SPECIMEN MATERIAL

**AQA**

Please write clearly, in block capitals.

Centre number ☐☐☐☐☐   Candidate number ☐☐☐☐

Surname _____

Forename(s) _____

Candidate signature _____

# AS
# FURTHER MATHEMATICS

Paper 1

Exam Date          Morning          Time allowed: 1 hour 30 minutes

**Materials**
For this paper you must have:
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

**Advice**
Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

**1** A reflection is represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

State the equation of the line of invariant points.

Circle your answer.

[1 mark]

$$x = 0 \qquad\qquad y = 0 \qquad\qquad y = x \qquad\qquad y = -x$$

**2** Find the mean value of $3x^2$ over the interval $1 \le x \le 3$

Circle your answer.

[1 mark]

$$8\frac{2}{3} \qquad\qquad 10 \qquad\qquad 13 \qquad\qquad 26$$

**3** Find the equations of the asymptotes of the curve $x^2 - 3y^2 = 1$

Circle your answer.

**[1 mark]**

$$y = \pm \, 3x \qquad y = \pm \frac{1}{3}x \qquad y = \pm \sqrt{3}\, x \qquad y = \pm \frac{1}{\sqrt{3}}x$$

**Turn over for the next question**

4    $A = \begin{bmatrix} 1 & 2 \\ 1 & k \end{bmatrix}$    $B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

**4 (a)** Find the value of $k$ for which matrix **A** is singular.

**[1 mark]**

**4 (b)** Describe the transformation represented by matrix **B**.

**[1 mark]**

**4 (c) (i)** Given that **A** and **B** are both non-singular, verify that $A^{-1}B^{-1} = (BA)^{-1}$.

**[4 marks]**

**4 (c) (ii)** Prove the result $\mathbf{M}^{-1}\mathbf{N}^{-1} = (\mathbf{NM})^{-1}$ for all non-singular square matrices $\mathbf{M}$ and $\mathbf{N}$ of the same size.

**[4 marks]**

**Turn over for the next question**

**5** The region bounded by the curve with equation $y = 3 + \sqrt{x}$ , the $x$-axis and the lines $x = 1$ and $x = 4$ is rotated through $2\pi$ radians about the $x$-axis.

Use integration to show that the volume generated is $\dfrac{125\pi}{2}$

**[5 marks]**

**6 (a)**  Use the definitions of $\sinh x$ and $\cosh x$ in terms of $\mathrm{e}^x$ and $\mathrm{e}^{-x}$ to show that

$x = \dfrac{1}{2} \ln \left( \dfrac{1+t}{1-t} \right)$ where $t = \tanh x$

**[4 marks]**

**Question 6 continues on the next page**

**Turn over ▶**

**6 (b) (i)** Prove $\cosh^3 x = \dfrac{1}{4}\cosh 3x + \dfrac{3}{4}\cosh x$

**[4 marks]**

**6 (b) (ii)** Show that the equation $\cosh 3x = 13\cosh x$ has only one positive solution.

Find this solution in exact logarithmic form.

**[4 marks]**

**Turn over for the next question**

**7**    A lighting engineer is setting up part of a display inside a large building. The diagram shows a plan view of the area in which he is working.

He has two lights, which project narrow beams of light.

One is set up at a point 3 metres above the point *A* and the beam from this light hits the wall 23 metres above the point *D*.

