AQA Qualifications

Level 2 Certificate

Further Mathematics

Paper 1 83601

Mark scheme

83601

June 2015

Version/Stage: 1 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q	Answer	Mark	Comments
1	(12, −14)	B2	B1 for each coordinate SC1 for (−14, 12)
2(a)	Q	B1	
2(b)	R	B1	
2(c)	P	B1	
2(d)	S	B1	
2(e)	P and S	B1	
3	$6x + 4x$ **or** $3 - 2$	M1	oe
	$x > \frac{1}{10}$	A1	oe
	Additional Guidance		
	If they divide by 2 first then $3x + 2x$ **or** $1.5 - 1$ score M1 Treating it as an equation, solving to get $x = \frac{1}{10}$ will be M0 A0 unless M1 already gained or they recover the inequality at the end and write $x > \frac{1}{10}$ for 2 marks		
4(a)	$2x - 5$	B2	B1 for one correct term $2x - 5 + c$ scores B1 fw is B1 only eg $2x - 5 = -3x$
4(b)	their $(2x - 5) = 1$	M1	
	$x = 3$ and $y = -6$	A1ft	ft from a linear expression eg $2x + 5 = 1$ will lead to $x = -2$, $y = 14$, eg $2x = 1$ will lead to $x = {}^1/_2$, $y = -2{}^1/_4$

	Additional Guidance		

5(a)	−2	B1	

5(b)	$-3 + 2 \times$ their k or $- 3x + 2 \times$ their kx	M1	Allow $2k - 3$ or $2kx - 3x$ either of which might be seen in their expansion in part (a)
	−7	A1ft	$-7x$ is M1 A0
	Additional Guidance		

6	**Alternative method 1**		
	$(0.8 \times 5x)$ **or** $4x$	M1	oe
	$(1.3 \times 2x)$ **or** $2.6x$	M1	oe
	their $4x -$ their $2.6x = 35$	M1dep	oe dep on at least M1 M0 or M0 M1
	25	A1	
	Alternative method 2		
	Two numbers in the ratio 5 : 2 and one correctly evaluated increase or decrease	M1	
	Both increase and decrease calculations correctly evaluated	M1dep	dep on the first M1
	Trial seen with 125 red and 50 blue or 100 red and 65 blue seen	M1dep	dep on both previous M marks
	25	A1	
	Additional Guidance		

7	Sight of ab^2 or cb^2 or ad^2 or cd^2 or $(3x......)(x ......)(x.......)$	M1	
	Two or three correct coefficients	A1	which may be embedded
	$a = 3$, $b = 1$, $c = 2$, $d = 7$	A1	which may be embedded SC2 for $(3x - 2)(x^2 - 49)$ SC1 for $(3x......)(x^2 - 49)$
	Additional Guidance		

8	**Alternative method 1**		
	common denominator $(x + 4)(x - 6)$	M1	oe allow $(x + 4)(x - 6)^2$
	(numerator) $5x - 3(x + 4)$	M1	oe allow $5x(x - 6) - 3(x + 4)(x - 6)$
	$\frac{2x-12}{(x+4)(x-6)}$	A1	$\frac{(2x-12)(x-6)}{(x+4)(x-6)^2}$
	$\frac{2}{(x+4)}$	A1	
	Alternative method 2		
	remove common factor of $\frac{1}{(x-6)}$ **and** common denominator $(x + 4)$	M1	
	numerator $5x - 3(x + 4)$	M1	
	$\frac{2x-12}{(x+4)(x-6)}$	A1	
	$\frac{2}{(x+4)}$	A1	
	Additional Guidance		

9	$3a - b$ **or** $2a + b$ seen	M1	oe
	$3a - b = b$	M1	oe
	$2a + b = a + 1$	M1	oe
	$a = \frac{2}{5}$	A1	
	$b = \frac{3}{5}$	A1	
	Additional Guidance		

10(a)	$3x^2 - 4x - 4 \quad = 0$ or < 0 or ≤ 0	M1	
	$(3x + 2)(x - 2)$ $(= 0$ or < 0 or $\le 0)$	M1	$(3x \pm a)(x \pm b)$ where $ab = \pm 4$ scores M1
	$-\frac{2}{3}$ and 2 seen as solutions	A1	
	$-\frac{2}{3} < x < 2$	A1	condone $-\frac{2}{3} \le x \le 2$ SC1 for either $x < 2$ or $x \le 2$ seen
	Additional Guidance		
	The 2nd M1 is for an attempt to factorise, they must have $3x$ and x but can have 1 and 4 for the values of a and b		
	Seeing solutions to the quadratic (whether correct or not) implies the first M mark ... they might not formally state $3x^2 - 4x - 4 = 0$		

