AQA

Level 2 Certificate
Further Mathematics

Paper 2 83602

Mark scheme

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Version 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q	Answer	Mark	Comments
1	$d = 12$ or $r = 6$ or $r = \sqrt{36}$	M1	oe eg $r^2 = 36$
	$x^2 + y^2 = 36$ or $x^2 + y^2 = 6^2$	A1	oe SC1 $x^2 + y^2 =$ their r^2
	Additional Guidance		
	M1 Must be clear that 12 is the diameter or that 6 or $\sqrt{36}$ is the radius		
	SC1 Must be clear that they are using their r eg1 $r = 12$ and $x^2 + y^2 = 144$ eg2 $x^2 + y^2 = 144$ eg3 $r = 6\pi$ and $x^2 + y^2 = (6\pi)^2$ eg4 $r = 6\pi$ and $x^2 + y^2 = 6\pi^2$		SC1 M0 A0 SC1 M0 A0
	$x^2 + y^2 = 12$ ($d = 12$ or $r = 6$ not seen) $x^2 + y^2 = 6$ ($r = 6$ or $d = 12$ not seen)		M0 A0 M0 A0
	$(x - 0)^2 + (y - 0)^2 = 36$ or $(x + 0)^2 + (y + 0)^2 = 6^2$		M1 A1
	(linear term)^2 + (linear term)^2 = 36 (or 6^2) implies M1 A0 eg1 $a^2 + b^2 = 36$ eg2 $(x - 6)^2 + y^2 = 6^2$		M1 A0 M1 A0
	Ignore subsequent incorrect evaluation of 6^2 after $x^2 + y^2 = 6^2$ seen $x^2 + y^2 = 6^2$ (in working) $x^2 + y^2 = 12$ (on answer line)		M1 A1

2	2 : 1	B2	B1	Ratio equivalent to 2 : 1 or 1 : 2 SC1 Ratio seen that is correctly converted to simplest form
	Additional Guidance			
	Equivalent ratios may involve decimals or fractions eg 1.8 : 0.9			B1
	Equivalent ratios must be a pair of values or a pair of single term expressions in the same variable eg1 36 : 18 eg2 $6b : 3b$ eg3 20 – 2 : 9			B1 B1 B0
	For B1 equivalent ratios to 2 : 1 can be seen as fractions eg $\dfrac{18}{9}$			B1

3	**Alternative method 1**			
	$5p - -10$ or $5p + 10$ or $-10 - 5p$ or $5p = 20$ or $5p = -40$	M1	oe	
	4	A1		
	–8	A1		
	Alternative method 2			
	$\dfrac{-10 + 30}{5}$ or $\dfrac{-10 - 30}{5}$	M1	oe	
	4	A1		
	–8	A1		
	Additional Guidance			
	Alt 1 M1 may be seen within Pythagoras (which does not have to be correct) eg $(5p + 10)^2 + (2 - 2)^2 = 30$ [$5p + 10$ seen]		M1	
	Only one value correct is likely to score 2 marks			

	Alternative method 1		
	$1 - a + 2a = 1 + a$ and $3(1 + a) = 3 + 3a$	B1	oe Allow $3(1 - a + 2a) = 3 + 3a$ if no incorrect working seen
	Alternative method 2		
	$\dfrac{3 + 3a}{3} = 1 + a$ and $1 + a - 2a = 1 - a$	B1	oe
	Additional Guidance		
	Allow $1a$ for a throughout		
4(a)	Alt 1 $a + 2a = 3a$ $3 \times 1 = 3$ $3 + 3a$ (incorrect working seen)		B0
	Alt 1 $-a + 2a = a$ $3 \times a = 3a$ $3 \times 1 = 3$ $3 + 3a$		B1
	$3(1 + a) = 3 + 3a$		B0
	Alt 1 $1 - a + 2a = 1 + a$ $3 \times 1 + a = 3 + 3a$ (incorrect working seen)		B0
	Alt 1 $1 - a + 2a = 1 + a$ $1 + a$ $\underline{\times 3}$ $3 + 3a$		B1
	Must use algebra		

	Alternative method 1		
	$9 + 15a$ or $3(3 + 5a)$ or $3(3 + 3a + 2a)$	M1	oe
	their $(9 + 15a) = 16$ and their $15a = 16 –$ their 9	M1	Must expand any brackets correctly and collect terms correctly their $(9 + 15a)$ must be at least two terms
	$\dfrac{7}{15}$ or $0.4\dot{6}$ or 0.47	A1ft	ft from M1 M0 or M0 M1 with 1 error Allow 0.466... or 0.467 SC1 $\dfrac{13}{3}$ or 4.33... oe
4(b)	**Additional Guidance**		
	$\dfrac{7}{15}$ (may be seen in working) with subsequent attempt at evaluation		M1 M1 A1
	$3(3 + 5a) = 16$ $9 + 5a = 16$ (error in expansion) $5a = 7$ $a = 1.4$ (1 error)		M1 M0 A1ft
	$3(3 + 5a) = 16$ $6 + 15a = 16$ (error in expansion) $15a = 22$ (error in collection) $a = \dfrac{22}{15}$ (2 errors)		M1 M0 A0ft
	May just state a 3rd term but cannot use $3 + 3a$ for the 3rd term $9 + 8a = 16$ $8a = 7$ (no brackets to expand and collects term correctly) $a = \dfrac{7}{8}$		M0 M1 A1ft
	For A1ft accept answers rounded to at least 2sf if not an integer		
	$3(3 + 5a) = 6 + 5a$ is two errors so not possible to award A1ft		
	$1 – a = 16$		M0 M0 A0

	Alternative method 2		
	$3(3 + 5a)$ or $3(3 + 3a + 2a)$	M1	oe
	their $(3 + 5a) = \dfrac{16}{\text{their } 3}$ and their $5a = \dfrac{16}{\text{their } 3} - \text{their } 3$	M1	Must divide by their 3 correctly and collect terms correctly their $(3 + 5a)$ must be at least two terms
	$\dfrac{7}{15}$ or $0.4\dot{6}$ or 0.47	A1	ft from M1 M0 or M0 M1 with 1 error Allow 0.466... or 0.467 SC1 $\dfrac{13}{3}$ or $4.33...$ oe

	Additional Guidance	
	$\dfrac{7}{15}$ (may be seen in working) with subsequent attempt at evaluation	M1 M1 A1
4(b)	$3(3 + 5a) = 16$ $9 + 5a = \dfrac{16}{3}$ (error in division by 3) $5a = \dfrac{16}{3} - 9$ $a = -\dfrac{11}{15}$ (1 error)	M1 M0 A1ft
	$3(3 + 5a) = 16$ $9 + 5a = \dfrac{16}{3}$ (error in division by 3) $5a = \dfrac{16}{3} + 9$ (error in collection) $a = \dfrac{43}{15}$ (2 errors)	M1 M0 A0ft
	For A1ft accept answers rounded to at least 2sf if not an integer	
	$3(3 + 5a) = 6 + 5a$ is two errors so not possible to award A1ft	

