



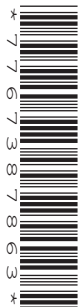
Oxford Cambridge and RSA

Monday 20 May 2019 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes



You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **12** pages.

Answer **all** the questions.

1 The graph of $y = 3.15x^3 + 0.75x^2 - 5.2x + 2.2$ is shown in Fig. 1.1.

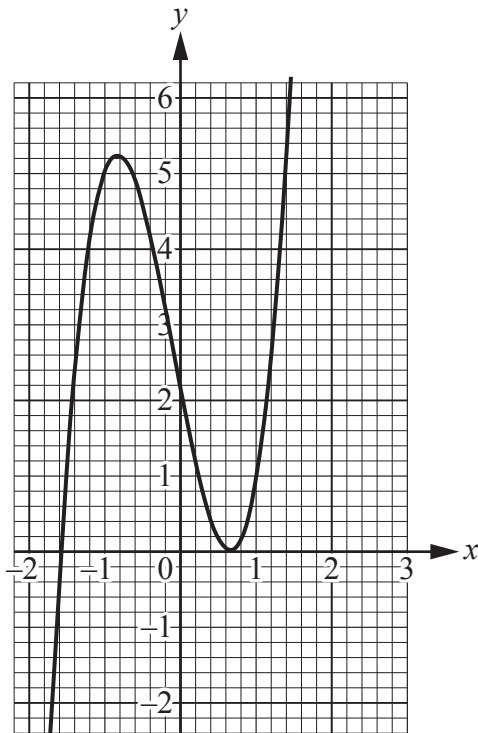


Fig. 1.1

There are three roots to the equation $3.15x^3 + 0.75x^2 - 5.2x + 2.2 = 0$, a repeated root, α , and β , where $\alpha > \beta$.

(a) Explain why the bisection method cannot be used successfully to find α . [1]

The bisection method is used to find the value of β using a spreadsheet. The output is shown in Fig. 1.2.

	A	B	C	D	E	F
1	a	$f(a)$	b	$f(b)$	x_{new}	$f(x_{\text{new}})$
2	-2	-9.6	-1	5	-1.5	1.05625
3	-2	-9.6	-1.5	1.05625	-1.75	-3.28516
4	-1.75	-3.28516	-1.5	1.05625	-1.625	-0.88623
5	-1.625	-0.88623	-1.5	1.05625	-1.5625	0.139758
6	-1.625	-0.88623	-1.5625	0.139758	-1.59375	-0.35926

Fig. 1.2

- (b) Use the information in Fig. 1.2 to find the value of β as accurately as you can. [1]
- (c) Deduce the maximum possible error in your answer to part (b). [1]

The formula in cell A3 is as follows.

=IF(F2<0,E2,A2)

- (d) Write down a similar formula for cell C3. [1]

The Newton-Raphson method is used to find α . The spreadsheet output, together with some further analysis, is shown in Fig. 1.3. The iterates are shown in column L, the difference between successive iterates is shown in column M and the ratio of successive differences is shown in column N.

	K	L	M	N
1	n	x_n	difference	ratio
2	0	0		
3	1	0.4230769	0.4230769	
4	2	0.5527929	0.129716	0.3066015
5	3	0.6112979	0.058505	0.4510239
6	4	0.6393379	0.02804	0.4792752
7	5	0.6530873	0.0137493	0.4903474
8	6	0.6598978	0.0068105	0.495331
9	7	0.6632874	0.0033896	0.4977027
10	8	0.6649783	0.0016909	0.4988604
11	9	0.6658228	0.0008445	0.4994324
12	10	0.6662448	0.000422	0.4997168
13	11	0.6664558	0.0002109	0.4998585
14	12	0.6665612	0.0001055	0.4999293
15	13	0.6666139	5.273E-05	0.4999647
16	14	0.6666403	2.636E-05	0.4999823
17	15	0.6666535	1.318E-05	0.4999911
18	16	0.6666601	6.59E-06	0.4999956
19	17	0.6666634	3.295E-06	0.4999976
20	18	0.666665	1.648E-06	0.499998
21	19	0.6666658	8.238E-07	0.4999929
22	20	0.6666663	4.119E-07	0.5000375