10(b)	substitutes $x = 1$ correctly into the expression for $\frac{dy}{dx}$	M1	
	$\left(\frac{dy}{dx} =\right)$ −5	A1	
	gradient normal = $\frac{1}{5}$	M1	ft their −5 if first M1 earned
	$y - -2 = \frac{1}{5}(x - 1)$ or $-2 = \frac{1}{5}(1) + c$	M1dep	ft their gradient of the normal dep on both previous M marks earned
	$y = \frac{1}{5}x - 2\frac{1}{5}$	A1 ft	oe ... it need not be in $y = mx + c$ form
	Additional Guidance		
	If they do not get −5 for the gradient of the tangent, they can still score 4 of the 5 marks if they follow through correctly with their value for the gradient of the normal, but it must be their gradient of the normal, not the gradient of the tangent.		
	If you see $y = -5x + 3$, they have given us the equation of the tangent and they score M1 A1 only.		

11	(f(x) =) $-x^2$	B1	
	(f(x) =) −4	B1	
	(f(x) =) $4x - 16$	B1	
	All domains correctly paired with the functions using the correct notation for the domains eg $-1 \le x < 2$	B1	Accept use of $<$ or $\le$ do not accept (eg) $-1 \le -x^2 < 2$
	Additional Guidance		

12	$\frac{3xy}{x+y} = 16$	B1	allow 4^2
	$3xy = 16(x + y)$ or $3xy = 16x + 16y$	M1	allow 'their 16' as obtained in first step
	$3xy - 16y = 16x$ or $y(3x - 16) = 16x$	M1	ft with 'their 16'
	$y = \frac{16x}{3x - 16}$	A1	oe eg $y = \frac{-16x}{16 - 3x}$
	Additional Guidance		
	They must get 4^2 or 16 to score B1 but 'their 16' is good enough to score the two M marks. For A1 it has to say 16, 4^2 is not acceptable		
	... any incorrect fw will lose the A mark		

13	**Alternative method 1**		
	$a = -5$	B1	
	$b = 25 - a$ or $x^2 - 10x + 25$ seen or $x^2 - 5x - 5x + 25$ seen	M1	
	$b = 30$	A1ft	ft using $b = 25 - a$ if M1 earned
	Alternative method 2		
	$a = -5$	B1	
	$(x + a)^2 - a^2\ (+b)$ or $b - a^2 = -a$ or $b = a^2 - a$	M1	
	$b = 30$	A1ft	ft using b = their $(a^2 - a)$
	Alternative method 3		
	$a = -5$	B1	
	Substituting one value of x into the identity, correctly, to give an equation connecting a and b	A1	eg $x = 0$, $a + b = 25$ $x = 1$, $3a + b = 15$ $x = 2$, $5a + b = 5$ $x = 3$, $7a + b = -5$
	$b = 30$	A1	
	Alternative method 4		
	Substituting two values of x into the identity, correctly, to give two simultaneous equations	M1	eg $x = 0$, $a + b = 25$ $x = 1$, $3a + b = 15$ $x = 2$, $5a + b = 5$ $x = 3$, $7a + b = -5$
	$a = -5$	A1	
	$b = 30$	A1	

14	$\frac{5\sqrt{2}\,(3\sqrt{6}+7)}{(3\sqrt{6}-7)(3\sqrt{6}+7)}$	M1	
	Numerator = $15\sqrt{2}\sqrt{6} + 35\sqrt{2}$	M1dep	oe eg $15\sqrt{12} + 35\sqrt{2}$ or $5\sqrt{2} \times 3\sqrt{6} + 35\sqrt{2}$ dep on the first M1
	Denominator = 54 − 49	M1dep	dep on the first M1
	$3\sqrt{12} + 7\sqrt{2}$	A1	oe eg $6\sqrt{3} + 7\sqrt{2}$
	$\sqrt{108} + \sqrt{98}$	A1	
	Additional Guidance		