5	Draws $y = -2x + 5$ for x-values from -3 to 3	B4	B3	Draws line with gradient -2 and intercept $\neq (0, 1)$ or Draws $y = -2x + 5$ but too short
			B2	Draws $y = -2x + 1$ or states $y = -2x + 5$ oe or states gradient -2 and (y-) intercept 5 or identifies a point other than $(3, -1)$ that lies on $y = -2x + 5$
			B1	States gradient -2 or states $y = -2x + c$ $c \neq 1$ $c \neq 5$ oe
			SC2	Draws $y = \dfrac{1}{2}x - \dfrac{5}{2}$
			SC1	Draws $y = \dfrac{1}{2}x + c$ $c \neq -\dfrac{5}{2}$

Additional Guidance	
Allow unruled lines if intention clear	
Allow B4 if correct line is too long	
Mark the better response between graph and working lines	
2 lines drawn (no working seen) eg1 $y = -2x + 5$ and $y = -2x + 1$ eg2 $y = -2x$ and $y = -2x + 1$ eg3 $y = -2x + 5$ and $y = -2x + 4$ (choice) eg4 $y = -2x + 5$ and $y = x + 3$ (choice) eg5 $y = -2x$ and $y = 2x$ (choice)	B4 B3 B3 B0 B0
Apart from B4 response, allow lines that do not span x-values from -3 to 3	
gradient $= -2x$ (no further valid work)	B0

6	$5x^6$ or $(-)6x^5$ or $ax^6 - bx^5$ with $a > 0$ and $b > 0$	M1	
	$5x^6 - 6x^5$	A1	
	Additional Guidance		
	$\dfrac{5x^6 - 6x^5}{1}$		M1 A0
	$\dfrac{5x^6}{1}$ or $(-)\dfrac{6x^5}{1}$		M1 A0

7	$6 \times \dfrac{2}{3} x^5$ or $4x^5$ or $-3 \times 8x^2$ or $-24x^2$	M1	oe
	$4x^5 - 24x^2$	A1	Fully correct and simplified
	-28	A1ft	ft M1 A0 and their gradient has at least two terms in x
	Additional Guidance		
	Second derivative used can score a maximum M1 A0 A0 unless recovered eg1 $4x^5 - 24x^2$ and $20x^4 - 48x$ eg2 $4x^5 - 24x^2$ $20x^4 - 48x$ $4 \times (-1)^5 - 24 \times (-1)^2 = -20$	M1 A0 A0 M1 A1 A0
	$4x^5 - 24x$ 20	M1 A0 A1ft
	For A1ft accept answers rounded to at least 1dp if not an integer	
	Condone $y = -28$	

8(a)	f(x) ≥ 16 or y ≥ 16	B1	Condone absence of (x) or absence of brackets
	Additional Guidance		
	x ≥ 16		B0
	f(x) > 16 or f(x) ≤ 16 or f(x) < 16		B0
	16		B0

8(b)	−1 ≤ g(x) ≤ 8 or −1 ≤ y ≤ 8	B2	B1 g(x) ≤ 8 or −1 ≤ g(x) or
			        y ≤ 8 or −1 ≤ y or
			        −1 and 8 chosen
			Condone absence of (x) or absence of brackets for B2 or B1

	Additional Guidance		
	Both inequalities g(x) ≤ 8 and −1 ≤ g(x) given as their answer		B2
	B1 may be seen with an incorrect inequality eg1 5 ≤ g(x) ≤ 8 eg2 −1 ≤ g(x) ≤ 2		B1 B1
	For B1 ignore incorrect notation if −1 and 8 seen eg1 −1 ≤ x ≤ 8 eg2 −1 < g < 8 eg3 −1 to 8 eg4 −1 0 1 2 3 4 5 6 7 8		B1 B1 B1 B0
	3 ≤ x ≤ 8		B0
	Allow g to be f		

8(c)	$5x - 3 < 1$ or $-2 < 5x - 3$ or $-2 < 5x - 3 < 1$	M1	oe eg $x < \dfrac{4}{5}$ or $\dfrac{1}{5} < x$ or $1 < 5x < 4$
	$\dfrac{1}{5} < x < \dfrac{4}{5}$ or $0.2 < x < 0.8$	A1	oe SC1 $\dfrac{1}{5} < \mathrm{h}(x) < \dfrac{4}{5}$ (condone absence of (x) or absence of brackets) or $\dfrac{1}{5} < y < \dfrac{4}{5}$ or $\dfrac{1}{5} \leq x \leq \dfrac{4}{5}$

Additional Guidance	
Both inequalities $x < \dfrac{4}{5}$ and $\dfrac{1}{5} < x$ given as their answer	M1 A1
M1 Must use correct inequality symbol unless recovered in the A mark $5x - 3 \leq 1$ or $5x - 3 > 1$ (answer not correct)	M0 A0
M1 If using equations award M0 unless recovered in the A mark $5x - 3 = 1$ $5x - 3 = -2$ $0.2 < x < 0.8$	M1 A1

9(a)	**Alternative method 1**		
	$12y - 18$	B1	
	$12y - 2y = 10 + 18$	M1	Collects terms Allow one sign or arithmetic error ft their expansion
	2.8 or $\dfrac{14}{5}$	A1ft	oe fraction Only ft an incorrect expansion
	Additional Guidance		
	For A1ft accept answers rounded to at least 2sf if not an integer		
	Omitting a term is not a sign or arithmetic error $12y - 18 = 2y$ $y = 1.8$		B1 M0 A0ft
	$12y - 9 - 10 = 2y$ $10y = 19$ $y = 1.9$		B0 M1 A1ft
	$12y - 3 - 10 = 2y$ $12y - 2y = 10 - 3$ (one sign error) $y = 0.7$		B0 M1 A0ft
	$8y - 18 - 10 = 2y$ $8y - 38 = 2y$ (one arithmetic error) $y = 6.3$ (M1 implied)		B0 M1 A0ft
	$12y - 18 - 10 = 2y$ $14y = 28$ (one sign error) $y = 2$ (no ft as their expansion is correct/cannot give full marks with an error)		B1 M1 A0ft

	Alternative method 2		
	$2y - 3 - \dfrac{10}{6} = \dfrac{2y}{6}$	B1	oe
	$2y - \dfrac{2y}{6} = 3 + \dfrac{10}{6}$	M1	Collects terms Must have at least one of $\dfrac{10}{6}$ or $\dfrac{2y}{6}$ oe Allow one sign or arithmetic error ft their division by 6
	2.8 or $\dfrac{14}{5}$	A1ft	oe fraction Only ft an incorrect division by 6
	Additional Guidance		
	For A1ft accept answers rounded to at least 2sf if not an integer		
9(a)	$2y - 3 - 10 = \dfrac{2y}{6}$ (error in division by 6) $2y - \dfrac{2y}{6} = 3 + 10$ 7.8	B0 M1 A1ft	
	$2y - 3 - 10 = \dfrac{2y}{6}$ (error in division by 6) $2y + \dfrac{2y}{6} = 3 + 10$ (one sign error) $\dfrac{39}{7}$ (only ft an incorrect division by 6)	B0 M1 A0ft	
	$2y - 3 - \dfrac{10}{6} = \dfrac{2y}{6}$ $2y - \dfrac{2y}{6} = 3 - \dfrac{10}{6}$ (one sign error) 0.8 (no ft as their division by 6 is correct/cannot give full marks with an error)	B1 M1 A0ft	

