

GCE

Further Mathematics A

Y533/01: Mechanics

Advanced Subsidiary GCE

Mark Scheme for November 2020

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
√ and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0,B1	Independent mark awarded 0, 1
SC	Special case
٨	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
a wrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

 Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such
 - Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
 - When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
 - NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g Rules for replaced work and multiple attempts:
 - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
 - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
 - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

	Question	Answer	Marks	AOs	Guid	ance
1	(a)	At constant velocity, $F = 250 = 10000/v$	M1	1.1	Tractive force = P/v = resistance	
		v = 40	A1	1.1		
			[2]			
1	(b)	$D = \frac{10000}{30}$	M1	1.1	Use of $P = Dv$ where D is	
		30			tractive force	
		$\frac{10000}{200} - 250 = 1200a$	M1	1.1	Attempt NII with 2 forces (one	
		30			of which could be just "D")	
		awrt 0.069 ms ⁻²	A1	1.1		
			[3]			
			D1	1 4 4		
2	(a)	$1.6 = e \times 2.4 = e = \frac{2}{3}$	B1	1.1		
		3	[11]			
	(1)	15.16.15.01	[1]	4 4	A	A 11
2	(b)	$4.5 \times -1.6 - 4.5 \times 2.4$	M1	1.1	Attempt at $mv - mu$	Allow sign confusion
		=-18 so 18 Ns (or kg ms ⁻¹)	A1	1.1	Allow ±18	e.g. 1.6 – 2.4 Ignore missing units
		in the final direction of motion of P	B1	2.2a	could be shown on a diagram	ignore missing units
		the final different of motion of 1	[3]	2.20	could be shown on a diagram	
2	(c)	1 × 4.5 × 2.4 ² 1 × 4.5 × 1.6 ²	M1	1.1	Attempt at $+ \begin{pmatrix} 1 & nv^2 - 1 & mu^2 \end{pmatrix}$	
		$\frac{1}{2} \times 4.5 \times 2.4^2 - \frac{1}{2} \times 4.5 \times 1.6^2$			Attempt at $\pm \left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right)$	
		7.2 J	A1	1.1		
			[2]			
2	(d)	Not perfectly elastic since KE is lost (due to the	B1	1.2	or $e < 1$ but valid reason must be	Must mention KE or collision,
		collision)			given.	Not just e.g. "energy lost"
			[1]			

	Questic	n	Answer	Marks	AOs	Guid	ance
3	(a)		Initial Energy = $\frac{1}{2} \times 5.6 \times 5^2$	B1	1.1	Using u to find P 's initial (kinetic) energy (=70J)	
			When $\theta = \frac{1}{4}\pi$, <i>P</i> 's PE 5.6 <i>g</i> × (2.1– 2.1cos $\frac{1}{4}\pi$)	*M1	1.1	Finding <i>P</i> 's PE when $\theta = \frac{1}{4}\pi$ (= 33.8J)	Allow 1 slip, but PE must not become negative
			So conservation of energy $\frac{1}{2} \times 5.6v^2 + 5.6g \times (2.1 - 2.1\cos\frac{1}{4}\pi) = 70$	M1 dep	1.1	Finding expression for <i>P</i> 's energy (KE + PE) and equating to initial energy	
			v = awrt 3.6	A1 [4]	1.1	to initial energy	Final speed must be < 3.6
3	(b)		When $v = 0$ P's energy = $5.6g \times (2.1 - 2.1\cos\theta) = 70$	M1	3.1b	Finding expression for <i>P</i> 's final (potential) energy and equating to initial energy.	
			θ = awrt 1.17 rads	A1 [2]	1.1	cao	Allow in degrees awrt 66.9°