Fig. 1.3

- (e) Write down suitable cell formulae for
- (i) cell M3,
 - (ii) cell N4. [2]
- (f) State what the values in column N tell you about the convergence of the sequence of estimates for α in column L, stating what is unusual about this. [2]

2 You are given that $f(x) = \log_{10}(2x + 1)$.

- (a) Calculate the absolute error when $f(5)$ is used to approximate $f(4.98)$. [2]
- (b) Hence find an approximation to $f'(4.98)$. [2]
- (c) Use the central difference method with $h = 0.02$, $h = 0.01$ and $h = 0.005$ to find three estimates of $f'(4.98)$. [3]
- (d) **Without** doing any further calculation, state the value of $f'(4.98)$ to an accuracy which seems justified. [1]

3 Sallysavers Building Society offers a “Bigsaver” account. Interest is payable at the end of each year. The rate of interest is 2.33% per annum.

A customer models the situation by assuming that interest earned is rounded to the nearest penny. This customer invests £375 at the start of the year.

- (a) Use this model to calculate the amount of interest earned on the customer’s investment after one year. [1]

In fact, the interest earned is not rounded to the nearest penny. Instead, the interest paid to each “Bigsaver” account is chopped to the nearest penny. There are 12 254 “Bigsaver” accounts, in which a range of amounts have been invested.

- (b) Stating the modelling assumption made, calculate an estimate of how much money in total the building society saves in interest payments over one year by chopping instead of rounding the interest payments on “Bigsaver” accounts. [3]

At Sallysavers there are 534 220 additional accounts on which interest is paid at the end of each year.

- (c) Use the modelling assumption in part (b) to calculate an estimate of how much money the building society saves in interest payments over one year to these additional accounts by chopping the interest paid to the nearest penny. [1]
- (d) State whether your answer to part (c) is more reliable or less reliable than your answer to part (b), explaining your reasoning. [1]

4 Fig. 4.1 shows the graph of $y = (0.8x)^x$.

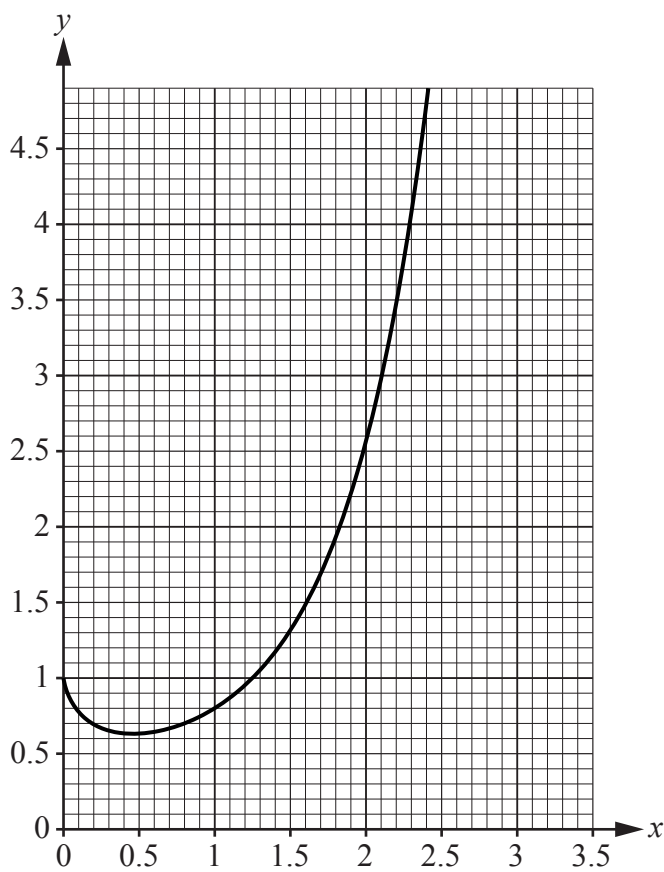


Fig. 4.1

Fig. 4.2 shows some spreadsheet output concerning the evaluation of $(0.8x)^x$ for various values of x .