9(b)	**Alternative method 1**		
	$\sqrt{w+4}$ = 6 × 2	M1	oe
	$w + 4 = (6 \times 2)^2$	M1dep	oe
	140	A1	SC1 68
	Alternative method 2		
	$\dfrac{w+4}{2^2} = 6^2$	M1	oe
	$w + 4 = 6^2 \times 2^2$	M1dep	oe
	140	A1	SC1 68
	Additional Guidance		
	Embedded correct answer		M1 M1 A0
	Alt 1 $\sqrt{w+4}$ = 12 followed by $w + 4 = \sqrt{12}$ or $w^2 + 4^2 = 12$ etc Second part is their next step so not a choice		M1 M0
	Alt 1 $\dfrac{\sqrt{w+4}}{2}$ × 2 = 6 × 2 does not score M1 unless correctly processed		

9(c)	$m^{\frac{1}{5}} = \dfrac{0-9}{3}$ or $m^{\frac{1}{5}} = -3$ or $\sqrt[5]{-3}^{\frac{1}{5}}$ or $(-3)^5$ or 243	M1	
	−243	A1	
	Additional Guidance		
	Condone -3^5 for $(-3)^5$		
	Allow $\sqrt[5]{m}$ for $m^{\frac{1}{5}}$		
	$m^{\frac{1}{5}} + 3 = 0$ or $3m^{\frac{1}{5}} = -9$ or $(3m^{\frac{1}{5}})^5 = (-9)^5$		M0
	$3^5 m = (-9)^5$ or $243m = -59049$		M1

	Alternative method 1		
	(grad CP =) $\dfrac{8 - 6}{2 - 3}$ or –2	M1	oe
	(grad PT =) $\dfrac{\pm 1}{\text{their} - 2}$ or $\pm\dfrac{1}{2}$	M1	oe
	$\dfrac{t - 8}{-4 - 2}$ = their grad PT	M1dep	oe dep on 2nd M1
	5	A1	
	Alternative method 2		
	(grad CP =) $\dfrac{8 - 6}{2 - 3}$ or –2	M1	oe
10	(grad PT =) $\dfrac{\pm 1}{\text{their} - 2}$ or $\pm\dfrac{1}{2}$	M1	oe
	y = (their grad PT) $x + c$ and substitutes (2, 8) to find c and substitutes x = –4 into their equation or y – 8 = their grad PT(x – 2) and substitutes x = –4 into their equation	M1dep	oe dep on 2nd M1
	5	A1	
	Additional Guidance		
	Answer of 5 gains full marks (could be a restart)		

	Alternative method 3		
	$(8 - 6)^2 + (2 - 3)^2$ or $(t - 8)^2 + (-4 - 2)^2$ or $(t - 6)^2 + (-4 - 3)^2$	M1	oe CP = $\sqrt{5}$ may be seen on the diagram
10	their CP^2 + their PT^2 = their CT^2 with at least two of CP^2, PT^2 and CT^2 correct	M1dep	oe their PT^2 and their CT^2 must both be in terms of t
	$(8 - 6)^2 + (2 - 3)^2 +$ $(t - 8)^2 + (-4 - 2)^2 =$ $(t - 6)^2 + (-4 - 3)^2$ or $t^2 - 8t - 8t + 64 + 36 + 4 + 1$ $= t^2 - 6t - 6t + 36 + 49$	M1	oe eg 20 = 4t Must be fully correct method
	5	A1	
	Additional Guidance		
	Answer of 5 gains full marks (could be a restart)		

	$3w^2 + 2wy – 12wy – 8y^2$	M1	oe 4 terms with 3 correct Terms may be seen in a grid May be implied eg1 $3w^2 – 10wy + 8y^2$ eg2 $w^2 – 10wy – 8y^2$
	$3w^2 + 2wy – 12wy – 8y^2$	A1	Fully correct Do not allow if only seen in a grid
	$3w^2 – 10wy – 8y^2$	A1ft	ft M1 A0

11(a)	**Additional Guidance**	
	Accept yw for wy throughout	
	A correct term must include a $–$ sign if it is negative	
	$3w^2 + 2wy – 12wy – 8y$ $3w^2 – 10wy – 8y$	M1 A0 A1ft
	$3w^2 + 2wy + 12wy – 8y^2$ $3w^2 + 14wy – 8y$ (does not ft from previous line)	M1 A0 A0ft
	$3w – 10wy – 8y^2$ (implied M1 and A1ft as terms collected)	M1 A0 A1ft
	$3w^2 + 2wy – 12wy – 8wy$ $3w^2 – 18wy$	M1 A0 A1ft
	$3w^2 + 10wy – 8y^2$	M0 A0 A0ft
	Penalise the 2nd A1 if further work seen $3w^2 – 10wy – 8y^2 = 3w^2 – 18wy^2$	M1 A1 A0ft

11(b)	$\dfrac{3x}{3x^2}$ or $\dfrac{9x^2}{x^2}$ or $(-)\dfrac{3}{x^2}$	M1	oe eg1 $\dfrac{3 \times x}{x^2 \times 3}$ eg2 9 One correct product, unsimplified or simplified
	$\dfrac{3x}{3x^2} + \dfrac{9x^2}{x^2} - \dfrac{3}{x^2}$ or $\dfrac{1}{x} + \dfrac{9x^2}{x^2} - 3x^{-2}$ or $\dfrac{3x + 27x^2}{3x^2} - \dfrac{3}{x^2}$ or $\dfrac{x}{x^2} + \dfrac{9x^2 - 3}{x^2}$ or $\dfrac{9x^2}{x^2} + \dfrac{3(x-3)}{3x^2}$ or $\dfrac{3x + 27x^2 - 9}{3x^2}$	A1	oe Fully correct expansion of given expression that requires further simplification Multiplication signs not allowed unless recovered eg $\dfrac{3 \times x}{x^2 \times 3} + \dfrac{9x^2}{x^2} - \dfrac{3}{x^2}$ M1 A0
	$\dfrac{1}{x} + 9 - \dfrac{3}{x^2}$ or $x^{-1} + 9 - 3x^{-2}$ or $\dfrac{1}{x} + \dfrac{9x^2 - 3}{x^2}$ or $x^{-1} + \dfrac{9x^2 - 3}{x^2}$ or $\dfrac{x-3}{x^2} + 9$ or $\dfrac{1+9x}{x} - \dfrac{3}{x^2}$ or $\dfrac{x + 9x^2 - 3}{x^2}$	A1	oe Any of these answers implies M1 A1 A1 Do not allow $\dfrac{9}{1}$ for 9 Multiplication signs or brackets that require expansion not allowed unless recovered After M1 A1 A1 penalise further work eg $\dfrac{x + 9x^2 - 3}{x^2}$ followed by $\dfrac{3x + 27x^2 - 9}{3x^2}$ M1 A1 A0
	Additional Guidance		
	3 mark responses with fractions must have fractions in their simplest form		

