places.

x	0.211111	0.411111	1.211111
$f(x) = e^{x^2}$	1.045576	1.184135	4.335296

Fig. 2.2

(a) For each of the values of x in Fig. 2.2, calculate the relative error in estimating the value of e^{x^2} when x is rounded to 1 decimal place, giving your answers correct to 3 significant figures. **[4]**

(b) With reference to Fig. 2.1, comment on the values obtained in part **(a)**. **[2]**

A similar procedure is carried out for $g(x) = (e^x)^2$. The values of x are exact and the values of $g(x)$ are correct to 6 decimal places. The results are shown in Fig. 2.3.

x	0.211111	0.411111	1.211111
$g(x) = (e^x)^2$	1.525347	2.275550	11.270875
x	0.2	0.4	1.2
$g(x) = (e^x)^2$	1.491825	2.225541	11.023176
relative error	–0.021977	–0.021977	–0.021977

Fig. 2.3

(c) Compare and contrast the relative errors shown in Fig. 2.3 with your answers to part **(a)**, explaining any significant discrepancies. **[2]**

3 Fig. 3 shows three values of x and the associated values of $f(x)$.

x	1	2	5
$f(x)$	2.53	7.71	47.37

Fig. 3

(a) Use Lagrange's method to construct the interpolating polynomial of degree 2 for the data points in Fig. 3, giving your answer in the form

$ax^2 + bx + c$,

where a, b and c are constants to be determined. **[4]**

You are given that the values of x are exact, but the values of $f(x)$ are rounded to 2 decimal places.

(b) Calculate the range of possible values of a, giving your answers correct to 4 decimal places. **[3]**

4 A small company leases a machine which they use for scanning, printing and photocopying. The cost of each job completed is displayed in pounds to the nearest tenth of a penny. A sample of these costs is shown in Fig. 4.

cost (£)
0.023
0.571
0.085
0.127
0.104

Fig. 4

Sue models the situation by assuming that the cost of each job is rounded correct to 3 decimal places before being displayed.

One day the machine is used for 412 jobs. Sue recorded the cost displayed for each job and found that the total of the displayed values was £45.730.

(a) Find the range of possible values of the true cost according to Sue's model, giving your answers correct to the nearest penny. **[3]**

Roxanne believes that the cost of each job is chopped to 3 decimal places before being displayed.

(b) Find the range of possible values of the true cost according to Roxanne's model, giving your answers correct to the nearest penny. **[2]**

The company received an invoice for £45.92 for the 412 jobs carried out on the machine on that day.

(c) Explain whether the company should use Roxanne's model or Sue's model for future budgeting. **[3]**

5 Arthur uses a spreadsheet to approximate $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$ using the trapezium rule. Some of the output is shown in Fig. 5.1.

3	C	D	E	F
4	x	y	T	
5	0	1.732051		
6	1	2.236068		
7			T_1	1.984059

Fig. 5.1

The formula in cell D5 is

=SQRT(C5^3+C5+3)

(a) Write the definite integral Arthur is approximating in standard mathematical notation. **[2]**

The entry in cell F7 is the approximation to $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$ using the trapezium rule with 1 strip.

(b) Write down a suitable cell formula for cell F7. **[1]**

Arthur obtains further approximations using the trapezium rule with 2, 4, 8 and 16 strips and these are shown in Fig. 5.2, together with T_1.

T_1	1.984059
T_2	1.944001
T_4	1.934406
T_8	1.932032
T_{16}	1.931439

Fig. 5.2

(c) Use values from Fig. 5.2 to obtain S_2, an estimate of the integral using one application of Simpson's rule, giving your answer correct to 6 decimal places. **[2]**

Arthur calculates S_2 using his spreadsheet and obtains the value 1.930649.

