



Oxford Cambridge and RSA

**Wednesday 15 May 2019 – Morning**

**AS Level Mathematics B (MEI)**

**H630/01 Pure Mathematics and Mechanics**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- Printed Answer Booklet

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **70**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

**Formulae AS Level Mathematics B (MEI) (H630)****Binomial series**

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Sample variance**

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation,  $s = \sqrt{\text{variance}}$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X=r) = {}^nC_r p^r q^{n-r}$  where  $q = 1-p$

Mean of  $X$  is  $np$

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Answer **all** the questions.

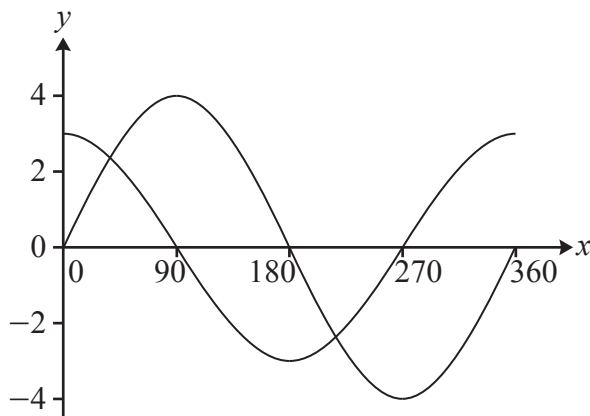
**1 In this question you must show detailed reasoning.**

Show that the equation  $x = 7 + 2x^2$  has no real roots.

**[3]**

**2 In this question you must show detailed reasoning.**

Fig. 2 shows the graphs of  $y = 4 \sin x^\circ$  and  $y = 3 \cos x^\circ$  for  $0 \leq x \leq 360$ .



**Fig. 2**

Find the  $x$ -coordinates of the two points of intersection, giving your answers correct to 1 decimal place.

**[3]**

**3** Given that  $k$  is an integer, express  $\frac{3\sqrt{2}-k}{\sqrt{8}+1}$  in the form  $a+b\sqrt{2}$  where  $a$  and  $b$  are rational expressions in terms of  $k$ .

**[4]**

**4** A triangle ABC has sides  $AB = 5$  cm,  $AC = 9$  cm and  $BC = 10$  cm.

(a) Find the cosine of angle BAC, giving your answer as a fraction in its lowest terms.

**[2]**

(b) Find the exact area of the triangle.

**[3]**

- 5 In this question, the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal and vertically upwards respectively.

A particle has mass 2.5 kg.

- (a) Write the weight of the particle as a vector. [1]

The particle moves under the action of its weight and two external forces  $(3\mathbf{i} - 2\mathbf{j})$  N and  $(-\mathbf{i} + 18\mathbf{j})$  N.

- (b) Find the acceleration of the particle, giving your answer in vector form. [2]

- 6 Fig. 6 shows a train consisting of an engine of mass 80 tonnes pulling two trucks each of mass 25 tonnes.

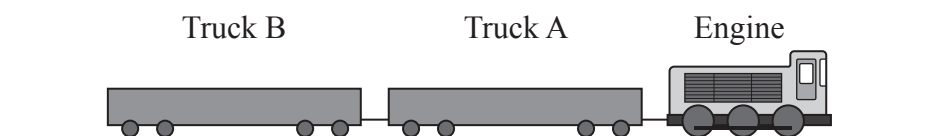


Fig. 6

The engine exerts a driving force of  $D$  N and experiences a resistance to motion of 2000 N. Each truck experiences a resistance of 600 N. The train travels in a straight line on a level track with an acceleration of  $0.1 \text{ m s}^{-2}$ .

- (a) Complete the force diagram in the Printed Answer Booklet to show all the forces acting on the engine and each of the trucks. [3]
- (b) Calculate the value of  $D$ . [2]
- (c) The tension in the coupling between the engine and truck A is larger than that in the coupling between the trucks. Determine how much larger. [2]

- 7 In this question you must show detailed reasoning.

- (a) Nigel is asked to determine whether  $(x + 7)$  is a factor of  $x^3 - 37x + 84$ . He substitutes  $x = 7$  and calculates  $7^3 - 37 \times 7 + 84$ . This comes to 168, so Nigel concludes that  $(x + 7)$  is not a factor.

Nigel's conclusion is wrong.

- Explain why Nigel's argument is not valid.
- Show that  $(x + 7)$  is a factor of  $x^3 - 37x + 84$ . [2]

- (b) Sketch the graph of  $y = x^3 - 37x + 84$ , indicating the coordinates of the points at which the curve crosses the coordinate axes. [5]

- (c) The graph in part (b) is translated by  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ . Find the equation of the translated graph, giving your answer in the form  $y = x^3 + ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are integers. [4]

**8 In this question you must show detailed reasoning.**

Show that the only stationary point on the graph of  $y = x^2 - 4\sqrt{x}$  is a minimum point at  $(1, -3)$ . [7]

**9 In this question you must show detailed reasoning.**

A car accelerates from rest along a straight level road. The velocity of the car after 8 s is  $25.6 \text{ m s}^{-1}$ .

In one model for the motion, the velocity  $v \text{ m s}^{-1}$  at time  $t$  seconds is given by  $v = 1.2t^2 - kt^3$ , where  $k$  is a constant and  $0 \leq t \leq 8$ .

(a) The model gives the correct velocity of  $25.6 \text{ m s}^{-1}$  at time 8 s. Show that  $k = 0.1$ . [2]

A second model for the motion uses constant acceleration.

(b) Find the value of the acceleration which gives the correct velocity of  $25.6 \text{ m s}^{-1}$  at time 8 s. [2]

(c) Show that these two models give the same value for the displacement in the first 8 s. [5]

**10 In this question you must show detailed reasoning.**

(a) Sketch the gradient function for the curve  $y = 24x - 3x^2 - x^3$ . [5]

(b) Determine the set of values of  $x$  for which  $24x - 3x^2 - x^3$  is decreasing. [2]

**11 David puts a block of ice into a cool-box. He wishes to model the mass  $m$  kg of the remaining block of ice at time  $t$  hours later. He finds that when  $t = 5$ ,  $m = 2.1$ , and when  $t = 50$ ,  $m = 0.21$ .**

(a) David at first guesses that the mass may be inversely proportional to time. Show that this model fits his measurements. [3]

(b) Explain why this model

(i) is not suitable for small values of  $t$ , [1]

(ii) cannot be used to find the time for the block to melt completely. [1]

David instead proposes a linear model  $m = at + b$ , where  $a$  and  $b$  are constants.

(c) Find the values of the constants for which the model fits the mass of the block when  $t = 5$  and  $t = 50$ . [3]

(d) Interpret these values of  $a$  and  $b$ . [2]

(e) Find the time according to this model for the block of ice to melt completely. [1]

**END OF QUESTION PAPER**



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