12	$\frac{1}{2}$ (×) x (×) y (×) sin 30 = x^2	M1	oe equation
	$y = 4x$	A1	oe Any unsimplified form but must have y as the subject

Additional Guidance			
$\frac{1}{2}$ (×) x (×) y (×) $\frac{1}{2}$ = x^2			M1
Unsimplified forms may involve fractions and/or sin 30 not evaluated eg $\dfrac{4x^2}{x}$ = y			M1 A1
If a 2 mark response is seen in the working lines, ignore any subsequent attempt to simplify unless the attempt produces an answer that does not have y as the subject eg1 $y = \dfrac{4x^2}{x}$ in working and $x = \dfrac{4}{y}$ on answer line			M1 A0

	Alternative method 1		
13	$OP = 3$ or P $(3, 0)$ and $PQ = 5$ or radius = 5	M1	May be seen on diagram May be implied eg $5^2 - 3^2$
	$(OQ =) \sqrt{\text{their PQ}^2 - \text{their OP}^2}$ or $\sqrt{5^2 - 3^2}$ or 4 or Q $(0, 4)$	M1	 May be seen on diagram
	$(\text{gradient} =) -\dfrac{\text{their OQ}}{\text{their OP}}$ or $\dfrac{0 \text{ - their } 4}{\text{their } 3 \text{ - } 0}$ or $-\dfrac{4}{3}$	M1	oe Gradient must be negative Allow $-1.33...$
	$4x + 3y - 12 = 0$ or $-4x - 3y + 12 = 0$	A1	$4x + 3y - 12$ or $-4x - 3y + 12$ imply M3 A0 Any correct equation not in required form implies M3 A0 eg $y = -\dfrac{4}{3}x + 4$ or $4x + 3y = 12$ or $\dfrac{4}{3}x + y - 4 = 0$
	Additional Guidance		
	$3y + 4x - 12 = 0$ etc $8x + 6y - 24 = 0$ etc		M3 A1 M3 A1
	$OQ = 4$ (implied by 4 next to Q on diagram)		M1 M1
	3rd M1 Gradient may be seen within working for an equation		
	3rd M1 Condone inclusion of x		

13	**Alternative method 2**		
	$(0 - 3)^2 + y^2 = 25$	M1	oe
	$(OQ =) \sqrt{25 - \text{their } (0-3)^2}$ or 4 or Q (0, 4)	M1	May be seen on diagram
	$(\text{gradient} =) -\dfrac{\text{their OQ}}{3}$ or $\dfrac{0 - \text{their } 4}{3 - 0}$ or $-\dfrac{4}{3}$	M1	oe Gradient must be negative Allow $-1.33...$
	$4x + 3y - 12 = 0$ or $-4x - 3y + 12 = 0$	A1	$4x + 3y - 12$ or $-4x - 3y + 12$ imply M3 A0 Any correct equation not in required form implies M3 A0 eg $y = -\dfrac{4}{3}x + 4$ or $4x + 3y = 12$ or $\dfrac{4}{3}x + y - 4 = 0$

Additional Guidance		
$3y + 4x - 12 = 0$ etc		M3 A1
$8x + 6y - 24 = 0$ etc		M3 A1
OQ = 4 (implied by 4 next to Q on diagram)		M1 M1
3rd M1 Gradient may be seen within working for an equation		
3rd M1 Condone inclusion of x		

14(a)	**Alternative method 1**			
	$3 \times \dfrac{3}{2}$	$3 \times \dfrac{5}{2}$	M1	$3 \div \dfrac{2}{3}$ is equivalent to $3 \times \dfrac{3}{2}$
	$6 + \dfrac{3}{2} \times 3 = 10.5$ or $3 + 3 + \dfrac{3}{2} \times 3$ $= 10.5$	$3 + \dfrac{5}{2} \times 3 = 10.5$	A1	$3 \div \dfrac{2}{5}$ is equivalent to $3 \times \dfrac{5}{2}$ $5 \times \dfrac{3}{2}$ is equivalent to $3 \times \dfrac{5}{2}$
	Additional Guidance			
	M1 Do not allow 4.5 or 7.5 unless correct method or scale factor also seen			
	$6 + 3 + \dfrac{3}{2}$			M0

14(a)	**Alternative method 2**			
	$10.5 - 6 = 4.5$ and $4.5 \div \dfrac{3}{2} = 3$	$10.5 - 3 = 7.5$ and $7.5 \div \dfrac{5}{2} = 3$	B2	May be seen in one step $4.5 \times \dfrac{2}{3} = 3$ is equivalent to $4.5 \div \dfrac{3}{2} = 3$ $7.5 \times \dfrac{2}{5} = 3$ is equivalent to $7.5 \div \dfrac{5}{2} = 3$
	Additional Guidance			
	Do not allow 4.5 and 3 unless correct method also seen			
	Do not allow 7.5 and 3 unless correct method also seen			
	B1 not possible for this method which is verification by working back to the x-coordinate of P			
	Allow further addition of 3 (to obtain x-coordinate of Q)			

14(a)	**Alternative method 3**				
	$\dfrac{10.5 - 6}{3} = 1.5$ and $\dfrac{6 - 3}{2} = 1.5$	$\dfrac{10.5 - 3}{5} = 1.5$ and $\dfrac{6 - 3}{2} = 1.5$	B2	oe eg $\dfrac{10.5 - 6}{3} = 1.5$ and $\dfrac{10.5 - 3}{5} = 1.5$	
	Additional Guidance				
	Do not allow 1.5 unless two correct methods also seen				
	B1 not possible for this method which is verification by working to 1.5 in two ways				

14(a)	**Alternative method 4**		
	$10.5 - 6 = 4.5$ and $\dfrac{6 - 3}{2} \times 3 = 4.5$	B2	
	Additional Guidance		
	Do not allow 4.5 unless two correct methods also seen		
	B1 not possible for this method which is verification by working to 4.5 in two ways		

14(a)	**Alternative method 5**		
	$10.5 - 3 = 7.5$ and $\dfrac{6 - 3}{2} \times 5 = 7.5$	B2	
	Additional Guidance		
	Do not allow 7.5 unless two correct methods also seen		
	B1 not possible for this method which is verification by working to 7.5 in two ways		

	Alternative method 6		
14(a)	Correct algebra using ratio 2 : 3 eg1 $\dfrac{a-3}{6-3} = \dfrac{5}{2}$ eg2 $\dfrac{a-6}{6-3} = \dfrac{3}{2}$ eg3 $\dfrac{a-3}{a-6} = \dfrac{5}{3}$ eg4 $\dfrac{3 \times 3 + 2 \times a}{5} = 6$	M1	oe
	Correct working leading to 10.5 eg1 $a-3 = 7.5$ and $a = 10.5$ eg2 $a-6 = 4.5$ and $a = 10.5$ eg3 $3(a-3) = 5(a-6)$ and $a = 10.5$ eg4 $9 + 2a = 30$ and $a = 10.5$	A1	Must see method for M1
	Additional Guidance		
	Equivalents for M1 include (eg1) $2a - 6 = 15$ (eg2) $\dfrac{6-3}{a-6} = \dfrac{2}{3}$ (eg3) $3(a-3) = 5(a-6)$ (eg4) $\dfrac{9+2a}{5} = 6$		
	For A1 there must be at least one correct working step seen (and no incorrect working)		

