Oxford Cambridge and RSA

Thursday 20 June 2019 – Morning

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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Answer **all** the questions.

1 Fig. 1 shows some spreadsheet output concerning the values of a function, f(x).

	A	B	C
1	x	f(x)	
2	1	0.367879441	0.367879441
3	2	0.018315639	0.38619508
4	3	0.00012341	0.38631849
5	4	1.12535E-07	0.386318602
6	5	1.38879E-11	0.386318602

Fig. 1

The formula in cell B2 is `=EXP(-(A2^2))`
and equivalent formulae are in cells B3 to B6.

The formula in cell C2 is `=B2`.

The formula in cell C3 is `=C2+B3`.

Equivalent formulae are in cells C4 to C6.

(a) Use sigma notation to express the formula in cell C5 in standard mathematical notation. **[2]**

(b) Explain why the same value is displayed in cells C5 and C6. **[2]**

Now suppose that the value in cell C2 is chopped to 3 decimal places and used to approximate the value in cell C2.

(c) Calculate the relative error when this approximation is used. **[1]**

Suppose that the values in cells B4, B5 and B6 are chopped to 3 decimal places and used as approximations to the original values in cells B4, B5 and B6 respectively.

(d) Explain why the relative errors in these approximations are all the same. **[1]**

2 Fig. 2.1 shows the graph of $y = x^2 e^{2x} - 5x^2 + 0.5$.

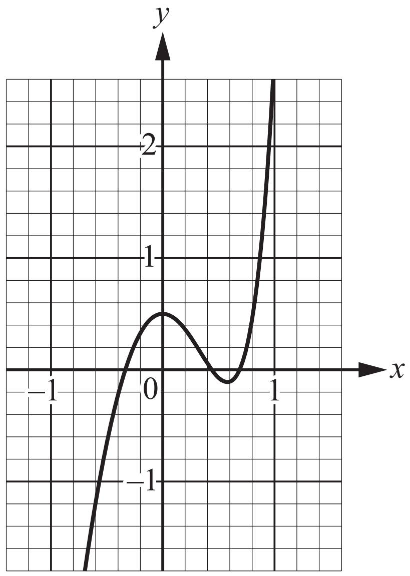

Fig. 2.1

There are three roots of the equation $x^2 e^{2x} - 5x^2 + 0.5 = 0$. The roots are α, β and γ, where $\alpha < \beta < \gamma$.

(a) Explain why it is not possible to use the method of false position with $x_0 = 0$ and $x_1 = 1$ to find β and γ. **[1]**

The graph of the function indicates that the root γ lies in the interval [0.6, 0.8]. Fig. 2.2 shows some spreadsheet output using the method of false position using these values as starting points.

	A	B	C	D	E	F
1	a	f(a)	b	f(b)	approx	
2	0.6	-0.10476	0.8	0.469941	0.636457	-0.07876
3	0.636457	-0.07876	0.8	0.469941	0.659931	-0.04748
4	0.659931	-0.04748	0.8	0.469941	0.672783	-0.0249
5	0.672783	-0.0249	0.8	0.469941	0.679184	-0.01211
6	0.679184	-0.01211	0.8	0.469941	0.682218	-0.00567
7	0.682218	-0.00567	0.8	0.469941	0.683623	-0.00261
8	0.683623	-0.00261	0.8	0.469941	0.684266	-0.00119
9	0.684266	-0.00119	0.8	0.469941	0.684559	-0.00054
10	0.684559	-0.00054	0.8	0.469941	0.684692	-0.00025
11	0.684692	-0.00025	0.8	0.469941	0.684753	-0.00011
12	0.684753	-0.00011	0.8	0.469941	0.68478	-5.1E-05

Fig. 2.2

(b) **Without** doing any further calculation, write down the smallest possible interval which is certain to contain γ. **[1]**

(c) State what is being calculated in column F. **[1]**

The formula in cell A3 is $\boxed{=\text{IF}(\text{F2}<0,\text{E2},\text{A2})}$.

(d) Explain the purpose of this formula in the application of the method of false position. **[2]**

The method of false position uses the same formula for obtaining new approximations as the secant method.

(e) Explain how the method of false position differs from the secant method. **[1]**

(f) Give one **advantage** and one **disadvantage** of using the method of false position instead of the secant method. **[2]**

3 In the first week of an outbreak of influenza, 9 patients were diagnosed with the virus at a medical practice in Pencaster. Records were kept of y, the total number of patients diagnosed with influenza in week n. The data are shown in Fig. 3.

n	1	2	3	4	5
y	9	32	63	96	125

Fig. 3

(a) Complete the difference table in the Printed Answer Booklet. **[3]**

(b) Explain why a cubic model is appropriate for the data. **[1]**

(c) Use Newton's method to find the interpolating polynomial of degree 3 for these data. **[4]**

In both week 6 and week 7 there were 145 patients in total diagnosed with influenza at the medical practice.

