AQA Qualifications

AQA CERTIFICATE

FURTHER MATHEMATICS

Paper 1 8360/1

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q	Answer	Mark	Comments
1	$10 = -2(-3) + c$ or $c = 4$	M1	$y - 10 = -2(x - (-3))$ or $y = -2x + c$
	$y = -2x + 4$	A1	
2	$\frac{dy}{dx} = 12x^2 - 7$	B2	B1 for each term
3	$\begin{pmatrix} 1 & a \\ 0 & 2 \end{pmatrix} \begin{pmatrix} b \\ 5 \end{pmatrix}$ or $\begin{pmatrix} b + 5a \\ 10 \end{pmatrix}$	M1	
	$b + 5a = 5$ **and** $b = 10$	M1	
	$a = -1$ and $b = 10$	A1	
4	$(20 + w <)\ 3w + 6$	M1	
	20 – their 6 $< 2w$	M1	oe
	$w > 7$ or $7 < w$	A1ft	ft from one error

5(a)	−6	B1	

5(b)	$f(x) \le 10$ or $10 \ge f(x)$	B1	Condone $y \le 10$ or $10 \ge y$

5(c)	$6a = 24$ (so $a = 4$)	B1	B1 for $2a \times 3 = 24$ B1 for $24 = (0 + 8)(0 + 3)$ $8 \times 3 = 24$ ...on its own ... is B0

5(d)	$10 - x^2 = (x + 8)(x + 3)$ or $10 - x^2 = x^2 + 2ax + 3x + 6a$	M1	oe
	$2x^2 + 11x + 14$ (= 0)	M1dep	oe allow one error
	$(2x + c)(x + d)$ (= 0)	M1dep	$cd = 14$ or $c + 2d = 11$ ft from their quadratic (factorising or correct substitution in quadratic formula)
	−3.5 and −2	A1	oe

6(a)	105 (numerator) or 145 (denominator)	M1	
	$\frac{21}{29}$	A1	

6(b)	**Alternative method 1**		
	$\dfrac{2 + \frac{7}{n^2}}{3 - \frac{2}{n^2}}$	M1	
	$\frac{7}{n^2}$ and $\frac{1}{n^2}$ both $\rightarrow 0$ as $n \rightarrow \infty$	A1	
	Alternative method 2		
	as $n \rightarrow \infty$ $2n^2 + 7 \rightarrow 2n^2$ and $3n^2 - 2 \rightarrow 3n^2$	B1	
	limiting value is $\dfrac{2n^2}{3n^2} = \dfrac{2}{3}$	B1	

7	Any **one** of these equations $2x + y + 20 = 180$ or $x + 2y + y + 40 = 180$ or $2x + y + 20 = x + 2y + y + 40$ or $2x + y + 20 + x + 2y + y + 40 = 360$	M1	oe
	Another of these equations $2x + y + 20 = 180$ or $x + 2y + y + 40 = 180$ or $2x + y + 20 = x + 2y + y + 40$ or $2x + y + 20 + x + 2y + y + 40 = 360$	M1	oe these simplify to ... $2x + y = 160$ or $x + 3y = 140$ or $x - 2y = 20$ or $3x + 4y = 300$
	equating coefficients and elimination of x or y for their equations eg $x + 3y = 140$ and $6x + 3y = 480$ or $2x + 6y = 280$ and $2x + y = 160$	M1dep	rearrangement and substitution for their equations eg $y = 160 - 2x$ and $x + 3(160 - 2x) = 140$ or $x = 140 - 3y$ and $2(140 - 3y) + y = 160$
	Allow one numerical error for the 3rd M1, but not an error in method (eg adding equations when they ought to be subtracted is an error in method, so M0)		
	$5x = 340$ or $5y = 120$	M1dep	ft their elimination or substitution
	$x = 68$ and $y = 24$	A1	

8(a)	$3(x + 2)(x - 2)$	B2	B1 for $3(x^2 - 4)$ or $(3x + 6)(x - 2)$ or $(x + 2)(3x - 6)$

8(b)	$(5x + ay)(x + by)$	M1	where $ab = \pm 12$ or $a + 5b = \pm 4$
	$(5x \pm 6y)(x \pm 2y)$	A1	for correct y terms in correct brackets, but with a sign error
	$(5x - 6y)(x + 2y)$	A1	

9	±25 or ±15 seen	B1	
	Using 80% or 20% in a correct calculation eg $0.8 \times (16 - -9)$ or 0.2×15 or $\frac{4}{5} \times 15$ or $\frac{1}{5} \times 25$ or $\frac{80}{100} \times 25$	M1	oe eg answers of 5 or 3 or 20 or 12 seen is evidence of a correct calculation
	(11, 6)	A2	A1 for each