	Alternative method 1			
14(b)	$\dfrac{8b}{2} \times 3$ or $12b$	$\dfrac{8b}{2} \times 5$ or $20b$	M1	oe
	$9b + \dfrac{8b}{2} \times 3 = 7$ or $21b = 7$	$b + \dfrac{8b}{2} \times 5 = 7$ or $21b = 7$	M1dep	oe
	$\dfrac{1}{3}$		A1	Allow 0.33...
	Additional Guidance			
	2nd M1 implies the 1st M1			
	If $\dfrac{1}{3}$ is clearly from incorrect method seen, do not award marks			

	Alternative method 2		
	Correct algebra using ratio 2 : 3 eg1 $\dfrac{7-b}{9b-b}=\dfrac{5}{2}$ eg2 $\dfrac{7-9b}{9b-b}=\dfrac{3}{2}$ eg3 $\dfrac{9b-b}{6-3}=\dfrac{7-b}{10.5-3}$ eg4 $\dfrac{7-9b}{10.5-6}=\dfrac{7-b}{10.5-3}$ eg5 $\dfrac{7-9b}{10.5-6}=\dfrac{9b-b}{3}$ eg6 $\dfrac{7-b}{7-9b}=\dfrac{5}{3}$ eg7 $\dfrac{3\times b+2\times 7}{5}=9b$	M1	oe
14(b)	Further correct simplification eg cross multiplication or expanding brackets eg1 $2(7-b)=5(9b-b)$ eg2 $14-18b=24b$ eg3 $60b=21-3b$ eg4 $52.5-67.5b=31.5-4.5b$ eg5 $21-27b=36b$ eg6 $21-3b=35-45b$ eg7 $3b+14=45b$	M1dep	oe
	$\dfrac{1}{3}$	A1	Allow 0.33...
	Additional Guidance		
	2nd M1 implies the 1st M1		
	If $\dfrac{1}{3}$ is clearly from incorrect method seen, do not award marks		

	Alternative method 1		
	$8(c^2 + 2)$ or $3(c^2 + 2)$	M1	
	$\dfrac{8(c^2 + 2)}{3(c^2 + 2)}$	A1	
	$\dfrac{8}{3} + \dfrac{1}{3} = 3$	A1	
	Alternative method 2		
15	Converts to a valid common denominator with at least one numerator correct eg1 $\dfrac{3(8c^2 + 16)}{3(3c^2 + 6)} + \dfrac{3c^2 + 6}{3(3c^2 + 6)}$ eg2 $\dfrac{8c^2 + 16 + c^2 + 2}{3c^2 + 6}$	M1	oe Other valid common denominators include $9c^2 + 18$ and $3(c^2 + 2)$
	Makes into a single fraction with terms collected eg1 $\dfrac{27c^2 + 54}{3(3c^2 + 6)}$ eg2 $\dfrac{9c^2 + 18}{3c^2 + 6}$	A1	oe
	Shows that fraction simplifies to 3 eg1 $\dfrac{9(3c^2 + 6)}{3(3c^2 + 6)} = 3$ eg2 $\dfrac{3(3c^2 + 6)}{3c^2 + 6} = 3$ eg3 $\dfrac{9(c^2 + 2)}{3(c^2 + 2)} = 3$	A1	oe Must see a correct common quadratic factor and = 3
	Additional Guidance		
	Answer of 3 does not gain marks without correct working for M1 A1 (1st) seen		
	Do not allow $\dfrac{3}{1}$ unless subsequently becomes 3		

16	**Alternative method 1**		
	$x(2x - 1) = 9$	M1	oe
	$2x^2 - x - 9 \ (= 0)$	A1	oe equation with brackets expanded
	$\dfrac{--1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -9}}{2 \times 2}$	M1	Allow one error ft their 3-term quadratic Allow ± to be + or – in formula (do not count as an error)
	$\dfrac{--1 \pm \sqrt{(-1)^2 - 4 \times 2 \times -9}}{2 \times 2}$ or $\dfrac{1 \pm \sqrt{73}}{4}$	A1ft	oe Fully correct substitution for their 3-term quadratic Only ft their 3-term quadratic Allow ± to be + or – in formula (do not count as an error)
	2.39	A1	A0 if negative solution also in answer SC5 1st M1 seen and answer 2.39 SC4 1st M1 seen and answer 2.38(6...) or 2.4

Additional Guidance	
$x \times 2x - 1 = 9$ is M0 unless recovered (eg followed by correct equation)	

3rd and 4th marks
If their 3-term quadratic factorises allow both marks if they factorise correctly (M1 A0 not possible)
eg1 $x^2 - x - 6$
          $(x + 2)(x - 3)$
          3
eg2 $x^2 - x - 6$
          $(x - 2)(x - 3)$
          3
	M0 A0
	M1 A1ft
	A0
	
	M0 A0
	M0 A0ft
	A0

Answer 2.39 with no equation seen	Zero
3rd mark Substituting incorrect value for b twice is 2 errors	
3rd mark Missing brackets is not an error if recovered	
3rd mark Omitting ± or $\sqrt{}$ or division line is always 2nd M0	

	Alternative method 2		
	$x(2x - 1) = 9$	M1	
	$2x^2 - x - 9 \ (= 0)$	A1	oe equation with brackets expanded
	$2[(x - \frac{1}{4})^2 \ ......]$	M1	Attempt to complete the square for their 3-term quadratic ft their 3-term quadratic
	$2[(x - \frac{1}{4})^2 - (\frac{1}{4})^2 - \frac{9}{2}] = 0$	A1ft	oe eg $2[(x - \frac{1}{4})^2 - \frac{73}{16}] = 0$ Fully correct equation for their 3-term quadratic
16	2.39	A1	A0 if negative solution also in answer SC5 1st M1 seen and answer 2.39 SC4 1st M1 seen and answer 2.38(6...) or 2.4
	Additional Guidance		
	$x \times 2x - 1 = 9$ is M0 unless recovered (eg followed by correct equation)		
	3rd and 4th marks They may divide (eg by 2) before attempting to complete the square		

	3rd and 4th marks
	If their 3-term quadratic factorises allow both marks if they factorise correctly
	(M1 A0 not possible)
	eg1 $x^2 - x - 6$
	       $(x + 2)(x - 3)$
	       3
	eg2 $x^2 - x - 6$
	       $(x - 2)(x - 3)$
	       3
		M0 A0
		M1 A1ft
		A0
		M0 A0
		M0 A0ft
		A0

	Answer 2.39 with no equation seen	Zero

Alternative method 1			
$(DB^2 =)$ $34^2 - 16^2$ or 900 or $(DB =)$ 30	M1	M2 $(DB^2 =)$ $34^2 - 16^2 - 18^2$	
their $DB^2 - 18^2$ or 576	M1		
24	A1		

