



Please write clearly, in block capitals.

Centre number

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Candidate number

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Surname

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Forename(s)

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Candidate signature

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# A-level

# FURTHER MATHEMATICS

## Paper 3 - Discrete

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Exam Date

Morning

Time allowed: 2 hours

### Materials

For this paper you must have:

- You must ensure you have the other optional question paper/answer booklet for which you are entered (**either** Mechanics **or** Statistics). You will have 2 hours to complete both papers.
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

### Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

### Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.

You do not necessarily need to use all the space provided.

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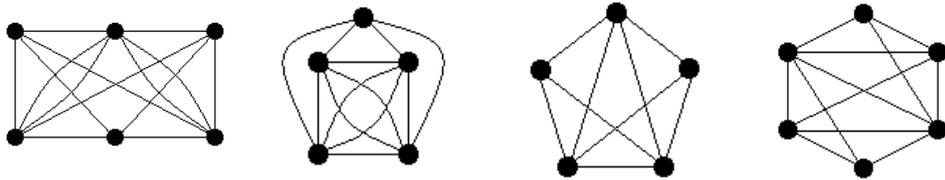
Answer **all** questions in the spaces provided.

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1 Which of the following graphs is **not** planar?

Circle your answer.

[1 mark]



2 The set  $\{1, 2, 4, 8, 9, 13, 15, 16\}$  forms a group under the operation of multiplication modulo 17.

Which of the following is a generator of the group?

Circle your answer.

**[1 mark]**

4

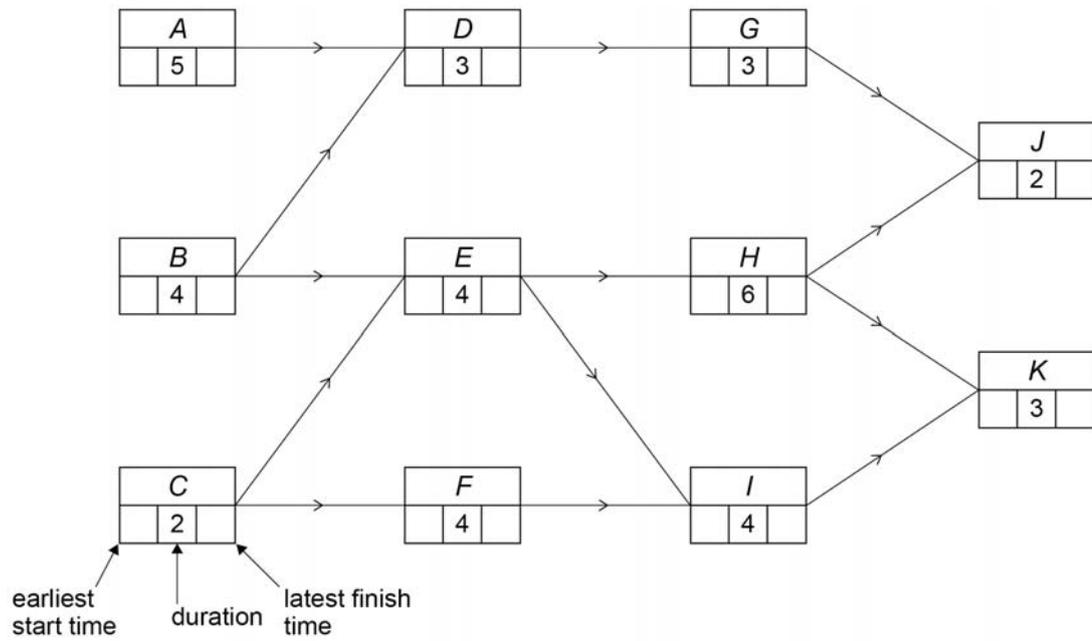
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- 3 Deva Construction Ltd undertakes a small building project. The activity network for this project is shown below in **Figure 1**, where each activity's duration is given in hours.

**Figure 1**



- 3 (a) Complete the activity network for the building project.

[2 marks]

- 
- 3 (b)** Deva Construction Ltd is able to reduce the duration of a single activity to 1 hour by using specialist equipment.

State, with a reason, which activity should have its duration reduced to 1 hour in order to minimise the completion time for the building project.

**[3 marks]**

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- 3 (c)** State one limitation in the building project used by Deva Construction Ltd.

Explain how this limitation affects the project.

**[2 marks]**

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4 Optical fibre broadband cables are being installed between 5 neighbouring villages.

The distance between each pair of villages in metres is shown in the table.

	<b>Alvanley</b>	<b>Dunham</b>	<b>Elton</b>	<b>Helsby</b>	<b>Ince</b>
<b>Alvanley</b>	-	2000	4000	750	5500
<b>Dunham</b>	2000	-	2500	2250	4000
<b>Elton</b>	4000	2500	-	3000	1250
<b>Helsby</b>	750	2250	3000	-	4250
<b>Ince</b>	5500	4000	1250	4250	-

The company installing the optical fibre broadband cables wishes to create a network connecting each of the 5 villages using the minimum possible length of cable.