(d) Explain why this is different to the value obtained using your calculator. **[2]**

(e) By obtaining two further Simpson's rule estimates, state the value of $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$ as accurately as you can, justifying the precision quoted. **[4]**

6 Fig. 6.1 shows the graph of $y = x^3 - 3x + 1$ for $-2 \leqslant x \leqslant 2$.

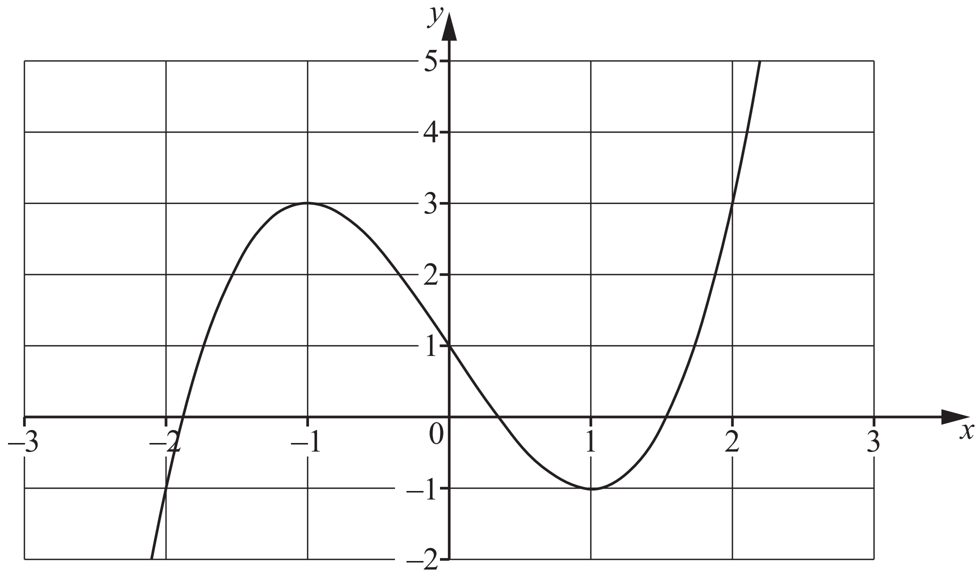

Fig. 6.1

The equation $x^3 - 3x + 1 = 0$ has three roots, α, β and γ, where $\gamma < \beta < \alpha$.

(a) Obtain the iterative formula $x_{n+1} = \mathrm{g}(x_n) = \sqrt[3]{3x_n - 1}$. **[2]**

(b) Starting with $x_0 = 1$, use the iterative formula to obtain x_1 and x_2, giving your answers correct to 6 decimal places. **[2]**

Fig. 6.2 shows the graph of $y = x$ and $y = \mathrm{g}(x)$ for $0 \leqslant x \leqslant 2$.