Alternative method 2			
$(DB =)$ $34 \times \cos(\sin^{-1}\frac{16}{34})$ or $34 \times \sin(\cos^{-1}\frac{16}{34})$ or 30 or $\frac{16}{\tan(\sin^{-1}\frac{16}{34})}$ or $16 \times \tan(\cos^{-1}\frac{16}{34})$ or 30	M1	Allow $34 \times \cos[28, 28.1]$ or $34 \times \sin[61.9, 62]$ or $\frac{16}{\tan[28, 28.1]}$ or $16\tan[61.9, 62]$	
their $DB \times \cos(\sin^{-1}\frac{18}{\text{their } DB})$ or their $DB \times \sin(\cos^{-1}\frac{18}{\text{their } DB})$ or $\frac{18}{\tan(\sin^{-1}\frac{18}{30})}$ or $18 \times \tan(\cos^{-1}\frac{18}{\text{their } DB})$	M1	Allow their $DB \times \cos[36.8, 36.9]$ or their $DB \times \sin 53.1...$ or $\frac{18}{\tan[36.8, 36.9]}$ or $18\tan 53.1...$	
24	A1		

17(a)

Additional Guidance		
Alt 1 576		M1 M1
Note that $\sqrt{16^2 + 18^2}$ = 24.08..... so do not award marks for 24 from this method		
Allow if they use correct Pythagoras for one M mark and correct trigonometry for the other M mark		
Marks may be gained from using correct cosine rule (up to $AB^2 =$) or correct sine rule (up to $AB =$)		

17(b)	**Alternative method 1**			
	$\sin x = \dfrac{16}{34}$	M1	oe eg $\sin^{-1}\dfrac{16}{34}$ or $90 - \cos^{-1}\dfrac{16}{34}$	
	[28, 28.0725] or 28.1	A1		
	Alternative method 2			
	$\cos x = \dfrac{\text{their DB}}{34}$ or $\cos x = \dfrac{\sqrt{34^2 - 16^2}}{34}$ or $\tan x = \dfrac{16}{\text{their DB}}$ or $\tan x = \dfrac{16}{\sqrt{34^2 - 16^2}}$	M1	oe eg $\cos^{-1}\dfrac{\sqrt{34^2 - 16^2}}{34}$ or $90 - \sin^{-1}\dfrac{\sqrt{34^2 - 16^2}}{34}$ Look back to (a) for their DB	
	[28, 28.0725] or 28.1	A1ft	Only ft their DB	
	Additional Guidance			
	x may be any letter			
	Condone $\sin = \dfrac{16}{34}$ etc			
	Only Alt 2 has A1ft (ft answers must be rounded to at least 1dp if not an integer)			
	Marks may be gained from using correct cosine rule or correct sine rule (Must have $\cos x$ or $\sin x$ as the subject)			

18(a)	(1 (or a) is) Midway between 0 and 2 or $\dfrac{2+0}{2} = 1$ or $\dfrac{2-0}{2} = 1$	B1	oe
	Minimum point (at $x = 1$ (or $x = a$)) or Symmetrical (about $x = 1$ (or $x = a$))	B1	oe
	Additional Guidance		
	For minimum allow stationary or turning or lowest or vertex		
	Line of symmetry		B1
	Do not award B2 if an error seen eg $\dfrac{2-0}{2} = 2$ is an error		
	Substitution of points in given equation does not score but ignore if other valid reason(s) seen		
	Ignore other non-contradictory reasons		

18(b)	$10 = 4(0-1)^2 + b$ or $10 = 4(2-1)^2 + b$	M1	oe eg $10 = 4 + b$
	6	A1	
	Additional Guidance		
	If expansion before substitution, expansion must be fully correct eg1 $4(x^2 - x - x + 1) + b$ $\quad 4(2^2 - 2 - 2 + 1) + b = 10$		M1
	eg2 $4x^2 - 2x + 1 + b$ $\quad 16 - 4 + 1 + b = 10$		M0 A0
	a must not be present for M1 or A1		

18(c)	$4(x^2 - x - x + 1) + b$ or $4(x^2 - x - x + 1) +$ their 6	M1	oe correct expansion eg $4x^2 - 8x + 10$ Value for b does not have to be used
	$y = 4x^2 - 8x + 10$	A1ft	Must have $y =$ Only ft their value for b

<table>
<tr><td colspan="3" align="center">Additional Guidance</td></tr>
<tr><td colspan="2"> A1ft is $y = 4x^2 - 8x + 4 +$ their value for b </td><td></td></tr>
<tr><td colspan="2"> a must not be present for M1 or A1 </td><td></td></tr>
<tr><td colspan="2"> $y = 4x^2 - 8x + 10$ seen in working with $4x^2 - 8x + 10$ on answer line </td><td>M1 A1</td></tr>
</table>

	Alternative method 1		
19	$3^3 - 10 \times 3 - 3$ or $27 - 30 - 3$	M1	oe Shows correct substitution (ignore any evaluation)
	Statement why this means it is not a factor eg1 –6 which is not zero eg2 $\neq 0$ eg3 Remainder is –6	A1	oe Must see correct working for M1

	Additional Guidance	
	Evaluation of f(3) is not needed but if shown must = –6 for A1	
	$27 - 30 - 3 \neq 0$	M1 A1
	$3^3 - 30 - 3 = -6 \neq 0$ $3^3 - 30 - 3 = 6 \neq 0$	M1 A1 M1 A0
	$27 - 30 - 3 = 0$ (condone as next line confirms their working) $-6 \neq 0$	M1 A1
	$3^3 - 30 - 3 = -6$	M1 A0
	$-6 \neq 0$ which means 3 is not a factor	M0 A0
	$3^2 - 30 - 3$	M0 A0

19	**Alternative method 2**		
	Attempt at division of $x^3 - 10x - 3$ by $(x - 3)$ correct up to $x^2 + 3x$ ...	M1	
	Division correct ie $x^2 + 3x - 1$ with remainder –6 seen and statement why this means it is not a factor eg there is a remainder	A1	
	Additional Guidance		
	For A1, –6 must be seen within the working or in the statement		

20(a)	Rotation and 270 (anti-clockwise) and centre O or Rotation and 90 clockwise and centre O	B2	oe B1 270 (anti-clockwise) or 90 clockwise Do not allow if reflection or translation or enlargement also stated
	Additional Guidance		
	270 is anti-clockwise by default so 'anti-clockwise' not required for B2 or B1		
	270		B1
	270 clockwise		B0
	Response that is not a single transformation is always B0 eg Rotation, 270 (anti-clockwise), centre O Scale factor 3 (enlargement)		B0
	Reflection 270 (anti-clockwise)		B0
	Rotation and 270 clockwise and centre O		B0
	Turn 90 clockwise centre O (B1 for 90 clockwise)		B1
	Do not allow a circular arrow for clockwise direction eg 90 with circular arrow indicating clockwise		B0
	Do not allow quarter turn etc eg Quarter turn clockwise		B0