x	$(0.8x)^x$
0.1	0.7768
0.01	0.952864
0.001	0.992894
0.0001	0.999057
0.00001	0.999883
0.000001	0.999986

Fig. 4.2

- (a) In order to calculate an estimate of $\int_0^1 (0.8x)^x dx$ using a numerical method it is necessary to make an important assumption. State this assumption. [1]

Some estimates of $\int_0^1 (0.8x)^x dx$ have been calculated using the midpoint rule, denoted by M_n , trapezium rule, denoted by T_n , and Simpson's rule, denoted by S_{2n} . These are shown in Fig. 4.3. The estimate given by T_1 is 0.9 and the other values have been given correct to 6 decimal places.

n	M_n	T_n	S_{2n}
1	0.632456	0.9	0.721637
2			0.705567
4	0.691727	0.720732	

Fig. 4.3

- (b) Complete the table in the Printed Answer Booklet, giving the values of M_2 , T_2 and S_8 correct to 6 decimal places. **[3]**
- (c) **Without** doing any further calculation, state the value of $\int_0^1 (0.8x)^x dx$ to an accuracy which seems justified, explaining your answer. **[1]**

- 5 (a) Explain what problem may arise in the evaluation of $\frac{1}{x^{10}} - \frac{1}{y^{10}}$ for large values of x and y . [2]
- (b) Explain why re-writing $\frac{1}{x^{10}} - \frac{1}{y^{10}}$ as $\frac{y^{10} - x^{10}}{(xy)^{10}}$ may alleviate the problem identified in part (a). [1]

A spreadsheet is used to evaluate $\frac{1}{x^{10}} - \frac{1}{y^{10}}$ for various values of x and y . The output from this spreadsheet is shown in Fig. 5. The calculation breaks down in row 7.

	A	B	C	D	E	F	G
1	x	y	x^{10}	y^{10}	$y^{10} - x^{10}$	$(xy)^{10}$	$(y^{10} - x^{10}) / (xy)^{10}$
2	800	805	1.07374E+29	1.14277E+29	6.9028E+27	1.22704E+58	5.62558E-31
3	800000	800005	1.07374E+59	1.07381E+59	6.7111E+54	1.153E+118	5.82057E-64
4	800000000	800000005	1.07374E+89	1.07374E+89	6.7109E+81	1.1529E+178	5.82077E-97
5	8E+11	8E+11	1.0737E+119	1.0737E+119	6.711E+108	1.1529E+238	5.8208E-130
6	8E+14	8E+14	1.0737E+149	1.0737E+149	6.746E+135	1.1529E+298	5.8512E-163
7	8E+17	8E+17	1.0737E+179	1.0737E+179	0	#NUM!	#NUM!

Fig. 5

The formulae in cells E2 and F2 are

=D2 - C2

and

=C2*D2

respectively. Equivalent formulae are entered in the cells below E2 and F2.

- (c) Write the value displayed in cell E2 using standard mathematical notation. [1]

From row 3 downwards, the values in column A have been found by multiplying the value in the cell above by 1000 and the values in column B have been found by adding 5 to the value in the adjacent cell in column A.

- (d) Comment on the behaviour of the sequence of values in column E. [1]
- (e) Explain why the same values are displayed in cells C4 and D4, but the value in cell E4 is non-zero. [1]
- (f) Explain why the value displayed in E7 is zero, even though the numerical value associated with the calculation is very large. [1]
- (g) State the meaning of the display in cell F7, explaining how it has arisen. [1]

- 6 Fig. 6 shows some spreadsheet output concerning the solution of an equation, together with some further analysis. The iterates are shown in column C, the difference between successive iterates is shown in column D and the ratio of successive differences is shown in column E.

