



Oxford Cambridge and RSA

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Question Paper

Tuesday 22 May 2018 – Afternoon

Time allowed: 1 hour 15 minutes



You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

- 1 The coordinates of point A are (1.05, 2.71) and the coordinates of point B are (1.07, 3.08). In each case the x and y values have been rounded to two decimal places. The gradient of the line AB is m .

Find the range of values of m , commenting on your answer. [4]

- 2 The table in Fig. 2 shows some values of $\log_3 x$ which are correct to 6 decimal places.

x	2	2.25	2.5	2.75	3
$\log_3 x$	0.630930	0.738140	0.834044	0.920799	1

Fig. 2

- (i) Use Simpson's rule to calculate an approximation to $\int_2^3 \log_3 x \, dx$, giving your answer correct to 6 decimal places. [3]
- (ii) Explain why it is unlikely that the answer to part (i) is in fact accurate to 6 decimal places. [1]
- 3 The spreadsheet output in Fig. 3 shows Table 1 and Table 2.

	A	B	C	D	E	F	G	
1	x	2	2.1	2.01	2.001	2.0001	2.00001	
2	$f(x)$	0.64	0.625876972	0.638573473	0.639857204	0.639985719	0.639998572	
3	Table 1							
4								
5	h	0.1	0.01	0.001	0.0001	0.00001		
6	dy/dx	-0.1412303	-0.142652654	-0.14279594	-0.142810279	-0.14281171		
7	difference	-0.0014224	-0.000143287	-1.4339E-05	-1.43402E-06			
8	ratio	0.1007378	0.100073651	0.10000709				
9	Table 2							

Fig. 3

Table 1 shows values of a function, $y = f(x)$, for different values of x . Table 2 shows approximations to $\frac{dy}{dx}$ at $x = 2$, along with the differences between successive approximations and the ratio of these differences.

The formula in cell C6 is

$$= (D2 - \$B2)/C5$$

Equivalent formulae are in cells D6, E6 and F6.

(i) Explain why the symbol $\$$ is used. [1]

(ii) State what method is being used to approximate $\frac{dy}{dx}$ at $x = 2$. [1]

(iii) Use extrapolation to find the value of $\frac{dy}{dx}$ at $x = 2$ as accurately as you can, justifying your answer. [4]

(iv) Calculate an approximation to the value of $f(2.05)$. [2]

4 The equation $e^x - x^2 - 2x = 0$ has a root α , where $0 < \alpha < 1$ and a root β , where $2 < \beta < 3$.

(i) Show how to obtain the iterative formula

$$x_{r+1} = \frac{e^{x_r} - x_r^2}{2}.$$

[2]

Fig. 4 shows part of the curve $y = \frac{e^x - x^2}{2}$ and part of the straight line $y = x$.

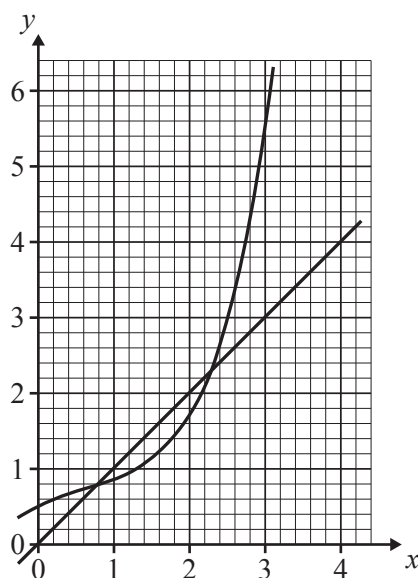


Fig. 4

(ii) Explain why the iteration in part (i) will

- successfully find α if a suitable starting value is chosen,
- fail to find β however close the starting value is to the root. [2]

(iii) Use the iteration in part (i) to find α correct to 6 significant figures. [2]

The relaxed iteration

$$x_{r+1} = (1 - \lambda)x_r + \lambda \left(\frac{e^{x_r} - x_r^2}{2} \right)$$

is used to find β .

- (iv) Use $x_0 = 2$ with $\lambda = -0.8$ to find the value of β correct to 6 decimal places. [3]
- (v) Determine what happens when the relaxed iteration is used with $\lambda = 0.8$ and a starting value of 2. [2]

5 The value of shares in Sunfield plc was £2.21 per unit on the first day it came under new management. One week later the value of one unit of shares was £4.00 and after three weeks the value of one unit was £7.34. The information is summarised in Fig. 5.1, where x is the value in pounds per unit, and t is the time in weeks since coming under new management.

t	0	1	3
x	2.21	4.00	7.34

Fig. 5.1

The Public Relations Director uses this information to propose a quadratic model connecting the value of a unit of shares, in pounds, and the time, in weeks, since coming under new management.

- (i) Use Lagrange’s interpolation formula to find a quadratic model for these data, giving your answer in the form $x = at^2 + bt + c$, where, a , b and c are constants to be found. [4]
- (ii) A shareholder notes that the value of a unit was £5.73 after two weeks. Determine whether the model is consistent with this information. [2]
- (iii) According to the model, how will the value of a unit change in the long run? [1]

After seven weeks the management analysed the data and presented the findings in a difference table. The results are shown in Fig. 5.2.

t	x	Δ	Δ^2	Δ^3
0	2.21			
		1.79		
1	4.00		−0.06	
		1.73		−0.06
2	5.73		−0.12	
		1.61		−0.07
3	7.34		−0.19	
		1.42		−0.08
4	8.76		−0.27	
		1.15		−0.07
5	9.91		−0.34	
		0.81		−0.06
6	10.72		−0.40	
		0.41		
7	11.13			

Fig. 5.2

A shareholder proposes a cubic model for the data.