Find the minimum length of cable required.

**[3 marks]**

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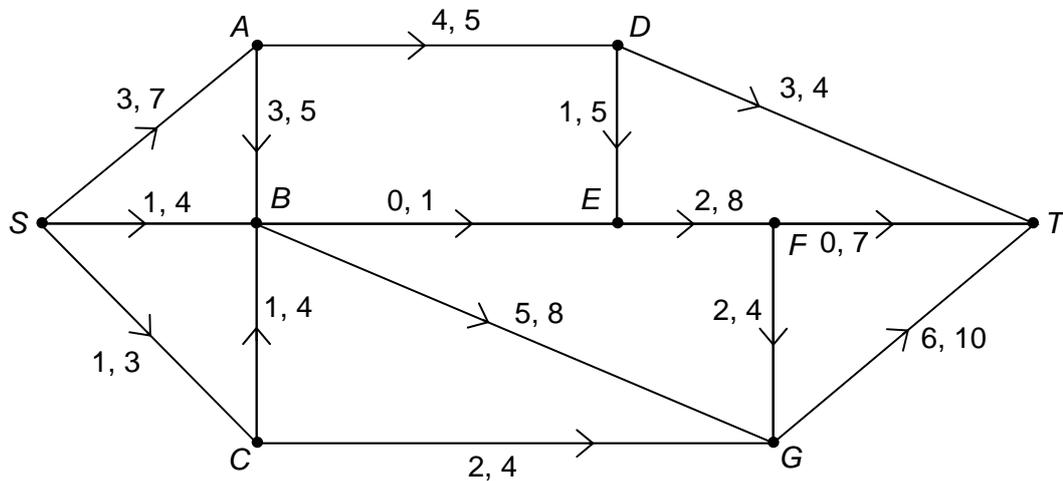
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- 6** The network shows a system of pipes, where  $S$  is the source and  $T$  is the sink.
- The lower and upper capacities, in litres per second, of each pipe are shown on each arc.



- 6 (a)** There is a feasible flow from  $S$  to  $T$ .

- 6 (a) (i)** Explain why arc  $AD$  must be at its lower capacity.

[1 mark]

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- 6 (a) (ii)** Explain why arc  $BE$  must be at its upper capacity.

[1 mark]

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- 6 (b)** Explain why a flow of 11 litres per second through the network is impossible.

[1 mark]

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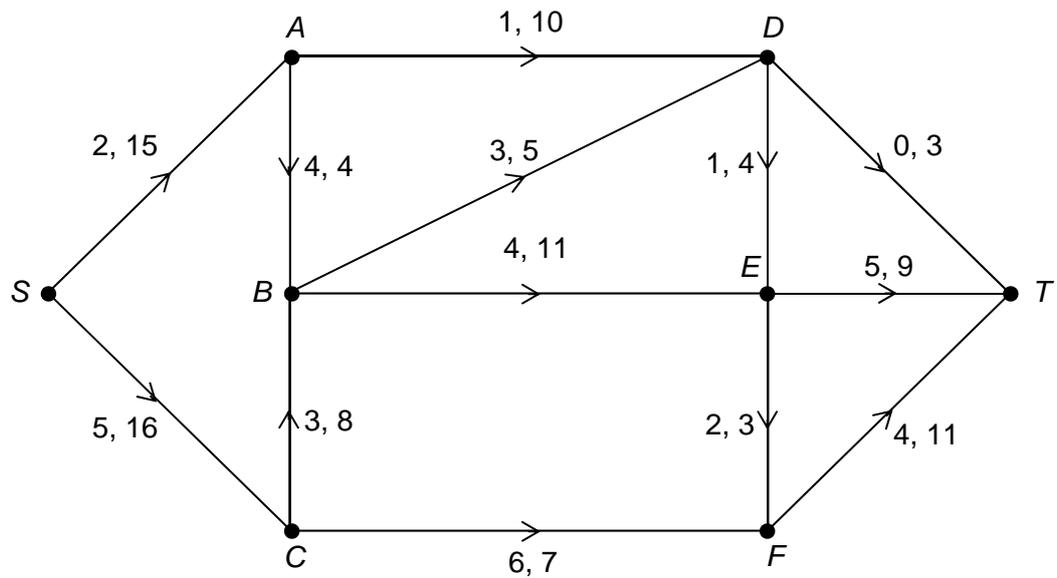


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- 6 (c) The network in **Figure 2** shows a second system of pipes, where  $S$  is the source and  $T$  is the sink.