20(b)	Rotation and 180 and centre O or Enlargement and scale factor –1 and centre O	B2	oe B1 Rotation and 180 or Enlargement and scale factor –1 or $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	
	Additional Guidance			
	Response that is not a single transformation is always B0 unless they give the two possible B2 answers			
	Rotation through 180 clockwise about O			B2
	Rotation through 180 anti-clockwise about O			B2
	For B2 or B1 ignore a circular arrow as direction not required			
	Do not allow half turn or turn eg1 Half turn eg2 Turn 180			B0 B0
	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ from multiplying given matrices in either order			B1
	Allow matrix to have brackets missing and/or commas but must be 2 by 2 array			
	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ scores B1 even if description of transformation is incorrect			
	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ seen followed by multiplication of matrix by a vector is not a choice			B1

21	2 – –4 or 6 or 10 – –26 or 36 or –4 – 2 or –6 or –26 – 10 or –36	M1	May be seen on diagram
	$\frac{1}{2}$ × (2 – –4) × (10 – –26) or $\frac{1}{2}$ × 6 × 36 or –108	M1	oe eg $\frac{1}{2}$ × 6 × 36 × sin 90 Allow (2 – –4) to be (–4 – 2) Allow (10 – –26) to be (–26 – 10)
	108	A1	SC2 Answer 108 but clearly used normal at A and tangent at B

Additional Guidance	
2nd M1 implies the 1st M1	
–108 is M1 M1 A0 unless recovered	
Diagram showing triangle with vertices in 2nd, 3rd and 4th quadrants and answer 108	SC2
Diagram showing rectangle or 2 triangles and answer 108	M1 M1 A1

	Alternative method 1			
	Second differences -4	M1	Implied by $-2n^2$	
	Subtracts $\dfrac{\text{their} -4}{2}\,n^2$ from given sequence or 304 608 912	M1	At least 3 correct values implies correct method (next term is 1216)	
	$-2n^2 + 304n$	A1	oe eg $n(304 - 2n)$ Allow any letter	
	Alternative method 2			
	Any 3 of $a + b + c = 302$ $4a + 2b + c = 600$ $9a + 3b + c = 894$ $16a + 4b + c = 1184$	M1	Using $an^2 + bn + c$	
22(a)	Correctly eliminates the same letter using two different pairs of equations eg $3a + b = 600 - 302$ and $5a + b = 894 - 600$	M1		
	$-2n^2 + 304n$	A1	oe eg $n(304 - 2n)$ Allow any letter Allow $a = -2$ $b = 304$ $c = 0$ if $an^2 + bn + c$ seen earlier	
	Additional Guidance			
	Condone mixed letters and/or inclusion of = 0 eg1 $-2n^2 + 304x$ eg2 $-2n^2 + 304n$ = 0			M1 M1 A1 M1 M1 A1
	Alt 1 2nd differences = 4 300 592 876 1152			M0 M1 A0

	Alternative method 3		
	$a = -2$	M1	Using $an^2 + bn + c$
	$3a + b = 600 - 302$ and substitutes their a	M1	oe eg $b = 304$ May also see $a + b + c = 302$ used to obtain c
	$-2n^2 + 304n$	A1	oe eg $n(304 - 2n)$ Allow any letter
	Alternative method 4		
	Second differences -4	M1	
22(a)	$302 + (600 - 302)(n - 1) +$ $0.5 \times$ their $-4(n - 1)(n - 2)$	M1	Using $a + d(n - 1) + 0.5c(n - 1)(n - 2)$ a is 1st term d is 2nd term – 1st term c is second differences
	$-2n^2 + 304n$	A1	oe eg $n(304 - 2n)$ Allow any letter
	Additional Guidance		
	Condone mixed letters and/or inclusion of = 0 eg1 $-2n^2 + 304x$ eg2 $-2n^2 + 304n = 0$		M1 M1 A1 M1 M1 A1

22(b)	$n(-2n + 304)$ or $2n(-n + 152)$ or $2n = 304$	M1	oe Factorises correctly to two linear factors or substitutes correctly in quadratic formula or correctly completes the square to a correct equation or simplifies to $an = b$ ft their quadratic
	152	A1	
	Additional Guidance		
	152 and 0		M1 A0
	M1 Factorising may be seen after division eg if (a) correct $n(-n + 152)$		M1
	Their quadratic must have at least two terms for M1		
	Only ft for M1 A0		
	If their quadratic in (a) is incorrect, check for M1 A0 using their answer (correct to at least 1dp) if method not shown		
	Do not award M1 if their quadratic from (a) has solution $n = 0$		

23	4th box indicated unambiguously	B1	

	Alternative method 1		
24	$(a + 2)(a – 2)$ or 2 and –2 identified	M1	2 and –2 may be seen on a graph or within inequalities
	$8 – 2b < 2$ or $b > 3$	M1	oe Allow any inequality symbol Allow inequality symbol to be = M3 $–2 < 8 – 2b < 2$
	$–2 < 8 – 2b$ or $b < 5$	M1	
	$3 < b < 5$	A1	SC3 $2 < b < 6$ or $–4 < b < 12$
	Additional Guidance		
	Both inequalities $b < 5$ and $3 < b$ given as their answer		M3 A1
	$a < 2$ $8 – 2b = 2$ $b = 3$		M0 M1 M0 A0
	Must use 2 in 2nd M1		
	Must use –2 in 3rd M1		
	3 or 5 identified implies M1		
	3 and 5 identified		M1 M1 M1
	Working with = throughout can gain a maximum of M1 M1 M1 A0 unless recovered		
	Condone use of any letter other than a		

	Alternative method 2		
24	$(8 - 2b)^2 < 4$	M1	Allow any inequality symbol Allow inequality symbol to be = Must see 4
	$64 - 16b - 16b + 4b^2$ or $64 - 32b + 4b^2$ or $60 - 16b - 16b + 4b^2$ or $60 - 32b + 4b^2$ or	M1	oe Correct expansion or correct expansion – 4
	$(2b - 10)(2b - 6)$ or $(b - 5)(b - 3)$ or 3 and 5 identified	M1	Correct factorisation of $60 - 32b + 4b^2$ or correctly substitutes into quadratic formula or correctly completes the square to an equation
	$3 < b < 5$	A1	SC3 $2 < b < 6$ or $-4 < b < 12$

Additional Guidance		
Both inequalities $b < 5$ and $3 < b$ given as their answer		M3 A1
Must expand correctly for 2nd M1		
Must factorise correctly for 3rd M1		
3 and 5 identified		M1 M1 M1
Working with = throughout can gain a maximum of M1 M1 M1 A0 unless recovered		
Condone use of any letter other than a		