15(a)	tangents (from an external point) are equal in length	B1	or $RT = RZ$
	angle RZT = angle $RTZ = b$ since they are base angles of an isosceles triangle	B1	mention of isosceles triangle is sufficient
	Additional Guidance		
	'tangents equal' and 'isosceles' are minimum requirements		

15(b)	**Alternative method 1**		
	angle *YXT* = a	B1	
	angle in alternate segment	B1dep	dep on the first B1 the reason why angle *YXT* = a must be stated for this mark to be awarded (angle *YXT* = angle *YTR*)
	angle *XTW* = angle *YXT* + angle *RZT* = $a + b$ **or** working leading to angle *XTY* = $180 - 2a - 2b$ **and** angle *XTW* = $180 - a - b - (180 - 2a - 2b) = a + b$	B1dep	(exterior angle property in triangle *XZT*) dep on the first B1 (angles on a straight line = 180 at point *T*)
	Alternative method 2		
	working leading to angle *XYT* = $a + 2b$ **and** angle *XTS* = $a + 2b$	B1	
	angle in alternate segment	B1dep	dep on the first B1 the reason why angle *XTS* = $a + 2b$ must be stated for this mark to be awarded (angle *XYT* = angle *XTS*)
	angle *WTS* = b **and** angle *XTW* = $a + 2b - b = a + b$	B1dep	(angle *WTS* is vert. opposite to angle *RTZ*) dep on the first B1
	Alternative method 3		
	Join *XW* angle *WTS* = b **and** angle *WXT* = b	B1	(angle *WTS* is vertically opposite to angle *RTZ*
	angle in alternate segment	B1dep	dep on the first B1 the reason why angle WXT = b must be stated for this mark to be awarded (angle *WXT* = angle *WTS*)
	working leading to angle *XWT* = $180 - a - 2b$ **and** angle *XTW* = $180 - b - (180 - a - 2b)$ $= a + b$	B1dep	dep on the first B1 (angle *XWT* is opposite angle *XYT* in cyclic quad *XYTW* and angle *XYT*= $a + 2b$)

	Additional Guidance
	they must use the alternate segment theorem ... the last B1 mark cannot be awarded unless the first B mark has been awarded, whichever scheme they use

16	$x^2(x^2 - x - 2)$ or $(x - 2)(x^3 + x^2)$ or $(x + 1)(x^3 - 2x^2)$ or $(x^2 + x)(x^2 - 2x)$	M1	
	$x^2(x + 1)(x - 2)$ seen in numerator	M1	allow $x(x + 1)x(x - 2)$ or $(x + 1)x^2(x - 2)$
	$(x^2 - 1)(x^2 - 4)$ seen in denominator	M1	
	$(x + 1)(x - 1)$ **or** $(x + 2)(x - 2)$	M1dep	dep on previous M mark
	$\frac{x^2}{(x-1)(x+2)}$	A1	accept $\frac{x^2}{x^2 + x - 2}$
	Additional Guidance		
	... any incorrect fw will lose the A mark		

17	**Alternative method 1**		
	LHS $\frac{2\sin^2\theta}{\cos^2\theta} + 1$	M1	
	$\frac{2\sin^2\theta + \cos^2\theta}{\cos^2\theta}$	M1	$\frac{\sin^2\theta + \sin^2\theta + \cos^2\theta}{\cos^2\theta}$
	$\frac{2\sin^2\theta + 1 - \sin^2\theta}{1 - \sin^2\theta}$ $\equiv \frac{\sin^2\theta + 1}{1 - \sin^2\theta}$	A1	$\frac{\sin^2\theta + 1}{1 - \sin^2\theta}$
	Alternative method 2		
	RHS $\frac{1 + \sin^2\theta}{\cos^2\theta}$	M1	accept $\frac{\cos^2\theta + \sin^2\theta + \sin^2\theta}{\cos^2\theta + \sin^2\theta - \sin^2\theta}$
	$\frac{\cos^2\theta + \sin^2\theta + \sin^2\theta}{\cos^2\theta}$	M1	or $\frac{\cos^2\theta + 2\sin^2\theta}{\cos^2\theta}$
	$\frac{\cos^2\theta}{\cos^2\theta} + \frac{2\sin^2\theta}{\cos^2\theta}$ $\equiv 1 + 2\tan^2\theta$	A1	
	Additional Guidance		
	Although not good practice, they might try to start from both sides and 'meet in the middle' ... the expressions given for the second M mark in each of the two schemes tries to take this into account.		