Q	Questio	n	Answer	Marks	AOs	Guid	ance
4	(a)		30×10	M1	1.1	Using Work done = Fd	
			300 J	A1 [2]	1.1		
4	(b)		$\frac{1}{2} \times 2.4 \times 12^2 + 300 = \frac{1}{2} \times 2.4 \times 18^2 + W$	M1	1.1	where W is energy loss (could be eg $10R$)	Allow 1 slip e.g. R instead of W
			10R = 84 R = 8.4	M1 A1 [3]	1.1 1.1	Use of energy loss = $10R$	Must be e.g. $10R$, not R
			Alternative method $a = \frac{18^2 - 12^2}{2 \times 10} = 9$	M1		Using $v^2 = u^2 + 2as$ to find a	
			$30 - R = 2.4 \times 9$	M1 A1		Use of $F = ma$ with their a	
4	(c)		$t = \frac{2 \times 10}{18 + 12}$	M1	1.1	Using $s = \left(\frac{v+u}{2}\right)^t t$	Or use Average power = Force × average speed, i.e. $P = F \frac{v+u}{2}$
			$t = \frac{2}{3}$	A1	1.1		2
			$\frac{84}{\frac{2}{3}}$ = 126 W	A1	1.1		
			Alternative method $t = \frac{18 - 12}{9}$	M1		Using $v = u + at$ to find t	NB $a = 9$ from (b) or N2L: $a = {30 - 8.4 \over 2.4} = 9$
			$t = \frac{2}{3}$	A1			2.1
			$\frac{84}{\frac{2}{3}} = 126 \text{ W}$	A1			
				[3]			

	Question		Answer	Marks	AOs	Guidance	
4	(d)	(i)	$17.28 = 1.6 \times v_Q$ $v_Q = 10.8 \text{ m s}^{-1}$	M1 A1 [2]	1.1 1.1		Do not allow -10.8
4	(d)	(ii)	$2.4 \times 18 = 2.4 \times v_P + 1.6 \times 10.8$ So $v_P = 10.8 = v_Q$, so the particles coalesce and the collision is therefore inelastic.	M1 A1 [2]	3.1b 2.1	Attempt conservation of momentum Find equal velocities and conclude	Do not allow use of KE

	Questic	on	Answer	Marks	AOs	Guid	ance
5	(a)		$[v^2] = [pu^{\alpha}]$ and $[v] = [u]$ and p is dimensionless or $L^2T^{-2} = L^{\alpha}T^{-\alpha} \Rightarrow \alpha = 2$	M1 A1 [2]	2.1	For the idea, that might be stated in words, that the dimensions of every term in the equation must be the same AG	Must relate LHS to RHS Must explicitly state that <i>p</i> is dimensionless to get this mark (this may be seen in part (b)).
5	(b)		$L^{2}T^{-2} = (LT^{-2})^{\beta}L^{\gamma}$ $-2 = -2\beta \Longrightarrow \beta = 1$ $\beta + \gamma = 2$ $\gamma = 1$	A1 M1 A1 A1 [4]	3.3 1.1 1.1	Equating dimensions of other term with $[v^2]$ with q gone and $[a]$ and $[s]$ used Ignore term in $[u^2]$ if present Equating powers of L	May be seen expanded ASM note – must be seen or strongly implied – see wording of question. SC2 for both correct
5	(c)		If $s = 0$ then $v = u$ so $u^2 = pu^2 + 0 => p = 1$	B1 [1]	3.4	Details must be shown	Do not allow use of prior knowledge of $v^2 = u^2 + 2as$.
5	(d)		$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}qmas = \frac{1}{2}qFs = \frac{1}{2}qW$ So we can see that this equation is a statement of the Work-Energy principle so $\frac{1}{2}q = 1$ so $q = 2$	M1 A1 [2]	2.4	Where F must be the force acting and W the work done by this force	Do not allow use of prior knowledge of $v^2 = u^2 + 2as$.