The other is set up 1 metre above the point *B* and the beam from this light hits the wall 29 metres above the point *C*.
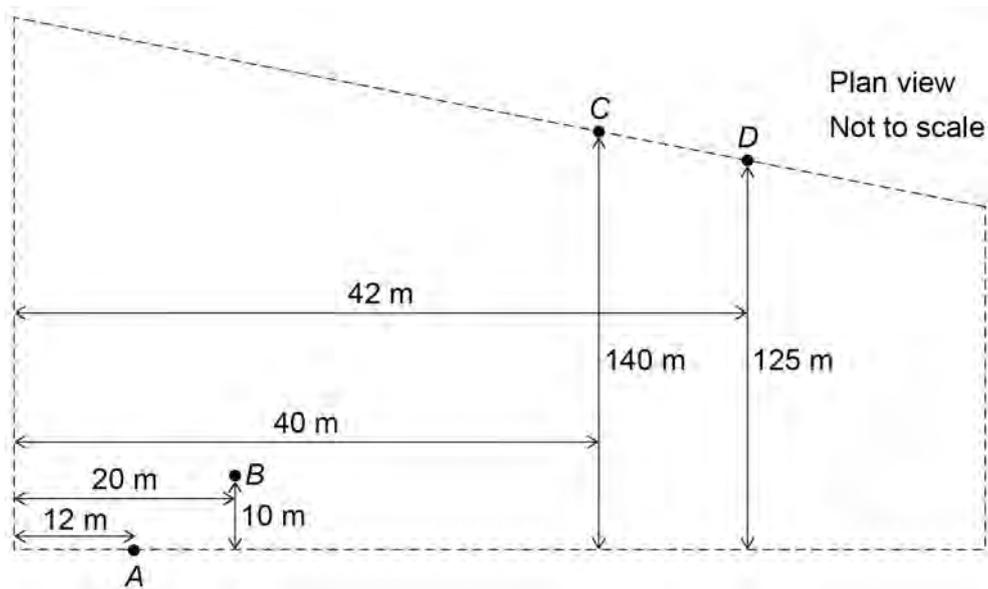


**7 (a)**    By creating a suitable model, show that the beams of light intersect.
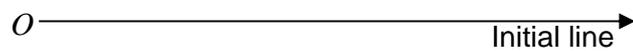
**[6 marks]**

**7 (b)**   Find the angle between the two beams of light.

[3 marks]

**7 (c)**   State one way in which the model you created in part **(a)** could be refined.

[1 mark]

**8**    A curve has polar equation $r = 3 + 2\cos\theta$, where $0 \le \theta < 2\pi$

**8 (a) (i)** State the maximum and minimum values of $r$.

**8 (a) (ii)** Sketch the curve.

$O$ —————————————————▶
                         Initial line

**8 (b)** The curve $r = 3 + 2 \cos \theta$ intersects the curve with polar equation $r = 8\cos^2 \theta$, where $0 \leq \theta < 2\pi$

Find all of the points of intersection of the two curves in the form $[r, \theta]$.
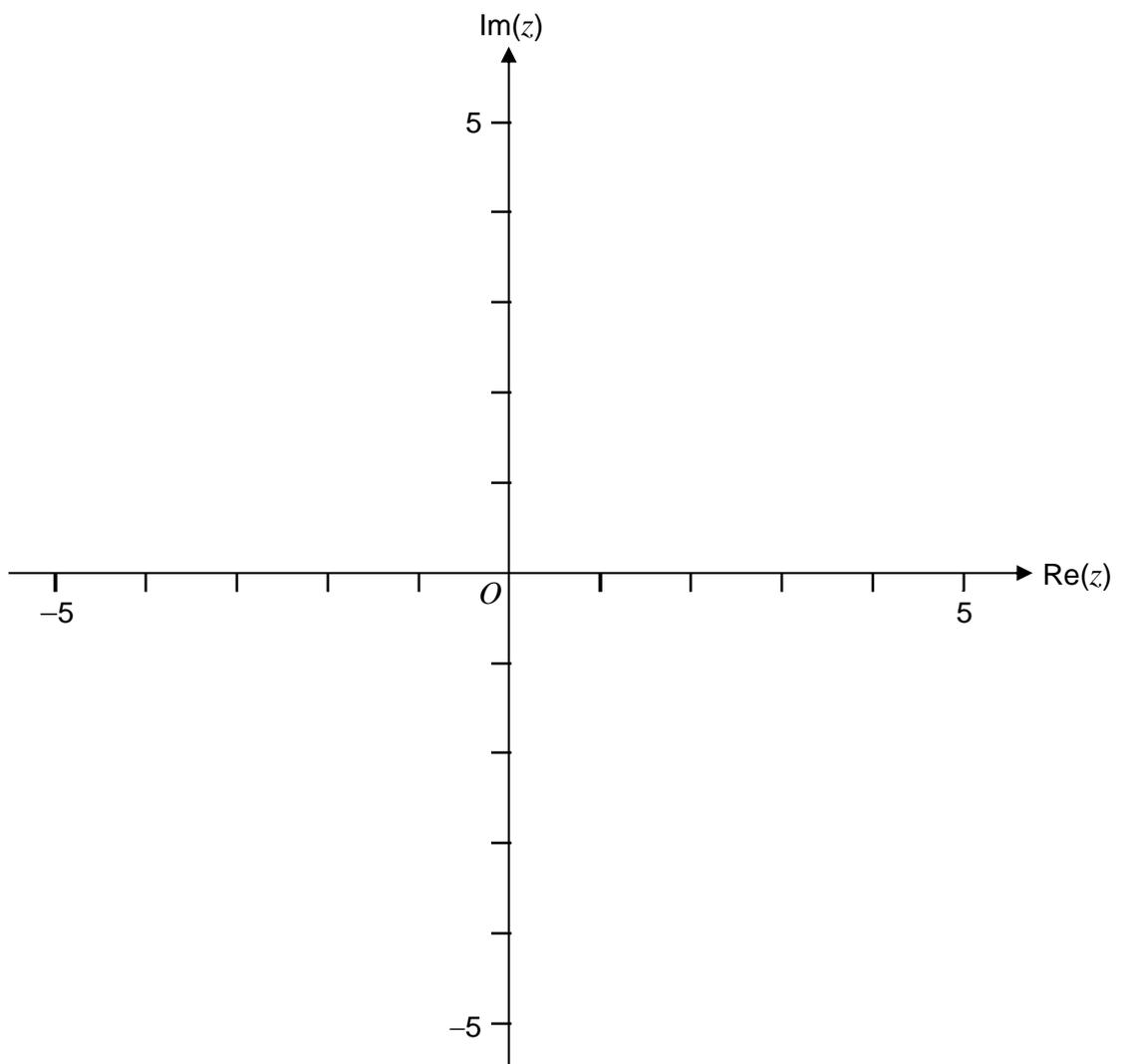
**[6 marks]**