(d) Determine whether the model is a good fit for these data. **[2]**

(e) Determine the maximum number of weeks for which the model could possibly be valid. **[1]**

4 Fig. 4 shows the graph of $y = x^5 - 6\sqrt{x} + 4$.

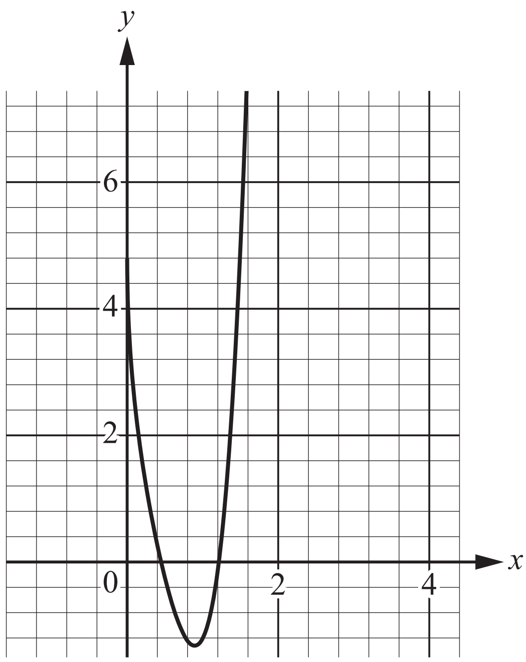

Fig. 4

There are two roots of the equation $x^5 - 6\sqrt{x} + 4 = 0$. The roots are α and β, such that $\alpha < \beta$.

(a) Show that $0 < \alpha < 1$ and $1 < \beta < 2$. **[2]**

(b) Obtain the Newton-Raphson iterative formula

$$x_{n+1} = x_n - \frac{x_n^{\frac{11}{2}} - 6x_n + 4\sqrt{x_n}}{5x_n^{\frac{9}{2}} - 3}.$$ **[3]**

(c) Use the iterative formula found in part **(b)** with a starting value of $x_0 = 1$ to obtain β correct to 6 decimal places. **[2]**

(d) Use the iterative formula found in part **(b)** with a starting value of $x_0 = 0$ to find x_1. **[1]**

(e) Give a geometrical explanation of why the Newton-Raphson iteration fails to find α in part **(d)**. **[1]**

(f) Obtain the iterative formula

$$x_{n+1} = \left(\frac{x_n^5 + 4}{6}\right)^2.$$ **[2]**

(g) Use the iterative formula found in part **(f)** with a starting value of $x_0 = 0$ to obtain α correct to 6 decimal places. **[2]**

5 Fig. 5 shows spreadsheet output concerning the estimation of the derivative of a function $f(x)$ at $x = 2$ using the forward difference method.

	A	B	C	D
1	*h*	estimate	difference	ratio
2	0.1	6.3050005		
3	0.01	6.0300512	-0.274949	
4	0.001	6.0030018	-0.027049	0.098379
5	0.0001	6.0003014	-0.0027	0.099835
6	0.00001	6.0000314	-0.00027	0.099983
7	0.000001	6.0000044	-2.7E-05	0.099994
8	1E-07	6.0000016	-2.71E-06	0.100352
9	1E-08	6.0000013	-3.02E-07	0.111457
10	1E-09	6.0000018	4.885E-07	-1.61765
11	1E-10	6.0000049	3.109E-06	6.363636
12	1E-11	6.0000005	-4.44E-06	-1.42857
13	1E-12	6.0005334	0.0005329	-120
14	1E-13	5.9952043	-0.005329	-10
15	1E-14	6.1284311	0.1332268	-25
16	1E-15	5.3290705	-0.799361	-6
17	1E-16	0	-5.329071	6.666667

Fig. 5

(a) Write down suitable cell formulae for

- cell C3,
- cell D4. **[2]**

(b) Explain what the entries in cells D4 to D8 tell you about the order of the convergence of the forward difference method. **[2]**

(c) Write the entry in cell A10 in standard mathematical notation. **[1]**

(d) Explain what the values displayed in cells D10 to D17 suggest about the values in cells B10 to B16. **[2]**

(e) Write down the value of the derivative of $f(x)$ at $x = 2$ to an accuracy that seems justified, explaining your answer. **[2]**

The formula in cell B2 is `=(LN(SQRT(SINH((2+A2)^3)))−LN(SQRT(SINH(2^3))))/A2`
and equivalent formulae are entered in cells B3 to B17.

(f) Write $f(x)$ in standard mathematical notation. **[1]**

The value displayed in cell B17 is zero, even though the calculation results in a non-zero answer.

(g) Explain how this has arisen. **[2]**

6 The spreadsheet output in Fig. 6 shows approximations to $\int_0^1 x^{-\sqrt{x}}\,dx$ found using the midpoint rule, denoted by M_n, and the trapezium rule, denoted by T_n.

	A	B	C
1	n	M_n	T_n
2	1	1.632527	1
3	2	1.641461	1.316263
4	4	1.623053	1.478862
5	8	1.610295	1.550957
6	16	1.604132	1.580626
7	32	1.601505	1.592379

Fig. 6

(a) Write down an efficient spreadsheet formula for cell C3. **[2]**

(b) By first completing the table in the Printed Answer Booklet using the Simpson's rule, calculate the most accurate estimate of $\int_0^1 x^{-\sqrt{x}}\,dx$ that you can, justifying the precision quoted. **[8]**

END OF QUESTION PAPER