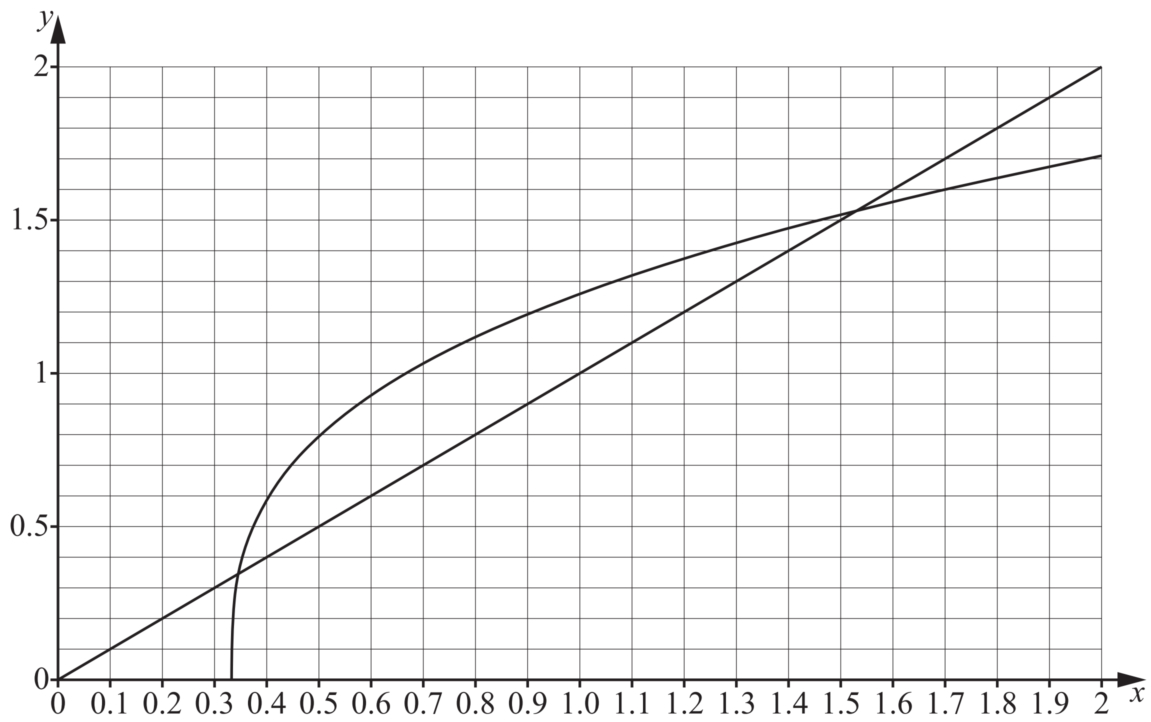

Fig. 6.2

(c) On the copy of Fig. 6.2 in the Printed Answer Booklet, starting with $x_0 = 1$, show how the iterative formula works to find x_1 and x_2. **[2]**

(d) With reference to Fig. 6.2, explain whether the iterative formula could be used to find β. **[1]**

The iterative formula in part **(a)** is to be used to approximate the largest of the three roots α. The spreadsheet in Fig. 6.3 shows 13 of the first 15 iterates obtained using this formula together with some further analysis.

n	x_n	difference	ratio
0	1		
1			
2			
3	1.4763961		
4	1.5079853	0.031589	
5	1.5217506	0.013765	0.436
6	1.5276718	0.005921	0.43
7	1.5302048	0.002533	0.428
8	1.5312858	0.001081	0.427
9	1.5317467	0.000461	0.426
10	1.5319431	0.000196	0.426
11	1.5320268	8.37E-05	0.426
12	1.5320624	3.57E-05	0.426
13	1.5320776	1.52E-05	0.426
14	1.5320841	6.47E-06	0.426
15	1.5320868	2.76E-06	0.426

Fig. 6.3

(e) Use extrapolation to obtain an improved approximation to α, justifying the precision quoted. **[4]**

7 The equation $x^3 - 2\sqrt{x} - 1 = 0$ has one root, which is close to $x = 1$.

(a) Use the secant method with $x_0 = 0.5$ and $x_1 = 0.8$ to obtain x_2, a first approximation to the root, giving your answer correct to 4 decimal places. **[2]**

When $x_0 = 0.5$ and $x_1 = 1$ are used as the starting values, x_2 is found to be 4.4577 correct to 4 decimal places.

(b) Explain why a small change in x_1 leads to such a significant change in the value obtained for x_2. **[1]**

The secant method with $x_0 = 0.5$ and $x_1 = 1$ is used to find a sequence of approximations to the root of the equation $x^3 - 2\sqrt{x} - 1 = 0$. These approximations, denoted by x_{new}, are shown in the spreadsheet output in Fig. 7.

r	x_r	$\mathrm{f}(x_r)$	x_{new}
0	0.5	–2.28921	
1	1	–2	4.457653
2	4.45765	83.35389	1.081019
3	1.08102	–1.81616	1.153022
4	1.15302	–1.61468	1.730063
5	1.73006	1.547644	1.447659
6	1.44766	–0.37249	1.502443
7	1.50244	–0.05997	1.512955
8	1.51295	0.003158	1.512429

Fig. 7

(c) By considering the values of x_{new} given in Fig. 7, determine the root of the equation $x^3 - 2\sqrt{x} - 1 = 0$ as accurately as you can. **[3]**

END OF QUESTION PAPER

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