(iv) Explain whether the information in the difference table supports this proposal. [1]

(v) Use Newton’s forward difference interpolation formula to show that the shareholder’s model is

$$x = -0.01t^3 + 1.8t + 2.21.$$
 [4]

(vi) Identify a limitation of the shareholder’s model. [1]

6 (i) Show that the equation $0.1x^3 - 2x + 3 = 0$ has a root α , where $3 < \alpha < 4$. [1]

The method of false position is used to find α .
The spreadsheet output in Fig. 6.1 shows some of the results.

	A	B	C	D	E	F	G
1	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$		
2	0	3	-0.3	4	1.4	3.176471	-0.14789
3	1	3.176471	-0.14789	4	1.4	3.255155	-0.06114
4	2	3.255155	-0.06114	4	1.4	3.286321	-0.02345
5	3	3.286321	-0.02345	4	1.4	3.298076	-0.00873
6	4	3.298076	-0.00873	4	1.4	3.302428	-0.00322
7	5	3.302428	-0.00322	4	1.4	3.304028	-0.00118
8	6	3.304028	-0.00118	4	1.4	3.304614	-0.00043
9	7	3.304614	-0.00043	4	1.4	3.304829	-0.00016
10	8	3.304829	-0.00016	4	1.4	3.304908	-5.8E-05
11	9	3.304908	-5.8E-05	4	1.4	3.304937	-2.1E-05

Fig. 6.1

The spreadsheet formula in cell F2 is

$$= (B2 * E2 - D2 * C2) / (E2 - C2) .$$

(ii) What is being calculated in this cell? [1]

(iii) Write down a suitable spreadsheet formula for cell G2. [1]

(iv) Explain why the values in column G are necessary. [1]

The spreadsheet formula in cell B3 is

$$= IF(G2 < 0, F2, B2) .$$

(v) Explain the purpose of this formula. [1]

(vi) Write down the value of α to an accuracy that appears justified. [1]

Further analysis is carried out. This is shown in the spreadsheet output in Fig. 6.2.

	J	K	L	M
1	x_r	$x_{r+1} - x_r$		
2	3			
3	3.176471	0.176471		
4	3.255155	0.078684	0.445876	2.526632
5	3.286321	0.031166	0.396089	5.033923
6	3.298076	0.011756	0.377196	12.10282
7	3.302428	0.004352	0.370184	31.48985
8	3.304028	0.0016	0.367603	84.47262
9	3.304614	0.000587	0.366657	229.2012
10	3.304829	0.000215	0.36631	624.5195
11	3.304908	7.87E-05	0.366183	1704.301

Fig. 6.2

The spreadsheet formula in cell L4 is

= K4/K3

and the spreadsheet formula in cell M4 is

= K4/(K3^2) .

Equivalent formulae are in cells L5 to L11 and M5 to M11.

- (vii) Explain what the values in columns L and M tell you about the order of convergence of the sequence of approximations to α found using the method of false position in this case. [2]

7 Fig. 7.1 shows part of the curve $y = 2^x - x^2 + 3$.

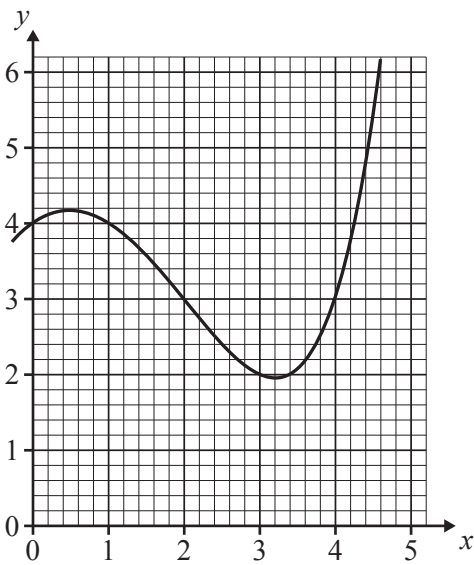


Fig. 7.1

- (i) State, with a reason, whether using the trapezium rule to approximate $\int_3^4 (2^x - x^2 + 3) dx$ will give an under-estimate or an over-estimate. [1]

The spreadsheet output in Fig. 7.2 was generated in order to evaluate $\int_3^4 (2^x - x^2 + 3) dx$.

The values in columns N and O are estimates of the integral using the midpoint rule and the trapezium rule respectively.

	M	N	O
1	n	Mn	Tn
2	1	2.063708499	2.5
3	2	2.171499782	2.281854249
4	4	2.199007398	2.226677016
5	8	2.205919726	2.212842207
6	16	2.207650029	2.209380966
7	32	2.208082743	2.208515498

Fig. 7.2

- (ii) Using only values from column N and/or column O, give a suitable spreadsheet formula for cell O3. [2]
- (iii) Use the entries in cells N7 and O7 to write down the value of $\int_3^4 (2^x - x^2 + 3) dx$ as accurately as you can, explaining your reasoning. [2]

Further analysis shows that the ratio of differences of the midpoint rule approximations and the trapezium rule approximations converge rapidly to the value expected from theory.

- (iv) Explain whether it is reasonable to assume that the ratio of differences of a sequence of approximations generated using Simpson's rule would also converge to the value predicted by theory. [1]
- (v) (A) Use the information in Fig. 7.2 to obtain two Simpson's Rule estimates of $\int_3^4 (2^x - x^2 + 3) dx$, S_{32} and S_{64} . [3]
- (B) Use extrapolation to find the value of $\int_3^4 (2^x - x^2 + 3) dx$ as accurately as you can, justifying the precision quoted. [3]

END OF QUESTION PAPER



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