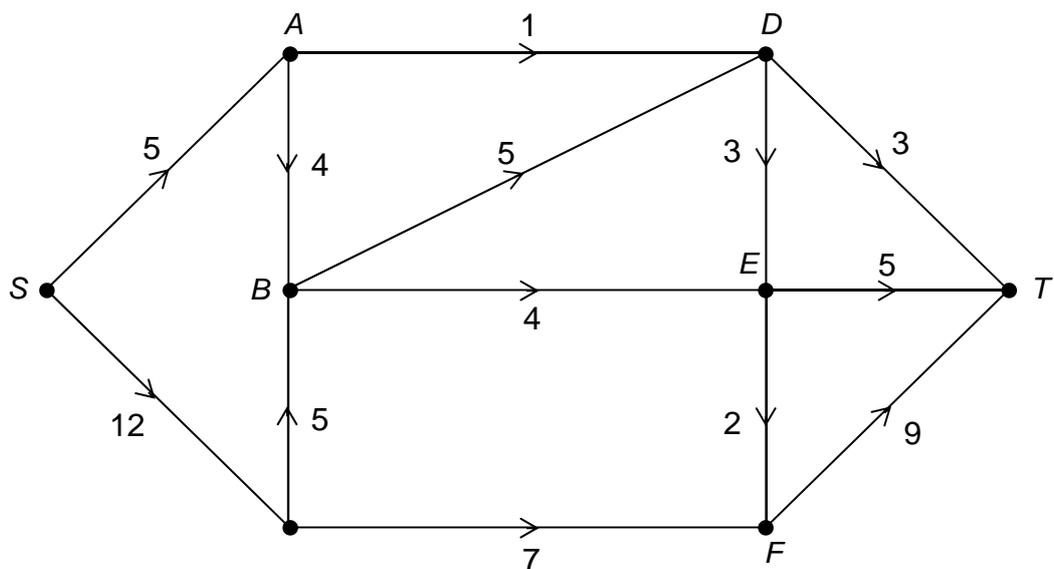
The lower and upper capacities, in litres per second, of each pipe are shown on each edge.

**Figure 2**



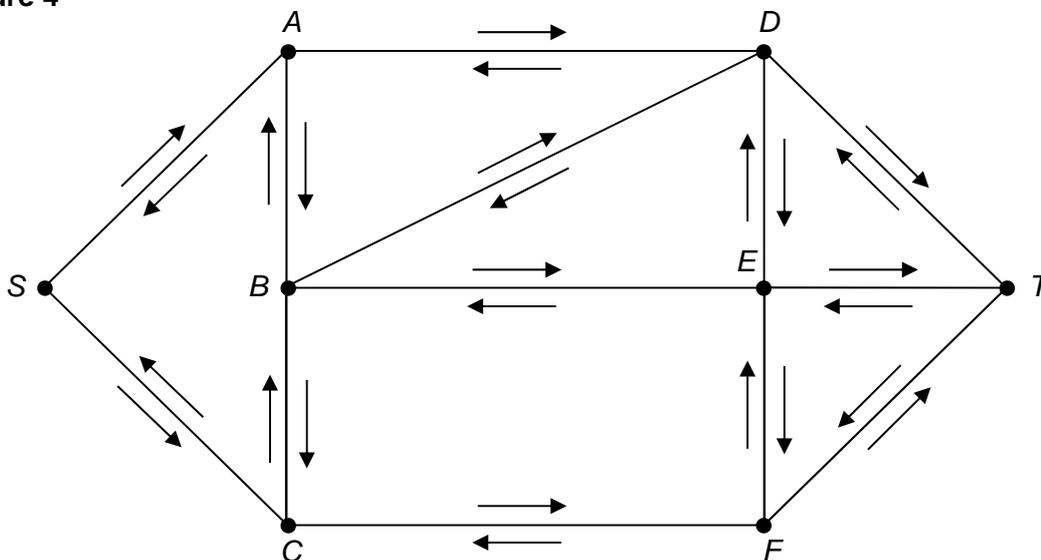
**Figure 3** shows a feasible flow of 17 litres per second through the system of pipes.

**Figure 3**



6 (c) (i) Using **Figures 2** and **3**, indicate on **Figure 4** potential increases and decreases in the flow along each arc. [2 marks]

**Figure 4**



6 (c) (ii) Use flow augmentation on **Figure 4** to find the maximum flow from S to T.

You should indicate any flow augmenting paths clearly in the table below and modify the potential increases and decreases of the flow on **Figure 4**. [3 marks]

Augmenting Path	Flow

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**6 (c) (iii)** Prove the flow found in part **(d) (ii)** is maximum.

**[1 mark]**

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**6 (c) (iv)** Due to maintenance work, the flow through node  $E$  is restricted to 9 litres per second.

Interpret the impact of this restriction on the maximum flow through the system of pipes.

**[2 marks]**

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**Turn over for the next question**

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