10	**Alternative method 1**		
	$\frac{8}{(3 - \sqrt{5})} \times \frac{(3 + \sqrt{5})}{(3 + \sqrt{5})}$	M1	
	Numerator = $24 + 8\sqrt{5}$ or Denominator = 9 – 5 or 4	A1	
	$6 + 2\sqrt{5}$	A1	
	Alternative method 2		
	$\frac{8}{(3 - \sqrt{5})} = a + b\sqrt{5}$ or $8 = (a + b\sqrt{5})(3 - \sqrt{5})$ or $8 = 3a - 5b + 3\sqrt{5}b - a\sqrt{5}$	M1	
	$3a - 5b = 8$ **and** $(3b - a)\sqrt{5} = 0$	A1	oe
	$a = 6$ **and** $b = 2$	A1	

11(a)	(cos B =) $\frac{(3\sqrt{2})^2 + (\sqrt{2})^2 - (\sqrt{14})^2}{2 \times 3\sqrt{2} \times \sqrt{2}}$	M1	$(\sqrt{14})^2 = (3\sqrt{2})^2 + (\sqrt{2})^2 - 2 \times 3\sqrt{2} \times \sqrt{2} \times \cos B$
	$\frac{18 + 2 - 14}{2 \times 3 \times 2}$ allow one error oe	M1dep	$14 = 18 + 2 - 12 \times \cos B$ allow one error oe
	$\cos B = \frac{6}{12} = \frac{1}{2}$ and $B = 60°$ or (B =) $\cos^{-1}(^1/_2) = 60°$	A1	

11(b)	$\sin 60 = \frac{\sqrt{3}}{2}$ seen	B1	
	$\frac{1}{2} \times 3\sqrt{2} \times \sqrt{2} \times \sin 60$	M1	oe
	$\frac{3\sqrt{3}}{2}$	A1	oe

12(a)	Any valid explanation eg 8 + 9 (=17) 8 − −9 (=17) −9 + 17 = 8 8 − 17 = −9	B1	

12(b)	**Alternative method 1**		
	Right-angle triangle formed with 8 and 17	M1	Correct use of Pythagoras' theorem eg $x^2 + 8^2 = 17^2$
	$AM = \sqrt{(17^2 - 8^2)}$ or $AM = 15$ where M is the midpoint of AB	M1	oe
	$AB = 2 \times AM = 2 \times 15 = 30$	A1	
	Alternative method 2		
	$(0 - 8)^2 + (y - 20)^2 = 17^2$	M1	ie. equation of circle applied to points A or B
	$(y - 20)^2 = 17^2 - 8^2$ or $(y - 20)^2 = 15^2$ or $y - 20 = 15$ or $y - 20 = -15$	M1	
	$y = 35$ and $y = 5$ and 35 − 5 seen $AB = 30$	A1	

13(a)	**Alternative method 1**		
	$\frac{dy}{dx} = 3x^2 - 6x$	M1	allow one error
	When $x = 2$, $3x^2 - 6x = 12 - 12 = 0$	A1	oe eg solving $3x^2 - 6x = 0$ to get $x = 2$
	Work out $\frac{dy}{dx}$ either side of $x = 2$ Using $0 < x < 2$ and $x > 2$	M1	ft their $3x^2 - 6x$ $x = 0$ is not a valid value to use for testing since the curve has a maximum point at $x = 0$
	One negative value and one positive value so $x = 2$ corresponds to minimum point	A1	
	Alternative method 2		
	$\frac{dy}{dx} = 3x^2 - 6x$	M1	allow one error
	When $x = 2$, $3x^2 - 6x = 12 - 12 = 0$	A1	oe eg solving $3x^2 - 6x = 0$ to get $x = 2$
	$d^2y/dx^2 = 6x - 6$	M1	ft their $3x^2 - 6x$
	$d^2y/dx^2 = 6$ (or positive) when $x = 2$ so minimum point	A1	

13(b)	Substitute $x = 2$ in equation of curve to find corresponding *y* value (y =) $(2)^3 - 3(2)^2 + 5$	M1	attempt to find equation of tangent at $x = 2$ oe
	(y =) 1	A1	
	Substitute $x = -1$ in equation of curve to verify that $y = 1$ (y =) $(-1)^3 - 3(-1)^2 + 5 = 1$	B1	oe or Solve $x^3 - 3x^2 + 5 = 1$ or $x^3 - 3x^2 + 4 = 0$ $(x + 1)(x - 2)(x - 2) = 0$ $x = -1$ is a solution $x = -1$ could be obtained by substitution in $x^3 - 3x^2 + 5 = 1$ or $x^3 - 3x^2 + 4 = 0$

14(a)	$a^3 + (2a \times a^2) - (a^2 \times a) - 16 = 0$ or $2a^3 - 16 = 0$ or $a^3 - 8 = 0$	M1	
	$a^3 = 8$ or $a = \sqrt[3]{8}$ (hence $a = 2$)	A1	clearly shown

