



Oxford Cambridge and RSA

**Thursday 13 June 2019 – Afternoon**

**A Level Further Mathematics A**

**Y542/01 Statistics**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1** A set of bivariate data  $(X, Y)$  is summarised as follows.

$$n = 25, \Sigma x = 9.975, \Sigma y = 11.175, \Sigma x^2 = 5.725, \Sigma y^2 = 46.200, \Sigma xy = 11.575$$

- (a) Calculate the value of Pearson's product-moment correlation coefficient. [1]
- (b) Calculate the equation of the regression line of  $y$  on  $x$ . [2]

It is desired to know whether the regression line of  $y$  on  $x$  will provide a reliable estimate of  $y$  when  $x = 0.75$ .

- (c) State one reason for believing that the estimate will be reliable. [1]
- (d) State what further information is needed in order to determine whether the estimate is reliable. [1]

- 2** The average numbers of cars, lorries and buses passing a point on a busy road in a period of 30 minutes are 400, 80 and 17 respectively.

- (a) Assuming that the numbers of each type of vehicle passing the point in a period of 30 minutes have independent Poisson distributions, calculate the probability that the total number of vehicles passing the point in a randomly chosen period of 30 minutes is at least 520. [3]
- (b) Buses are known to run in approximate accordance with a fixed timetable.

Explain why this casts doubt on the use of a Poisson distribution to model the number of buses passing the point in a fixed time interval. [1]

- 3** Six red counters and four blue counters are arranged in a straight line in a random order.

Find the probability that

- (a) no blue counter has fewer than two red counters between it and the nearest other blue counter, [3]
- (b) no two blue counters are next to one another. [3]

- 4 The greatest weight  $WN$  that can be supported by a shelving bracket of traditional design is a normally distributed random variable with mean 500 and standard deviation 80.

A sample of 40 shelving brackets of a new design are tested and it is found that the mean of the greatest weights that the brackets in the sample can support is 473.0N.

- (a) Test at the 1% significance level whether the mean of the greatest weight that a bracket of the new design can support is less than the mean of the greatest weight that a bracket of the traditional design can support. [7]
- (b) State an assumption needed in carrying out the test in part (a). [1]
- (c) Explain whether it is necessary to use the central limit theorem in carrying out the test. [1]

- 5 Five runners,  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$ , take part in two different races.

Spearman's rank correlation coefficient for the orders in which the runners finish is calculated and a test for positive agreement is carried out at the 5% significance level.

- (a) State suitable hypotheses for the test. [1]
- (b) Find the largest possible value of  $\sum d^2$  for which the result of the test is to reject the null hypothesis. [3]
- (c) In the first race, the order in which the five runners finished was:  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ . In the second race, three of the runners finished in the same positions as in the first race. The result of the test is to reject the null hypothesis.

Find a possible order for the runners to finish in the second race. [3]

- 6 Yusha investigates the proportion of left-handed people living in two cities,  $A$  and  $B$ . He obtains data from random samples from the two cities. His results are shown in the table, in which  $L$  denotes "left-handed".

	$L$	$L'$
$A$	14	9
$B$	26	51

- (a) Test at the 10% significance level whether there is association between being left-handed and living in a particular city. [7]

A person is chosen at random from one of the cities  $A$  and  $B$ .  
Let  $A$  denote "the person lives in city  $A$ ".

- (b) State the relationship between  $P(L)$  and  $P(L|A)$  according to the model implied by the null hypothesis of your test. [1]
- (c) Use the data in the table to suggest a value for  $P(L|A)$  given by an improved model. [2]

- 7 The random variable  $D$  has the distribution  $\text{Geo}(p)$ . It is given that  $\text{Var}(D) = \frac{40}{9}$ .

Determine

- (a)  $\text{Var}(3D + 5)$ , [1]  
 (b)  $E(3D + 5)$ , [6]  
 (c)  $P(D > E(D))$ . [3]

- 8 A university course was taught by two different professors. Students could choose whether to attend the lectures given by Professor  $Q$  or the lectures given by Professor  $R$ . At the end of the course all the students took the same examination.

The examination marks of a random sample of 30 students taught by Professor  $Q$  and a random sample of 24 students taught by Professor  $R$  were ranked. The sum of the ranks of the students taught by Professor  $Q$  was 726.

Test at the 5% significance level whether there is a difference in the ranks of the students taught by the two professors. [10]

- 9 The continuous random variable  $T$  has cumulative distribution function

$$F(t) = \begin{cases} 0 & t < 0, \\ 1 - e^{-0.25t} & t \geq 0. \end{cases}$$

- (a) Find the cumulative distribution function of  $2T$ . [3]  
 (b) Show that, for constant  $k$ ,  $E(e^{kt}) = \frac{1}{1 - 4k}$ .

You should state with a reason the range of values of  $k$  for which this result is valid. [7]

- (c)  $T$  is the time before a certain event occurs.

Show that the probability that no event occurs between time  $T = 0$  and time  $T = \theta$  is the same as the probability that the value of a random variable with the distribution  $\text{Po}(\lambda)$  is 0, for a certain value of  $\lambda$ . You should state this value of  $\lambda$  in terms of  $\theta$ . [4]

### END OF QUESTION PAPER

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