25	**Alternative method 1**		
	$\cos x = \sqrt{\dfrac{9}{25}}$ or $\cos x = \dfrac{3}{5}$ or 53.1 or 306.9	M1	oe
	53.1 and 306.9	A1	
	$\cos x = -\sqrt{\dfrac{9}{25}}$ or $\cos x = -\dfrac{3}{5}$	M1	oe 126.9 alone (or with 53.1 or 306.9) is 2nd M0 233.1 alone (or with 53.1 or 306.9) is 2nd M0
	126.9 and 233.1	A1	
	Additional Guidance		
	$\cos x$ must be the subject for M marks eg1 5 $\cos x = 3$ (no further valid work) eg2 $\cos x = \pm\sqrt{\dfrac{9}{25}}$ (no further valid work)		M0 A0 M0 A0 M1 A0 M1 A0
	'Correct' answers rounded or truncated to nearest integer or given to greater accuracy than 1 dp are penalised 1 accuracy mark eg1 53, 306, 127, 233 eg2 53, 307 eg3 53 eg4 53.13, 306.87, 126.87, 233.33		M1 A1 M1 A0 M1 A0 M0 A0 M1 A0 M0 A0 M1 A1 M1 A0
	Ignore any solutions outside of [0, 360] eg −53.1		
	All four answers with extra answers are penalised the final accuracy mark eg1 53.1 306.9 126.9 233.1 90 eg2 53.13 306.87 126.87 233.33 90 (loses 2 accuracy marks as accuracy error as well)		M1 A1 M1 A0 M1 A0 M1 A0
	53.2 or 306.8 (condone for M marks)		M1 A0 M0 A0
	53.2, 306.8, 126.8, 233.2 (condone for M marks)		M1 A0 M1 A0
	Answer line blank, award any marks gained from working lines		

	If angles are found in working lines but only some are listed on answer line	
	award any method marks gained from the working lines	
	award any accuracy marks gained from the answer line	
	eg1 Working lines $\cos x = \pm\sqrt{\dfrac{9}{25}}$ 53.1 306.9 126.9 233.1	M1 A1 M1 A0
	Answer line 53.1 306.9 233.1	
	eg2 Working lines $\cos x = \dfrac{3}{5}$ 53.1 306.9	M1 A0 M0 A0
	Answer line 53.1	
	eg3 Working lines $\cos x = \dfrac{3}{5}$ 53.1 306.9 $\cos x = -\dfrac{3}{5}$ 233.1	M1 A0 M1 A0
	Answer line 233.1	
	Answers only of 53.1 and 126.9	
	If it is clear which method they are using, mark using the scheme for that method	
	If no method is seen, award M1 A1 (alt 2)	

	Alternative method 2		
	$\sin x = \sqrt{\dfrac{16}{25}}$ or $\sin x = \dfrac{4}{5}$ or 53.1 or 126.9	M1	oe
	53.1 and 126.9	A1	
	$\sin x = -\sqrt{\dfrac{16}{25}}$ or $\sin x = -\dfrac{4}{5}$	M1	oe 233.1 alone (or with 53.1 or 126.9) is 2nd M0 306.9 alone (or with 53.1 or 126.9) is 2nd M0
	233.1 and 306.9	A1	

25	**Additional Guidance**	
	$\sin x$ must be the subject for M marks eg1 $5 \sin x = 4$ (no further valid work)	M0 A0 M0 A0
	eg2 $\sin x = \pm\sqrt{\dfrac{16}{25}}$ (no further valid work)	M1 A0 M1 A0
	'Correct' answers rounded or truncated to nearest integer or given to greater accuracy than 1 dp are penalised 1 accuracy mark eg1 53, 127, 233, 306	M1 A1 M1 A0
	eg2 53, 127	M1 A0 M0 A0
	eg3 53	M1 A0 M0 A0
	eg4 53.13, 126.87, 233.33, 306.87	M1 A1 M1 A0
	Ignore any solutions outside of [0, 360] eg –53.1	
	All four answers with extra answers are penalised the final accuracy mark eg1 53.1 126.9 233.1 306.9 90	M1 A1 M1 A0
	eg2 53.13 126.87 233.33 306.87 90 (loses 2 accuracy marks as accuracy error as well)	M1 A0 M1 A0
	53.2 or 126.8 (condone for M marks)	M1 A0 M0 A0
	53.2, 126.8, 233.2, 306.8 (condone for M marks)	M1 A0 M1 A0
	Answer line blank, award any marks gained from working lines	

	If angles are found in working lines but only some are listed on answer line	
	award any method marks gained from the working lines	
	award any accuracy marks gained from the answer line	
	eg1 Working lines $\sin x = \pm\sqrt{\dfrac{16}{25}}$ 53.1 126.9 233.1 306.9 Answer line 53.1 126.9 233.1	M1 A1 M1 A0
	eg2 Working lines $\sin x = \dfrac{4}{5}$ 53.1 126.9 Answer line 53.1	M1 A0 M0 A0
	eg3 Working lines $\sin x = \dfrac{4}{5}$ 53.1 126.9 $\sin x = -\dfrac{4}{5}$ 233.1 Answer line 233.1	M1 A0 M1 A0
	Answers only of 53.1 and 306.9 If it is clear which method they are using, mark using the scheme for that method If no method is seen, award M1 A1 (alt 1)	

26(a)	$2\pi r^2 = \pi r l$ leading to $2r = l$ or $\dfrac{4\pi r^2}{2} = \pi r l$ leading to $2r = l$	B1	oe Allow verification	
	Additional Guidance			
	$2\pi r^2 = \pi r l$ with appropriate cancelling shown			B1
	Any incorrect working			B0
	Verification example (Cone =) $\pi r l = \pi r \times 2r = 2\pi r^2$ Hemisphere is $2\pi r^2$ (Must link $2\pi r^2$ with the hemisphere)			B1

26(b)	$(2r)^2 = r^2 + h^2$	M1	oe
	$h = r\sqrt{3}$ or $h = \sqrt{3r^2}$	A1	
	$\dfrac{2}{3}\pi r^3\ (+)\dfrac{1}{3}\pi r^2 \times$ their $r\sqrt{3}$	M1	Must replace h with an expression in terms of r Allow $\dfrac{2}{3}\pi r^3$ to be $\dfrac{4}{3}\pi r^3$ or $\dfrac{8}{3}\pi r^3$
	$\dfrac{1}{3}\pi r^3(2+\sqrt{3})$ with correct method seen	A1	

Additional Guidance	
$2r^2 = r^2 + h^2$ is M0 unless recovered	
$2r^2 = r^2 + h^2$ $h = r$ $\dfrac{8}{3}\pi r^3 + \dfrac{1}{3}\pi r^3$ $3\pi r^3$	M0 A0 M1 A0
Ignore units	

27	8 seen as 2^3 or 16 seen as 2^4	M1	oe eg 2^{3a}
	2^{3a} and 2^4 seen	M1	oe eg 2^{3a+4}
	$a^2 - 3a - 4 \; (= 0)$	M1	oe equation eg $a^2 = 3a + 4$ ft if all three terms expressed as powers of 2 and a^2 term correct
	–1 and 4 with correct method seen	A1	

Additional Guidance		
Trial and improvement or answer(s) only		Zero
First 2 M marks can be awarded even if subsequent method is not clear		
2nd M1 may be implied eg $\quad 2^{a^2} = 2^{2a} \quad 2^3 = 8 \quad 2^4 = 16$ $\qquad 2a = 3a + 4$ (3a + 4 implies 2nd M1) (a^2 term not correct so 3$^{\text{rd}}$ mark is M0) $\qquad a = -4$		M1 M1 M0 A0
$16 = 2^4 \quad (2^3)^a = 2^{a^3}$ $a^2 = a^3 + 4$		M1 M0 M1 A0