

SPECIMEN MATERIAL

Please write clearly, in block capitals.	
Centre number	Candidate number
Surname	
Forename(s)	
Candidate signature	

# A-level MATHEMATICS

Paper 1

Exam Date Morning Time allowed: 2 hours

#### **Materials**

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

#### **Instructions**

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer all questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

#### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

### **Advice**

Unless stated otherwise, you may quote formulae, without proof, from the booklet. You do not necessarily need to use all the space provided.

## Answer all questions in the spaces provided.

1 Find the gradient of the line with equation 2x + 5y = 7

Circle your answer.

[1 mark]

$$\frac{2}{5}$$

$$-\frac{2}{5}$$

$$-\frac{5}{2}$$

2 A curve has equation  $y = \frac{2}{\sqrt{x}}$ 

Find  $\frac{\mathrm{d}y}{\mathrm{d}x}$ 

Circle your answer.

[1 mark]

$$\frac{\sqrt{x}}{3}$$

$$\frac{1}{x\sqrt{x}}$$

$$-\frac{1}{x\sqrt{x}}$$

$$-\frac{1}{2x\sqrt{x}}$$

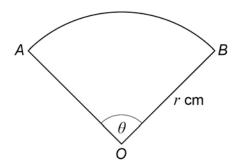
When $\theta$ is small, find an approximation for $\cos 3\theta + \theta \sin 2\theta$ , giving your answer form $a+b\theta^2$	in the
	marks]

4 
$$p(x) = 2x^3 + 7x^2 + 2x - 3$$

4 (a)	Use the factor theorem to prove that $x+3$ is a factor of $p(x)$	[2 marks]

Simplify the expression		[4
		<b>.</b>

5 The diagram shows a sector AOB of a circle with centre O and radius r cm.



The angle AOB is  $\theta$  radians

The sector has area 9 cm<sup>2</sup> and perimeter 15 cm.

5 (a) Show that r satisfies the equation  $2r^2 - 15r + 18 = 0$ 

			[4 marks]

5

Find the value of $ heta$	heta . Explain why it is	the only possibl	e value.	[4 marks

6		Sam goes on a diet. He assumes that his mass, $m$ kg after $t$ days, decreases a that is inversely proportional to the cube root of his mass.	nt a rate
6	(a)	Construct a differential equation involving $m$ , $t$ and a positive constant $k$ to mo situation.	[3 marks]
6	(b)	Explain why Sam's assumption may not be appropriate.	[1 mark]

Find the values of k for	which the equation (2)	$(k-3)x^2 - kx + (k-1) =$	
			[4

8 (a)	Given that $u = 2^x$ , write down an expression for	$\frac{\mathrm{d}u}{\mathrm{d}x}$
8 (a)	Given that $u = 2^x$ , write down an expression for	$\frac{\mathrm{d}u}{\mathrm{d}x}$

[1 mark]

8 (b) Find the exact value of	$\int_0^1 2^x \sqrt{3 + 2^x} \ \mathrm{d}x$
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Fully justify your answer.

[6 marks]

2 marks]
2 marks]
4 marks]

[2

10	The function $f$ is defined by $f\left(x\right)=4+3^{-x} \ \ , \ \ x\in\mathbb{R}$	
10 (a)	Using set notation, state the range of f	[2 marks]
10 (b)	The inverse of $f$ is $f^{-1}$	
10 (b) (i)	Using set notation, state the domain of f <sup>-1</sup>	[1 mark]
10 (b) (ii	) Find an expression for $f^{-1}(x)$	[3 marks]

10	(c)	The function g is defined by $g\left(x\right)=5-\sqrt{x}\  \  ,\  \left(x\in\mathbb{R}:x>0\right)$	
10	(c) (i)	Find an expression for gf (x)	[1 mark]
10	(c) (ii)	Solve the equation $\operatorname{gf}(x) = 2$ , giving your answer in an exact form.	[3 marks]

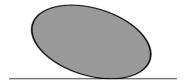
11	A circle with centre C has equation $x^2 + y^2 + 8x - 12y = 12$	
11 (a)	Find the coordinates of C and the radius of the circle.	[3 marks

11 (b)	The points P and Q lie on the circle.	
11 (5)	The origin is the midpoint of the chord <i>PQ</i> .	
	Show that $PQ$ has length $n\sqrt{3}$ , where $n$ is an integer.	[5 marks]
		[o marks]

A sculpture formed from a prism is fixed on a horizontal platform, as shown in the diagram.

The shape of the cross-section of the sculpture can be modelled by the equation  $x^2 + 2xy + 2y^2 = 10$ , where x and y are measured in metres.

The x and y axes are horizontal and vertical respectively.



Find the maximum vertical height above the platform of the sculpture.	[8 marks]

Prove the ide	Positive $\cot^2 \theta - \cos^2 \theta$	$\theta = \cot^2 \theta \cos^2 \theta$	a	
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Prove the ide	entity $\cot^2 \theta - \cos^2$	$\theta \equiv \cot^2 \theta \cos^2 \theta$	9	

14	An open-topped fish tank is to be made for an aquarium.
	It will have a square horizontal base, rectangular vertical sides and a volume of 60 m <sup>3</sup>
	The materials cost:
	• £15 per m <sup>2</sup> for the base
	• £8 per m <sup>2</sup> for the sides.

**14 (a)** Modelling the sides and base of the fish tank as laminae, use calculus to find the height of the tank for which the overall cost of the materials has its minimum value.

Fully justify your answer.	[8 marks]

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14	(b) (i)	In reality, the thickness of the base and sides of the tank is 2.5 cm	
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	( ) ( )	Briefly explain how you would refine your modelling to take account of the thickness	
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15	The height $x$ metres, of a column of water in a fountain display satisfies the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{8\sin 2t}{3\sqrt{x}}$ , where $t$ is the time in seconds after the display begins.
15 (a)	Solve the differential equation, given that initially the column of water has zero height. Express your answer in the form $x = f(t)$ [7 marks]

45 (b)	
15 (b)	Find the maximum height of the column of water, giving your answer to the nearest cm.
15 (b)	
15 (b)	Find the maximum height of the column of water, giving your answer to the nearest cm.  [1 mark]
15 (b)	

16 16 (a)	A student argues that when a rational number is multiplied by an irrational number the result will always be an irrational number.  Identify the rational number for which the student's argument is not true.
	[1 mark]
16 (b)	Prove that the student is right for all rational numbers other than the one you have identified in part (a).  [4 marks]

Using differentiation from first principles find the exact value of $f'\left(\frac{\pi}{6}\right)$	
Fully justify your answer.	
	[6

There are no questions printed on this page

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