14(b)	**Alternative method 1**		
	$(x - 2)(x^2 + \ldots.. + 8)$ (= 0)	M1	
	$(x - 2)(x^2 + 6x + 8)$ (= 0)	A1	
	$(x + m)(x + n)$ (= 0)	M1	where $mn = 8$ and $m + n = 6$
	2, −2, −4	A1	
	Alternative method 2		
	$(x^3 + 4x^2 - 4x - 16) \div (x - 2)$ $= x^2 + ax + \ldots$	M1	Attempt at long division of polynomials a need not be correct to score M1
	$x^2 + 6x + 8$	A1	
	$(x + m)(x + n)$ (= 0)	M1	where $mn = 8$ and $m + n = 6$
	2, −2, −4	A1	
	Alternative method 3		
	$(x + 4)(x^2 + \ldots)$ (= 0)	M1	
	$(x + 4)(x^2 - 4)$ (= 0)	A1	
	$(x + 4)(x + 2)(x - 2)$ (= 0)	M1	or $(x + 4) = 0$ or $(x^2 - 4) = 0$
	2, −2, −4	A1	
	Alternative method 4		
	$x = 2$	B1	
	testing a value of x ($x \neq 2$) to see if $f(x) = 0$	M1	
	one of −2 or −4	A1	
	2, −2, −4	A1	

15	**Alternative method 1**		
	$\dfrac{\sin\theta - \sin^3\theta}{\cos^3\theta} \equiv \dfrac{\sin\theta\,(1 - \sin^2\theta)}{\cos^3\theta}$	M1	
	$\dfrac{\sin\theta \times \cos^2\theta}{\cos^3\theta}$	M1	oe eg $\sin\theta\,(\sin^2\theta + \cos^2\theta - \sin^2\theta)$
	$\dfrac{\sin\theta\cos^2\theta}{\cos^3\theta} \equiv \dfrac{\sin\theta}{\cos\theta} \equiv \tan\theta$	A1	
	Alternative method 2		
	$\dfrac{\sin\theta - \sin^3\theta}{\cos^3\theta} \equiv \dfrac{\sin\theta\,(1 - \sin^2\theta)}{\cos^3\theta}$	M1	
	$\dfrac{\sin\theta\,(1 - \sin^2\theta)}{\cos\theta\,(1 - \sin^2\theta)}$	M1	
	$\dfrac{\sin\theta\,(1 - \sin^2\theta)}{\cos\theta\,(1 - \sin^2\theta)} \equiv \dfrac{\sin\theta}{\cos\theta} \equiv \tan\theta$	A1	

16	**Alternative method 1**		
	$2x^2 - 4ax + 2a^2$ (+ 3)	M1	or $2(x^2 - 2ax + a^2)$ (+ 3) allow one error
	$2a^2 + 3 = 7a$ or $2a^2 - 7a + 3 = 0$	M1	oe for equating constant terms
	$(2a - 1)(a - 3)$ (= 0)	A1	
	$a = {}^1/_2$ and $a = 3$	A1	
	$-2bx = -4ax$ or $2b = 4a$ or $b = 2a$	M1	oe for equating x terms
	$b = 1$ when $a = {}^1/_2$ and $b = 6$ when $a = 3$	A1ft	ft their a values
	Alternative method 2		
	when $x = 0$ $7a = 2a^2 + 3$ or $2a^2 - 7a + 3 = 0$	M1	oe
	$(2a - 1)(a - 3)$ (= 0)	A1	
	$a = {}^1/_2$ and $a = 3$	A1	
	when $x = 1$ $2 - 2b + 7a = 2(1 - a)^2 + 3$ or $2b = 7a - 1 - 2(1 - a)^2$	M1	oe
	substituting $a = {}^1/_2$ and $a = 3$ in the expression for $2b$ (or b)	M1	
	$b = 1$ when $a = {}^1/_2$ and $b = 6$ when $a = 3$	A1ft	ft their a values
	Alternative method 3		
	when $x = 0$ $7a = 2a^2 + 3$ or $2a^2 - 7a + 3 = 0$	M1	oe
	$(2a - 1)(a - 3)$ (= 0)	A1	
	$a = {}^1/_2$ and $a = 3$	A1	
	Correctly substitute a second value of x into the identity	M1	eg if $x = 2$, $8 - 4b + 7a = 2(2 - a)^2 + 3$

	Correctly substitute a third value of x into the identity	M1	eg if $x = 3$, $18 - 6b + 7a = 2(3 - a)^2 + 3$
	$b = 1$ when $a = {}^1/_2$ and $b = 6$ when $a = 3$	A1ft	ft their a values
	Alternative method 4		
	$2[(x - b/2)^2 - b^2/4 + 7a/2]$ or $2(x - b/2)^2 - b^2/2 + 7a$	M1	
	$a = b/2$ or $3 = - b^2/2 + 7a$	M1	
	$2a^2 - 7a + 3 = 0$ or $b^2 - 7b + 6 = 0$	M1	oe
	$(2a - 1)(a - 3) (= 0)$ or $(b - 1)(b - 6) (= 0)$	A1	
	$a = {}^1/_2$ and $a = 3$ or $b = 1$ and $b = 6$	A1	
	$b = 1$ when $a = {}^1/_2$ and $b = 6$ when $a = 3$	A1ft	ft from the values they calculate first