**9 (a)** Sketch on the Argand diagram below, the locus of points satisfying the equation $|z - 2| = 2$

**[2 marks]**

**9 (b)** Given that $\left|z - 2\right| = 2$ and $\arg(z - 2) = -\dfrac{\pi}{3}$ , express $z$ in the form $a + b\mathrm{i}$ , where $a$ and $b$ are real numbers.

**[3 marks]**

**Turn over for the next question**

**10 (a)** Prove that

$$6 + 3\sum_{r=1}^{n} (r + 1)(r + 2) = (n + 1)(n + 2)(n + 3)$$

**[6 marks]**

**10 (b)** Alex substituted a few values of $n$ into the expression $(n + 1)(n + 2)(n + 3)$ and made the statement:

"For all positive integers $n$,

$$6 + 3\sum_{r=1}^{n} (r + 1)(r + 2)$$

is divisible by 12."

Disprove Alex's statement.

**[2 marks]**

**Turn over for the next question**

**11**  The equation  $x^3 - 8x^2 + cx + d = 0$  where $c$ and $d$ are real numbers, has roots $\alpha, \beta, \gamma$.

When plotted on an Argand diagram, the triangle with vertices at $\alpha, \beta, \gamma$ has an area of 8.

Given $\alpha = 2$, find the values of $c$ and $d$.

Fully justify your solution.

**[5 marks]**

**12** A curve, $C_1$ has equation $y = f(x)$, where $f(x) = \dfrac{5x^2 - 12x + 12}{x^2 + 4x - 4}$

The line $y = k$ intersects the curve, $C_1$

**12 (a) (i)** Show that $(k + 3)(k - 1) \geq 0$

**[5 marks]**

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**Question 12 continues on the next page**

**12 (a) (ii)** Hence find the coordinates of the stationary point of $C_1$ that is a maximum point.

**[4 marks]**

**12 (b)** Show that the curve $C_2$ whose equation is $y = \dfrac{1}{f(x)}$ , has no vertical asymptotes.

**[2 marks]**

**12 (c)** State the equation of the line that is a tangent to both $C_1$ and $C_2$.

**[1 mark]**

**END OF QUESTIONS**