C	Questio	n	Answer	Marks	AOs	Guid	ance
6	(a)		1st Collision: $3.3u = 3.3v_A + 2.2v_B$	M1	3.1b	Conservation of momentum	Must be seen
			1st Collision: $\pm e = \frac{v_B - v_A}{v_B}$	M1	3.1b	NEL	Must be seen
			u				
			$3u = 3v_A + 2v_B$ and $2eu = 2v_B - 2v_A$ u(3 - 2e)	A1	1.1	AG	
			$\Rightarrow 5v_A = 3u - 2eu \Rightarrow v_A = \frac{u(3 - 2e)}{5}$				
			$3u = 3v_A + 2v_B$ and $3eu = 3v_B - 3v_A$				AFF 1:0 : (1)
			$\Rightarrow 5v = 3u + 3eu \Rightarrow v = \frac{3u(1+e)}{5}$	A1	1.1	find v_B by elimination or substitution	AEF - award if seen in (b)
			5	[4]		Substitution	
6	(b)		2 nd Collision: $2.2 \times \frac{3u(1+e)}{5} = 2.2 \frac{V}{B} + \frac{V}{C}$	M1ft	3.3	Conservation of momentum	May be in terms of V_A
						(ft their value of V_B)	
			2^{nd} Collision: $\pm e = \frac{V_C - V_B}{(2\pi(1+\epsilon))}$	M1ft	3.3	NEL (ft their value of V_B)	
			$\left(\frac{3u(1+e)}{5}\right)$				
			2 nd Collision: $\pm e = \frac{V_C - V_B}{\left(\frac{3u(1+e)}{5}\right)}$ $\frac{33}{25}u(1+e) = \frac{11}{5}V_B + V_C & \frac{3eu(1+e)}{5} = V_C - V_B$				
			$\frac{1}{25}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$				$V_C = \frac{33u(1+e)^2}{80}$
			$\Rightarrow \frac{16}{5}V_B = \frac{33}{25}u(1+e) - \frac{3eu(1+e)}{5}$	M1ft	3.1b	Attempt to eliminate V_C	$V_C = \phantom{aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa$
			$\Rightarrow V_B = \frac{3u(1+e)(11-5e)}{80}$	A1	1.1	oe	Must be in terms of e and u only.
			80				
6	(a)		(2 22) 2(1+2)(11 52)	[4] M1ft	3.4	Correct condition for further	
0	(c)		$\frac{u(3-2e)}{5} > \frac{3u(1+e)(11-5e)}{80}$	WIIIt	3.4	collision (ft their V_B from (b)	
			$3e^2 - 10e + 3 > 0$	M1	1.1	Rearranging to 3 term quadratic	
			A and B collide again $=> e \neq 0$	B1	2.2a	inequality in e	
			A and B confide again $-> e \neq 0$ $(3e-1)(e-3) > 0$ and $0 \le e \le 1$ and $e \ne 0$	DI	2.2a	1.	If B1 not awarded then award
			$\Rightarrow 0 < e < \frac{1}{2}$	A1	1.1	$e < \frac{1}{3}$ is not sufficient for A1	A1 for $0 \le e < \frac{1}{e}$
			$\frac{3}{3}$				3
				[4]			

	Questio	n	Answer	Marks	AOs	Guid	ance
7	(a)		$\uparrow C\cos 30^{\circ} = mg$	M1*	3.3	where C is the (normal) contact force between the car and the track	Allow sin/cos confusion Allow θ instead of 30°.
			$C \sin 30^\circ = \frac{mv^2}{r}$	M1*	3.3	NII with centripetal acceleration	Allow sin/cos confusion
			$-C \sin 30^\circ = \frac{mv^2}{r}$ $\Rightarrow \frac{C \sin 30^\circ}{C \cos 30^\circ} = \frac{mv^2}{r}$	M1ft	1.1	Dividing so that C and m will cancel. May see $\tan\theta$ or $\tan 30$ instead of \sin/\cos	Or rearrange one equation for <i>C</i> and substitute into the other one
			$\Rightarrow \tan 30^{\circ} = \frac{1}{\sqrt{3}} = v \Rightarrow \frac{1}{rg} = v \Rightarrow 3v^{2} \neq rg$	A1 [4]	1.1	AG	θ must be clearly stated and correctly used to gain this mark
7	(b)		$\sqrt{3}v^2 = 24 \times 9.8$	M1	3.4	Using the formula from (a)	
			v = awrt 11.7	A1 [2]	1.1		
7	(c)		The model implies that only a single value for the speed is possible for a given radius	B1	3.5b	Or equivalent	Do not allow discussion of the assumptions here
			so any change in speed should cause the car to move in a different circle	B1	2.2b	e.g. to slide sideways	Or any equivalent comment about a possible consequence according to the model of a change in the speed or the radius
			The track should be modelled as resisting sideways motion	B1 [3]	3.5c	Accept 'model track as rough' or 'include friction' etc without explicit reference to 'sideways'	Must be relevant to the question. Do not accept references that ignore friction

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