



Oxford Cambridge and RSA

**Tuesday 18 June 2019 – Morning**

**A Level Further Mathematics A**

**Y543/01 Mechanics**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

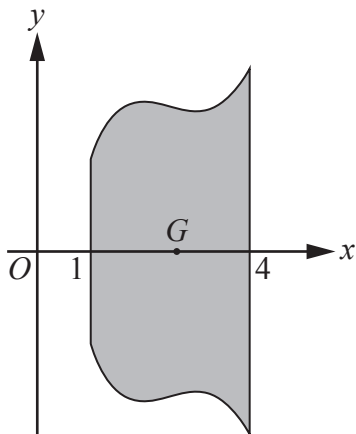
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 The region bounded by the  $x$ -axis, the curve  $y = \sqrt{2x^3 - 15x^2 + 36x - 20}$  and the lines  $x = 1$  and  $x = 4$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a uniform solid of revolution  $R$ . The centre of mass of  $R$  is the point  $G$  (see diagram).



- (a) Explain why the  $y$ -coordinate of  $G$  is 0. [1]
- (b) Find the  $x$ -coordinate of  $G$ . [4]

$P$  is a point on the edge of the curved surface of  $R$  where  $x = 4$ .  $R$  is freely suspended from  $P$  and hangs in equilibrium.

- (c) Find the angle between the axis of symmetry of  $R$  and the vertical. [3]

- 2 A solenoid is a device formed by winding a wire tightly around a hollow cylinder so that the wire forms (approximately) circular loops along the cylinder (see diagram).



When the wire carries an electrical current a magnetic field is created inside the solenoid which can cause a particle which is moving inside the solenoid to accelerate.

A student is carrying out experiments on particles moving inside solenoids. His professor suggests that, for a particle of mass  $m$  moving with speed  $v$  inside a solenoid of length  $h$ , the acceleration  $a$  of the particle can be modelled by a relationship of the form  $a = km^\alpha v^\beta h^\gamma$ , where  $k$  is a constant. The professor tells the student that  $[k] = \text{MLT}^{-1}$ .

- (a) Use dimensional analysis to find  $\alpha$ ,  $\beta$  and  $\gamma$ . [6]
- (b) The mass of an electron is  $9.11 \times 10^{-31} \text{ kg}$  and the mass of a proton is  $1.67 \times 10^{-27} \text{ kg}$ .

For an electron and a proton moving inside the same solenoid with the same speed, use the model to find the ratio of the acceleration of the electron to the acceleration of the proton. [3]

- (c) The professor tells the student that  $a$  also depends on the number of turns or loops of wire,  $N$ , that the solenoid has.

Explain why dimensional analysis **cannot** be used to determine the dependence of  $a$  on  $N$ . [1]

- 3 A particle  $Q$  of mass  $m$  kg is acted on by a single force so that it moves with constant acceleration  $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ms}^{-2}$ . Initially  $Q$  is at the point  $O$  and is moving with velocity  $\mathbf{u} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \text{ms}^{-1}$ .

After  $Q$  has been moving for 5 seconds it reaches the point  $A$ .

(a) Use the equation  $\mathbf{v} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{u} + 2\mathbf{a} \cdot \mathbf{x}$  to show that at  $A$  the kinetic energy of  $Q$  is  $37m$  J. [5]

(b) (i) Show that the power initially generated by the force is  $-8m$  W. [2]

(ii) The power in part (b)(i) is negative. Explain what this means about the initial motion of  $Q$ . [1]

(c) (i) Find the time at which the power generated by the force is instantaneously zero. [3]

(ii) Find the minimum kinetic energy of  $Q$  in terms of  $m$ . [2]

- 4 A right circular cone  $C$  of height 4 m and base radius 3 m has its base fixed to a horizontal plane. One end of a light elastic string of natural length 2 m and modulus of elasticity 32 N is fixed to the vertex of  $C$ . The other end of the string is attached to a particle  $P$  of mass 2.5 kg.