	A	B	C	D	E	F
1	n	x_n	$f(x_n)$	difference	ratio	
2	0	-1	-0.63212056			
3	1	0	1			
4	2	-0.61269984	0.1664848	-0.6126998		
5	3	-0.73507938	-0.0608743	-0.1223795	0.19973817	-0.3259968
6	4	-0.70231285	0.00219476	0.03276653	-0.26774516	2.1878261
7	5	-0.7034531	2.7229E-05	-0.0011403	-0.03479939	-1.0620408
8	6	-0.70346743	-1.2445E-08	-1.432E-05	0.01256215	-11.016965
9	7	-0.70346742	7.0499E-14	6.5436E-09	-0.00045683	31.89219

Fig. 6

The cell formula in cell C2 is as follows.

`=EXP(B2) - B2^2`

- (a) Using standard mathematical notation, write down the equation being solved. [1]

The cell formula in cell B4 is as follows.

`=(B3*C2-B2*C3)/(C2-C3)`

- (b) State the method being used to solve the equation. [1]

- (c) Verify that the value shown in cell B9 is the root of the equation correct to 8 decimal places. [1]

- (d) Explain what the values in column E tell you about the sequence of estimates in column B. [1]

The cell formula in cell F5 is as follows.

`=D5/D4^2`

- (e) Write down a suitable entry for cell F1 which could be used as a column heading for column F. [1]

- (f) Explain what the values in column F tell you about the sequence of estimates in column B. [1]

7 Fig. 7 shows values of a function $f(x)$ for integer values of x from -2 to 3 inclusive.

x	-2	-1	0	1	2	3
$f(x)$	-9	2	1	0	11	46

Fig. 7

- (a) Complete the difference table in the Printed Answer Booklet. [2]
- (b) Explain why it is reasonable to assume that $f(x)$ is a polynomial of degree 3. [1]
- (c) Use Newton’s forward difference interpolation formula to construct a polynomial of degree 3 for these data, giving your answer in the form $ax^3 + bx^2 + cx + d$, where a , b , c and d are constants to be determined. [4]

- 8 Fig. 8.1 shows estimates of $\int_0^1 \sin(\sqrt{x}) dx$ found using the trapezium rule, denoted by T_n , together with some further analysis of these estimates.

n	T_n	difference	ratio
1	0.42073549		
2	0.53518622	0.114450723	
4	0.57788949	0.042703272	0.37311492
8	0.59352512	0.01563563	0.36614596
16	0.59918122	0.0056561	0.36174428
32	0.60121166	0.002030438	0.35898194
64	0.60193696	0.0007253	0.35721365

Fig. 8.1

- (a) Use extrapolation to find the value of $\int_0^1 \sin(\sqrt{x}) dx$ to an accuracy which seems justified, explaining your answer. [5]

The work is extended to include Simpson's rule estimates, denoted by S_{2n} . These are shown in Fig. 8.2, together with some further analysis of these estimates.

	A	B	C	D	E
1	n	T_n	S_{2n}	difference	ratio
2	1	0.42073549	0.573336457		
3	2	0.53518622	0.592123912	0.01878746	
4	4	0.57788949	0.598736995	0.00661308	0.351994627
5	8	0.59352512	0.601066585	0.00232959	0.352269826
6	16	0.59918122	0.601888468	0.00082188	0.352801881
7	32	0.60121166	0.602178723	0.00029025	0.353157287
8	64	0.60193696			

Fig. 8.2

- (b) Write down a suitable cell formula for cell C2. [2]
- (c) Use extrapolation to find the value of $\int_0^1 \sin(\sqrt{x}) dx$ to an accuracy which seems justified, explaining your answer. [5]

END OF QUESTION PAPER



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