

Monday 24 June 2019 – Morning

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

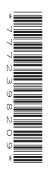
• a scientific or graphical calculator



- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \, \text{m} \, \text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.

INFORMATION

- The total mark for this paper is 75.
- The marks for each question are shown in brackets [].
- · You are reminded of the need for clear presentation in your answers.
- The Printed Answer Booklet consists of 12 pages. The Question Paper consists of 4 pages.



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Answer all the questions.

- 1 The sequence $\{u_n\}$ is defined by $u_0 = 2$, $u_1 = 5$ and $u_n = \frac{1 + u_{n-1}}{u_{n-2}}$ for $n \ge 2$. Prove that the sequence is periodic with period 5.
- 2 A surface has equation z = f(x, y) where $f(x, y) = x^2 \sin y + 2y \cos x$.

(a) Determine
$$f_x$$
, f_y , f_{xx} , f_{yy} , f_{xy} and f_{yx} . [5]

- (b) (i) Verify that z has a stationary point at $(\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{1}{4}\pi^2)$. [3]
 - (ii) Determine the nature of this stationary point. [3]
- 3 (a) Solve $7x \equiv 6 \pmod{19}$. [2]
 - **(b)** Show that the following simultaneous linear congruences have no solution.

$$x \equiv 3 \pmod{4}, \ x \equiv 4 \pmod{6}.$$

- 4 (a) Solve the second-order recurrence relation $T_{n+2} + 2T_n = -87$ given that $T_0 = -27$ and $T_1 = 27$. [8]
 - (b) Determine the value of T_{20} . [2]

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- 5 The group G consists of a set S together with \times_{80} , the operation of multiplication modulo 80. It is given that S is the smallest set which contains the element 11.
 - (a) By constructing the Cayley table for G, determine all the elements of S. [5]

The Cayley table for a second group, H, also with the operation \times_{80} , is shown below.

| \times_{80} | 1 | 9 | 31 | 39 |
|---------------|----|----|----|----|
| 1 | 1 | 9 | 31 | 39 |
| 9 | 9 | 1 | 39 | 31 |
| 31 | 31 | 39 | 1 | 9 |
| 39 | 39 | 31 | 9 | 1 |

- (b) Use the two Cayley tables to explain why G and H are not isomorphic. [2]
- (c) (i) List
 - all the proper subgroups of *G*,
 - all the proper subgroups of *H*. [3]
 - (ii) Use your answers to (c) (i) to give another reason why G and H are not isomorphic. [1]
- 6 (a) For the vectors $\mathbf{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$, calculate
 - $\mathbf{p} \cdot \mathbf{q} \times \mathbf{r}$,
 - $\mathbf{p} \times (\mathbf{q} \times \mathbf{r})$,

•
$$(\mathbf{p} \times \mathbf{q}) \times \mathbf{r}$$
. [6]

(b) State whether the vector product is associative for three-dimensional column vectors with real components. Justify your answer. [1]

It is given that **a**, **b** and **c** are three-dimensional column vectors with real components.

(c) Explain geometrically why the vector $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ must be expressible in the form $\lambda \mathbf{b} + \mu \mathbf{c}$, where λ and μ are scalar constants.

It is given that the following relationship holds for a, b and c.

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c}) \mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \mathbf{c}$$
 (*)

(d) Find an expression for $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ in the form of (*).

4

7 The points $P\left(\frac{1}{2}, \frac{13}{24}\right)$ and $Q\left(\frac{3}{2}, \frac{31}{24}\right)$ lie on the curve $y = \frac{1}{3}x^3 + \frac{1}{4x}$.

The area of the surface generated when arc PQ is rotated completely about the x-axis is denoted by A.

(a) Find the exact value of A. Give your answer as a rational multiple of π . [4]

Student X finds an approximation to A by modelling the arc PQ as the straight line segment PQ, then rotating this line segment completely about the x-axis to form a surface.

(b) Find the approximation to A obtained by student X. Give your answer as a rational multiple of π .

Student Y finds a second approximation to A by modelling the original curve as the line y = M, where M is the mean value of the function $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$, then rotating this line completely about the x-axis to form a surface.

- (c) Find the approximation to A obtained by student Y. Give your answer correct to four decimal places. [4]
- 8 In this question you must show detailed reasoning.

(a) Prove that
$$2(p-2)^{p-2} \equiv -1 \pmod{p}$$
, where p is an odd prime. [4]

(b) Find two odd prime factors of the number $N = 2 \times 34^{34} - 2^{15}$. [7]

END OF QUESTION PAPER



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