$P$  moves in a horizontal circle with constant speed and in contact with the smooth curved surface of  $C$ . The extension of the string is 1.5 m.

(a) Find the tension in the string. [2]

(b) Find the speed of  $P$ . [7]

- 5 A particle  $P$  of mass 4.5 kg is free to move along the  $x$ -axis. In a model of the motion it is assumed that  $P$  is acted on by two forces:

- a constant force of magnitude  $f$  N in the positive  $x$  direction;
- a resistance to motion,  $R$  N, whose magnitude is proportional to the speed of  $P$ .

At time  $t$  seconds the velocity of  $P$  is  $v \text{ms}^{-1}$ . When  $t = 0$ ,  $P$  is at the origin  $O$  and is moving in the positive direction with speed  $u \text{ms}^{-1}$ , and when  $v = 5$ ,  $R = 2$ .

(a) Show that, according to the model,  $\frac{dv}{dt} = \frac{10f - 4v}{45}$ . [2]

(b) (i) By solving the differential equation in part (a), show that  $v = \frac{1}{2}(5f - (5f - 2u)e^{-\frac{4}{45}t})$ . [5]

(ii) Describe briefly how, according to the model, the speed of  $P$  varies over time in each of the following cases.

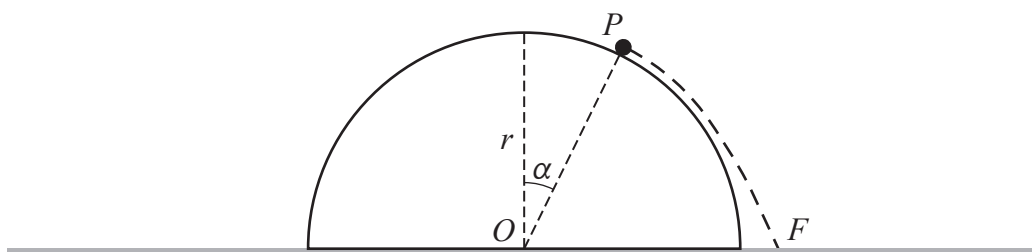
- $u < 2.5f$
- $u = 2.5f$
- $u > 2.5f$

[3]

(c) In the case where  $u = 2f$ , find in terms of  $f$  the exact displacement of  $P$  from  $O$  when  $t = 9$ . [4]

- 6 Two particles  $A$  and  $B$ , of masses  $m$  kg and  $1$  kg respectively, are connected by a light inextensible string of length  $d$  m and placed at rest on a smooth horizontal plane a distance of  $\frac{1}{2}d$  m apart.  $B$  is then projected horizontally with speed  $v$  m s<sup>-1</sup> in a direction perpendicular to  $AB$ .
- (a) Show that, at the instant that the string becomes taut, the magnitude of the instantaneous impulse in the string,  $I$  N s, is given by  $I = \frac{\sqrt{3}mv}{2(1+m)}$ . [4]
- (b) Find, in terms of  $m$  and  $v$ , the kinetic energy of  $B$  at the instant after the string becomes taut. Give your answer as a single algebraic fraction. [3]
- (c) In the case where  $m$  is very large, describe, with justification, the approximate motion of  $B$  after the string becomes taut. [2]

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The flat surface of a smooth solid hemisphere of radius  $r$  is fixed to a horizontal plane on a planet where the acceleration due to gravity is denoted by  $\gamma$ .  $O$  is the centre of the flat surface of the hemisphere.

A particle  $P$  is held at a point on the surface of the hemisphere such that the angle between  $OP$  and the upward vertical through  $O$  is  $\alpha$ , where  $\cos \alpha = \frac{3}{4}$ .

$P$  is then released from rest.  $F$  is the point on the plane where  $P$  first hits the plane (see diagram).

- (a) Find an exact expression for the distance  $OF$ . [11]

The acceleration due to gravity on and near the surface of the planet Earth is roughly  $6\gamma$ .

- (b) Explain whether  $OF$  would increase, decrease or remain unchanged if the action were repeated on the planet Earth. [1]

### END OF QUESTION PAPER

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